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APPLICATION OF A NON ISOTROPIC TURBULENCE MODEL TO STABLE ATMOSPHERIC FLOWS OVER 3D TOPOGRAPHY

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SUMMARY

A non-isotropic turbulence model is extended and applied to three dimensional stably stratified flows. The model is derived from the algebraic stress model (including wall proximity effects), but it retains the simplicity of the "eddy viscosity" concept of first order models. The "modified k-ε model" is implemented in a three dimensional numerical code. To evaluate the model, various steady state numerical solutions are compared with wind tunnel experiments which were conducted at the wind tunnel of Mitsubishi Heavy Industries, in Japan. Stably stratified flows over three distinct idealized complex topographies are studied. Vertical profiles of velocity and turbulent kinetic energy are shown and discussed. Also, comparisons are made against the results obtained with the standard k-ε model.

INTRODUCTION

Atmospheric boundary layer flows are object of intense study over the last years. A more comprehensive understanding of the complex phenomena involved in this particular type of flow is being sought, aiming the analysis of structural implications due to strong winds (neutral atmosphere), the pollutant dispersion under neutral or stable conditions and also for meteorological purposes. The phenomenal increase in computer power over the last two decades has led to the possibility of computing such flows by the integration of the (modeled, time-averaged) Navier-Stokes equations.

Raithby *et al* (1987) employed the k-ε model (with modification in the C_μ value) to calculate the neutrally buoyant flow over the Askervein hill, and compared their numerical results with the experiment made over the real terrain in Scotland. Dawson *et al* (1991) also used the k-ε model (with some modification in the constants of the dissipation equation) to simulate the flow and dispersion over Steptoe Butte (Washington, USA) under neutrally and stably stratified atmosphere. Their results were favorably compared with experimental data, indicating that mathematical models using the eddy viscosity assumption in the turbulence closure could be used to predict the flow and pollutant dispersion over complex terrain. Koo (1993) developed a non-isotropic modified k-ε model to account for different eddy diffusivities in the lateral and vertical directions in the atmosphere. His model is derived from the algebraic stress model and was applied in one-dimensional problems to predict the vertical profiles of velocity, potential temperature and turbulence variables for horizontal flow in a homogeneous boundary layer. Also, the model was applied in two-dimensional problems to simulate the sea breeze circulation and the manipulation of the atmospheric boundary layer by a thermal fence. Koo's model is similar to the level 2.5 model of Mellor and Yamada (1982). Recently, Castro and Apsley (1997) compared numerical (using a "dissipation modification" k-ε model, as named by the authors) and laboratory data for two-dimensional flow and dispersion over topography. Also, Boçon and Maliska (1997a, 1997b) extended the non-isotropic k-ε model of Koo (1993) to numerically simulate the flow and pollutant dispersion over complex idealized topography, under neutral stratification. Computational results were compared with experimental data obtained from a wind tunnel simulation.

In the present work we extend the application of Koo's modified k-ε model to predict three-dimensional stably stratified flows over complex terrain. Our final objective is to calculate the dispersion of pollutants in the atmosphere. The task of computing the concentration field downstream from a pollutant source is obtained from the solution of the concentration equation. To do so, it's necessary firstly to calculate the velocity field and eddy viscosities in the region of interest.

FLOW MODELING

The governing equations for the flow are the conservation of mass, momentum and energy, written below in the usual tensor notation.

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\left(-\overline{u'_i u'_j}\right) \tag{2}$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(-\overline{u'_j T'}\right) \tag{3}$$

where p is the pressure deviation with respect to the hydrostatic pressure. Primed variables denote turbulent fluctuations. As we are simulating wind tunnel flows, the Coriolis effect is neglected. Modeling of fluctuation terms are described in the next section.

