A NUMERICAL FORMULATION USING UNSTRUCTURED GRIDS FOR MODELING TWO-PHASE FLOWS IN POROUS MEDIA CONSIDERING HETEROGENEITIES AND CAPILLARY EFFECTS

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ABSTRACT
This paper briefly describes a two-dimensional numerical formulation using unstructured grids, developed for simulating two-phase immiscible displacements in porous media. The Element-based Finite Volume Method (EbFVM) is used for discretizing the model differential equations.

NOMENCLATURE
K Absolute permeability
k_r Relative permeability
s Saturation
ϕ Porosity
Subscripts
C Capillary
D Displaced phase
e Related to an element
F Generic fluid phase
i Related to an integration point
I Injected phase
p Related to a node
Superscripts
n Time level
P Related to the pressure discretized equation
s Related to the saturation discretized equation

INTRODUCTION
This paper describes a two-dimensional numerical formulation using unstructured grids, developed for simulating two-phase immiscible displacements in porous media. Although this formulation is as general as possible, our primary interest is to simulate displacements in core samples for solving the inverse problem of determining relative permeability curves by means of a parameter estimation method. The discretization of differential equations is carried out employing the Element-based Finite Volume Method (EbFVM). This approach follows the basic guidelines of the conventional finite volume method, namely the integration of differential equations over control volumes in a way that conservation is automatically enforced. However, a significant improvement in flexibility is introduced through the concept of elements as basic geometrical entities for the solution domain discretization. Porous media intrinsic properties are associated also to elements, so accurate spatial variations obtained by means of imaging methods can be easily included into the numerical model.

MATHEMATICAL MODEL
The mathematical model considered is based on the standard macroscopic description of the flow of two immiscible fluid phases through a porous medium [2]. The main equations of the model are the phase mass conservation differential equations and the multiphase extension of Darcy's law:

$$\phi \, \partial_t (\rho_F s_F) + \vec{\nabla} \cdot (\rho_F \vec{v}_F) = 0 \, ; \qquad F = I, D \qquad (1)$$

$$\vec{v}_F = -\frac{K k_{rF}}{\mu_F}(\vec{\nabla} P_F - \rho_F \vec{g}) \, ; \qquad F = I, D \qquad (2)$$

The two fluid phases are designated as injected phase (I) and displaced phase (D), respectively. The coupling between the preceding equations results from the volumetric restriction equation and the definition of capillary pressure:

$$s_I + s_D = 1 \qquad (3)$$

$$P_C = P_D - P_I \qquad (4)$$

As boundary conditions for modeling core displacements, at inlet face usually are prescribed the volumetric flow-rate of both fluid faces. Alternatively, can be prescribed the injection pressure and be assumed the non-existence of countercurrent flow of the displaced phase. At the outlet face, the so-called

capillary end effect is modeled by means of the following boundary condition:

$$\begin{cases} (\vec{\mathbf{v}}_I \cdot \hat{\mathbf{n}})_{out} = 0 & if \ (P_C)_{out} > 0 \\ (\vec{\mathbf{v}}_D \cdot \hat{\mathbf{n}})_{out} = 0 & if \ (P_C)_{out} < 0 \\ (s_I)_{out} = s_I^{P_C = 0} & since \ the \ time \ when \ (P_C)_{out} = 0 \end{cases} \quad (5)$$

This condition comes from the fact that simultaneous flow of both phases through the outlet face can occur only if the phase pressures are continuous across that surface. This condition is satisfied when capillary pressure decreases to zero, in any other situation only the non-wetting phase (the phase with higher pressure) will be able to flow out of the sample [2]. This situation causes the accumulation of the wetting phase near the outlet face, until local saturation attains the value for which capillary pressure becomes zero ($s_I^{P_C = 0}$).

DISCRETIZATION METHOD

For application of the EbFVM, the solution domain must be broken up into much smaller sub-domains, called *elements*, which in this work have quadrilateral shape. These entities are the basis for defining the domain geometry, as well as for storing the physical properties of the medium. The problem unknowns are computed at points called *nodes*, located at element corners. Around every node is built a *control volume*, formed by portions of the elements that share the node. Every control volume is delimited by a certain number of faces, where fluxes are usually approximated by the midpoint rule, thus face center points become so important and are called as *integration points*. All these geometrical entities are depicted in Fig. 1.

The integration of the mass conservation equation (1) over a generic control volume and a finite time interval leads to discretized equation:

$$(\rho_F^{n+1} s_F