TURBULENCE MODELING

In environmental flows the non isotropic character of turbulence is notable, specially in the case of dispersion of a scalar (pollutant) in the flow. For the case of stably stratified flows, for instance, vertical fluctuations are much inhibited due to buoyancy forces (arising from the positive vertical temperature gradient), while horizontal fluctuations are not. Even neutrally stratified flows feature some anisotropy. So, it's not expected that isotropic turbulence models may well reproduce the non isotropic turbulent diffusion. However, standard k-ε model is successfully applied for environmental flows calculation where horizontal gradients (of velocity, temperature and turbulence variables) are smaller than the vertical gradients. In these situations, turbulent diffusion is significant only in the vertical direction, and an isotropic model can handle it appropriately. On the contrary, in the problem of pollutant dispersion from a point source, both vertical and horizontal concentration gradients are significant, so are the corresponding turbulent diffusion. For this situations, a better description of the anisotropy in turbulent exchanges is necessary.

In his Ph.D. thesis, Koo (1993) proposed a modification on the classic k-ε model, through use of algebraic stress model including wall proximity effects. The resulting model was compared to data and higher order simulations for one and two-dimensional atmospheric flows. The modified k-ε model reproduced well the observed behaviors.

In our work we extend the application of the Koo's modified k-ε model to three dimensional flow and to dispersion problems. A description of the turbulence model is given below. Detailed description of derivation of the model can be seen in Koo (1993). Following the Boussinesq's eddy viscosity concept, Reynolds stresses are related to the gradient of the velocity components as

$$-\overline{u_i' u_j'} = K_m^j\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3} k \delta_{ij} \tag{4}$$

where K_m^j is the turbulent eddy viscosity in the j direction. Analogously, turbulent heat exchange is expressed by

$$-\overline{u_j' T'} = K_h^j \frac{\partial T}{\partial x_j} \tag{5}$$

where K_h^j is the eddy diffusivity in the j direction. Eddy viscosities (for momentum) and eddy diffusivities (for energy) are expressed as functions of turbulent kinetic energy and its dissipation rate. For the vertical direction:

$$K_m^z = C_m \frac{k^2}{\varepsilon} \tag{6}$$

$$K_h^z = C_h \frac{k^2}{\varepsilon} \tag{7}$$

And for the horizontal directions:

$$K_m^x = K_m^y = C_\mu \frac{k^2}{\varepsilon} \tag{8}$$

$$K_h^x = K_h^y = \frac{K_m^x}{Pr_t} \tag{9}$$

C_m and C_h are, respectively, the proportionality coefficients for eddy viscosity and eddy diffusivities in the vertical direction. They are defined by functions of flow structure (from the algebraic stress model). Pr_t is the turbulent Prandlt number (=0.5).

$$C_m = \frac{2}{3}\frac{(c_1-1)(E_7 - AG_H)}{E_4 + \frac{E_4 E_8}{C_{1T}} G_H - E_5 E_7 G_M + E_5 A G_H G_M} \tag{10}$$

$$C_h = \frac{2}{3}\frac{(c_1-1) + E_5 G_M C_m}{(c_{1T} + c_{1T}' f)E_4 + \left(\frac{2E_4 E_9}{E_{10}} + E_6\right) G_H} \tag{11}$$

$$f = \frac{l}{k_v z} = \frac{C_\varepsilon k^{3/2}}{k_v z \varepsilon} \tag{12}$$

f is the wall function which reflects the effect of the ground proximity on the Reynolds stresses and turbulent heat flux, l is the turbulence length scale, k_v is the von Karman constant (=0.4), z is the distance from the ground and $C_\varepsilon = 0.13$. Other constants in equations (10) and (11) can be found in Koo (1993).

The C_m is function of G_M, the production of turbulent kinetic energy by mean velocity shear

$$G_M = \left(\frac{k}{\varepsilon}\right)^2\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \tag{13}$$

The C_h is function of G_H, the production (or destruction) of turbulent kinetic energy by buoyancy effects

$$G_H = g\beta\left(\frac{k}{\varepsilon}\right)^2 \frac{\partial \theta}{\partial z} \tag{14}$$

Turbulent kinetic energy and its dissipation rate are computed from their well known prognostic equations:

$$\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{K_m^j}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + P + G - \varepsilon \tag{15}$$

$$\frac{\partial \varepsilon}{\partial t} + u_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{K_m^j}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + C_{\varepsilon 1}(P+G)\frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{16}$$

P is the production term due to mean velocity gradients

$$P = -\overline{u_i' u_j'}\frac{\partial u_i}{\partial x_j} = K_m^j\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \tag{17}$$

G is the production (or destruction) term due to buoyancy

$$G = g\beta\overline{w'T'} = -g\beta K_h^z \frac{\partial T}{\partial z} \tag{18}$$

Constants in equations (15) and (16) are those from the standard k-ε model ($C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$).

NUMERICAL METHOD

The finite volume method is employed to solve the governing equations, in a non-orthogonal, generalized curvilinear coordinate system. Co-located arrangement is used for variables storage in the grid, and the QUICK interpolation scheme with source deferred correction term Lien (1994) is applied on the convection terms, except for turbulence variables where a hybrid scheme (WUDS of Raithby and Torrance, 1967) is adopted. Our own code NAVIER (1991) is used to solve the governing equations.

In order to verify grid dependent errors, the computations are made in a coarse and in a fine grid. Figure 1 illustrates one of the coarse grids used (inflow boundary at left). Coarse and fine grids are, respectively, 42x18x18 and 95x41x41. Only half domain is resolved, because of symmetry. Results for coarse and fine grids are nearly identical, as one can notice in figures 2, 3, 5 and 6 where vertical profiles of velocity and turbulent kinetic energy are shown.

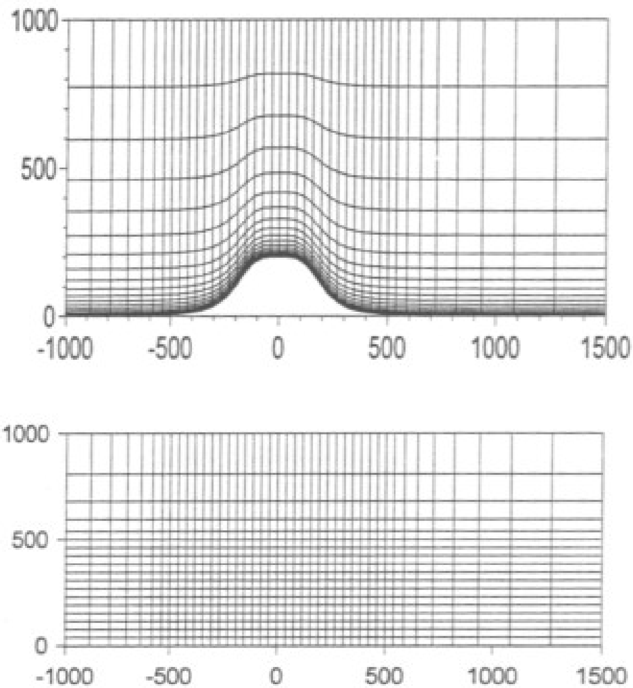

Figure 1 - Vertical (at the xz symmetry plane, above) and horizontal (below) views of the coarse grid for hill height 200mm (42x18x18 volumes)

To verify the model performance, in a first step, the above described modified k-ε model is applied to simulate wind tunnel experiments. A second series of tests, this time for a full scale experiment, will further be performed.

THE WIND TUNNEL EXPERIMENT

Pollutant dispersion wind tunnel experiments were conducted at the Mitsubishi Heavy Industries, in Nagasaki, Japan, 1991. A report containing the results was obtained directly from that company. Wind tunnel test section is 2.5m wide, 1m high and 10m long. Axisymetric hills of different heights (0, 100 and 200mm), were positioned with the top located at (x,y)=(0,0). Hill shape can be seen in fig. 1. Streamwise direction is x, lateral is y and vertical is z. Source of tracer gas was positioned at (x,y,z)=(-500 mm, 0, 50mm) for hill heights 0 and 100mm, and at (x,y,z)=(-500mm, 0, 100mm) for hill height 200mm. Cases of neutral ($\Delta T = 0$, Pasquill class D) and stable atmosphere ($\Delta T = 20°C$, Pasquill class E) were performed. Streamwise velocity, velocity fluctuations, temperature and concentration were measured at various locations.

NUMERICAL EXPERIMENTS AND BOUNDARY CONDITIONS

Three different wind tunnel experiments were computationally simulated. They are designated with a letter - indicating stability class - followed by a number indicating hill height in mm. Hill heights of 0, 100 and 200mm were simulated. Neutral flows (Pasquill class D) over these topographies were already numerically studied by Boçon and Maliska (1997a, 1997b). At this time, stably stratified flows (Pasquill class E) are considered. At the inflow boundary, velocity, temperature and turbulent kinetic energy are specified according to experimental measured values. As the dissipation rate of turbulent kinetic energy was not measured during the experiment, its inflow profile is calculated according to a prescribed turbulence length scale. For neutral boundary layer flows, this length scale increases linearly with the distance from the wall (height above the surface, in the present problems). However, in the case of stably stratified flows the turbulence length scale does not increase linearly with the height, but it is limited to a maximum value (Castro and Apsley, 1997).

$$l = \frac{k_v z}{1 + \frac{k_v z}{0.085L}} \tag{19}$$

where z is the distance from the ground, k_v is the von Karman constant (= 0.4) and L is the Monin-Obukhov length (= 0.13m), which was calculated from the experimental values of velocity and temperature near the ground.

Outflow conditions are that of zero-gradient for all variables. For velocity, lateral and upper boundaries are impermeable, with zero tangential stresses. For all other variables, lateral and upper boundary conditions are of zero-gradient. Symmetry conditions are applied at the boundary coincident with the plane of symmetry (y = 0).

Wall functions are invoked to apply boundary conditions appropriate to a rough wall (z_0 = 1.5e-4 m) at the ground. Namely, diffusive fluxes of momentum and heat are related to the velocity and temperature at the volumes adjacent to the ground by (Launder and Spalding, 1974)

$$\frac{V_p}{u_*} = \frac{1}{k_v}\ln\left(\frac{E h_p u_*}{\nu}\right) \tag{20}$$

$$\frac{T_p - T_0}{T_*} = \frac{Pr_t}{k_v}\ln\left(\frac{F h_p u_*}{\nu}\right) \tag{21}$$

where the subscript p refers to the volumes adjacent to the ground, E = 26 and F = 38.6 (airflow over a smooth wall and Pr_t= 0.5). Turbulent kinetic energy and its dissipation rate are calculated at the volumes adjacent to the ground by assuming local equilibrium between production and dissipation.

RESULTS AND DISCUSSION

In order to better evaluate the modified (anisotropic) k-ε model, computations were also made using the standard model and are presented along with the experimental results.

Figures 2 and 3 show vertical profiles of the streamwise component (u) of velocity on the symmetry plane (y = 0) for the cases E100 and E200 (hill heights 100 and 200 mm respectively). For both cases, the modified and the standard

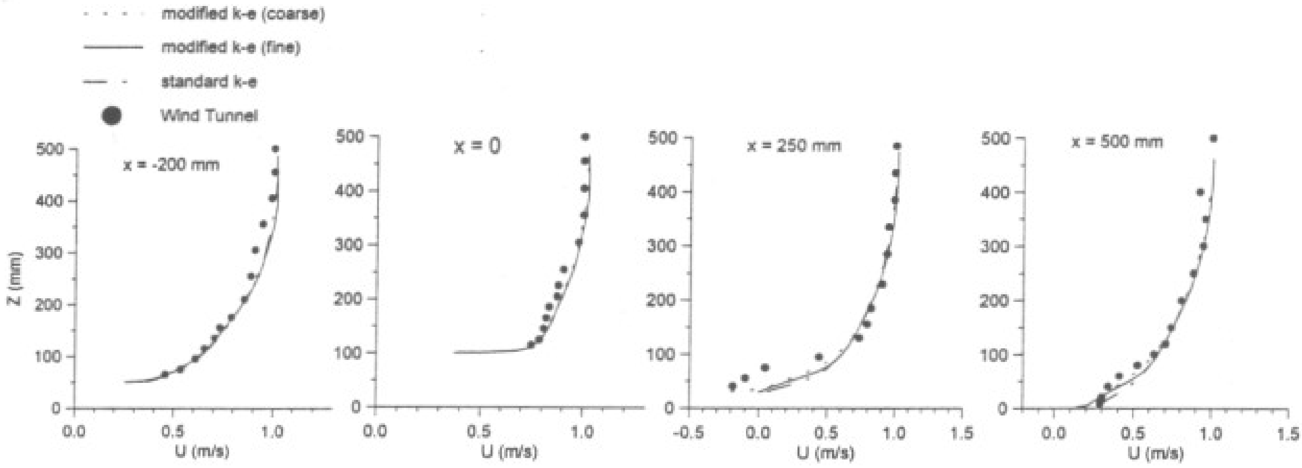

Figure 2 - Case E100 - vertical profiles of streamwise component of velocity (u) at the symmetry plane (y = 0) for different positions upstream and downstream the hill top (x = 0)

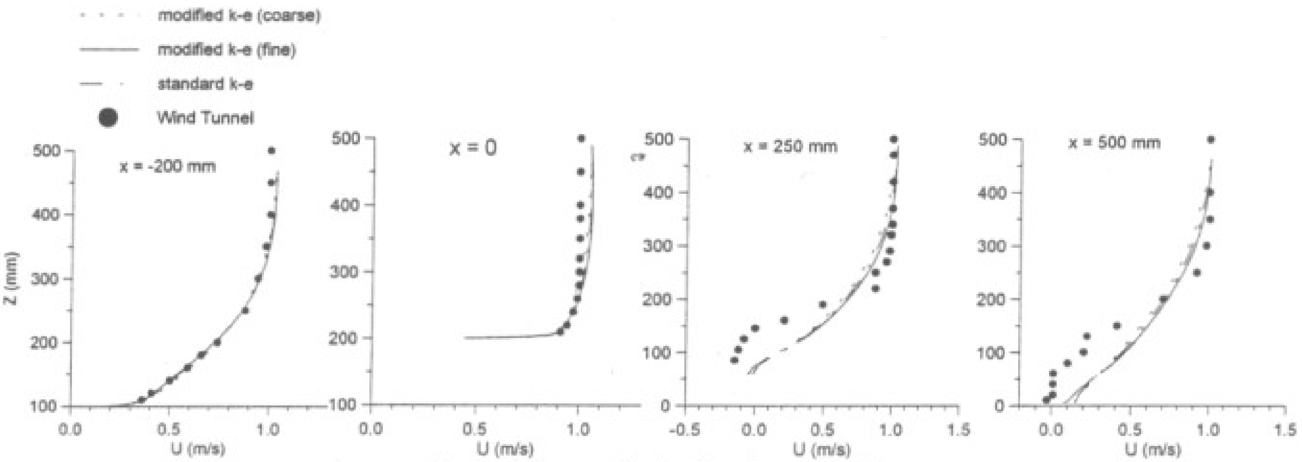

Figure 3 - Same as for fig. 3 but for case E200

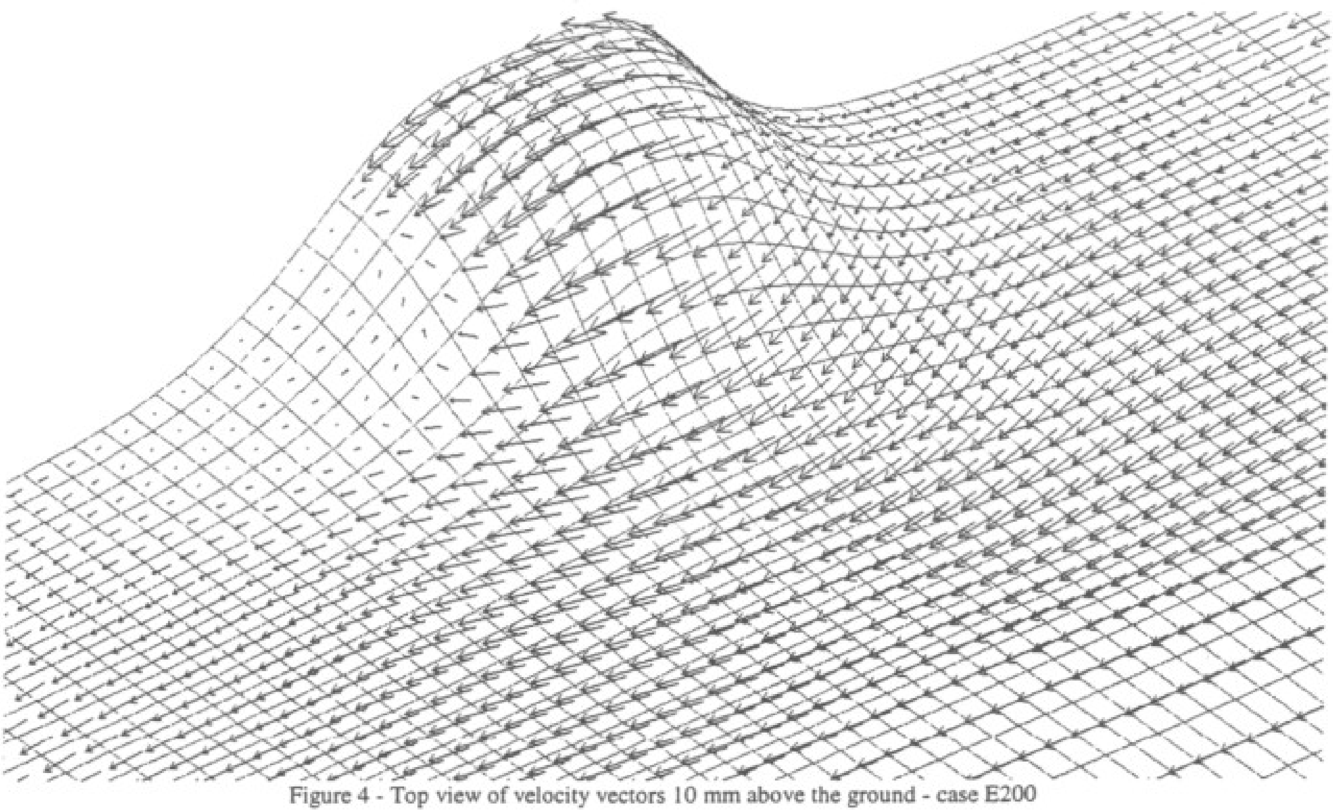

Figure 4 - Top view of velocity vectors 10 mm above the ground - case E200

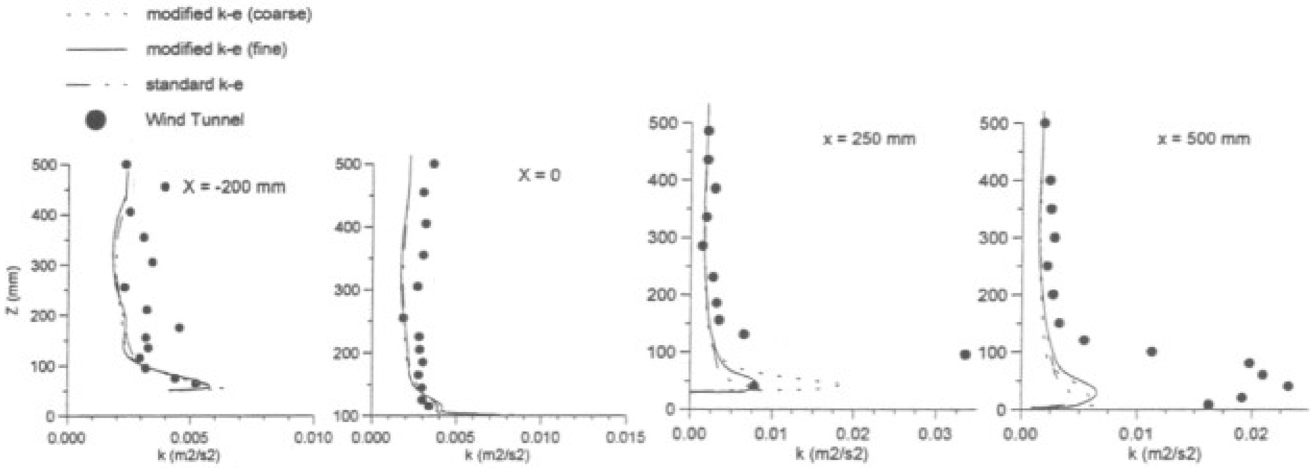

Figure 5 - Case E100 - vertical profiles of turbulent kinetic energy (k) at the symmetry plane (y = 0) for different positions upstream and downstream the hill top (x = 0)

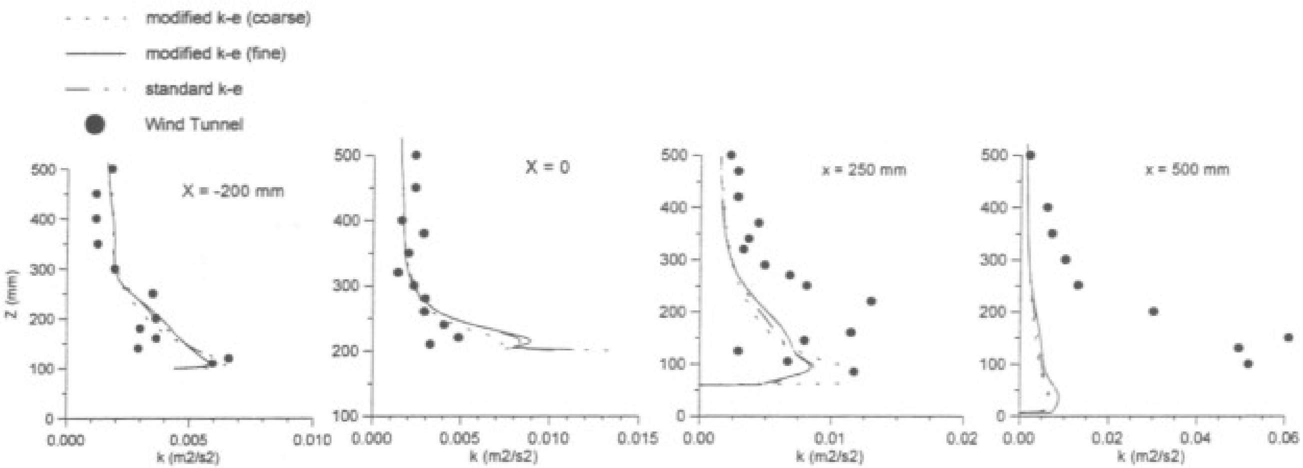

Figure 6 - Same as for fig. 5 but for case E200

models produced nearly the same velocity profiles. In case E200 the three dimensional open recirculation zone in the lee side of the hill was underestimated (by roughly 50%). Different inflow turbulent length scales were tested at inflow to verify a possible influence, but it was noticed that the flow after the hill top is essentially determined by local conditions. A possible explanation for this model defect would be that the pronounced velocity gradients in this region, due to the 3D open recirculation zone (see figure 4), increase the production of turbulent kinetic energy and consequently enhance the eddy viscosities there, thus diminishing the size of the recirculation. Figures 5 and 6 show vertical profiles of turbulent kinetic energy (k). For the case E200 (fig. 6) it seems that, at the position x=300 mm (which is in the recirculation zone), the level of turbulent kinetic energy is not overestimated by the mathematical model (although there arefew experimental values available). Thus, regarding to the above cited problem of high eddy viscosities in that region, the drawback should be attributed to the dissipation equation, which is underestimating ε, and not to an overestimation of the production of turbulent kinetic energy (P).

Figure 7 shows vertical profiles of the eddy diffusivities (for momentum) in the horizontal and vertical directions computed with the modified anisotropic k-ε model. Also it is shown the eddy diffusivity (isotropic) calculated with the standard model. Despite the different vertical turbulent diffusivity, the results for the modified model were practically identical to those for the standard model. Velocity and turbulent kinetic energy were almost not affected by the anisotropy in the eddy viscosities. This is explained by the fact that the vertical gradients for velocity and turbulent kinetic energy are not great, and turbulent diffusion is nearly the same for both isotropic and anisotropic models.

Although the anisotropy did not cause significant alterations in the flow results, this is not expected to be the case in the concentration calculation. In fact, dispersion is very sensitive to the lateral and vertical turbulent diffusion, which are the main mechanisms of the plume spread. For this reason, even in neutral atmospheres, the turbulence anisotropy is responsible for the different lateral and vertical spread rates. In a further work, the non isotropic modified model will be applied for the calculation of pollutant dispersion.

Figure 8 shows vertical profiles of eddy diffusivities for turbulent mass transfer. In the modified k-ε model, the mass diffusivities for the vertical and horizontal directions are given respectively by

$$K_c^z = C_c \frac{k^2}{\varepsilon} \tag{22}$$

$$K_c^x = K_c^y = \frac{K_m^x}{Sc_t} \tag{23}$$

Sc_t is the turbulent Schmidt number (= 0.7) and C_c is the proportionality coefficient of vertical eddy diffusivity for scalars, a function of the mean velocity shear (G_M), buoyancy effects (G_H)

and wall proximity (f), above defined. Panofsky (1984) recomends the application of the same eddy diffusivities for all scalars, in the absence of better information. Thus we take

$$C_c = C_h \qquad (24)$$

where C_h is the proportionality coefficient for turbulent heat diffusion in the vertical direction, given by (11).

For the standard model, the eddy diffusivity (isotropic) for the concentration is given by the usual relation

$$K_c = \frac{K_m}{Sc_t} \qquad (25)$$

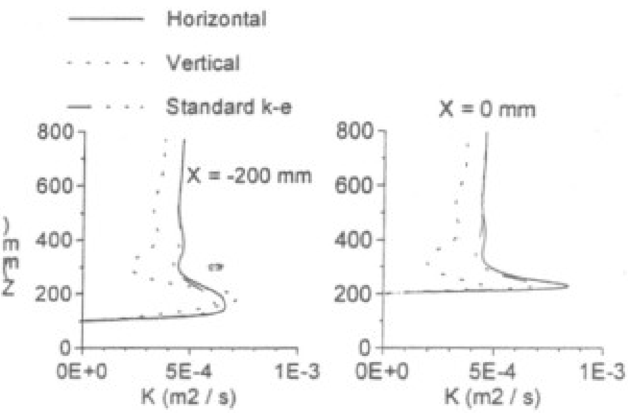

Figure 7 - Case E200 - Profiles of eddy diffusivity for momentum

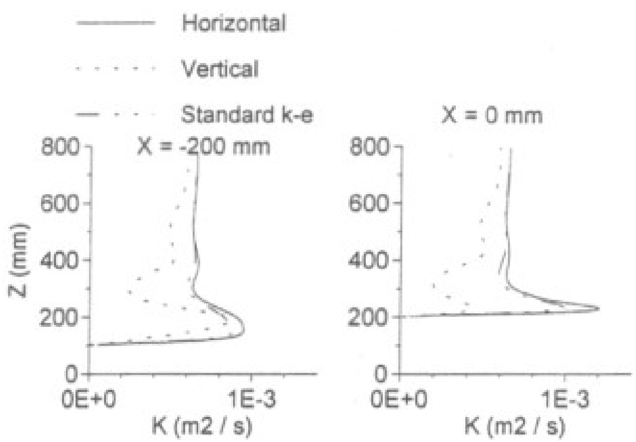

Figure 8 - Case E200 - Profiles of eddy diffusivity for concentration

CONCLUSIONS

A modified non-isotropic k-ε model is applied to simulate three dimensional stably stratified flows (Pasquill class E) over an idealized complex terrain. The results for velocity and turbulent kinetic energy are similar to those obtained with the standard model, because the vertical gradients of these variables are not great, resulting nearly the same turbulent diffusion for both the standard (isotropic) and the modified (non isotropic) k-ε models. The agreement against the wind tunnel results is in general good for the velocity profiles and reasonable for the turbulent kinetic energy. In the recirculation zones, the eddy viscosities are overestimated and thus the size of the recirculation is underpredicted.

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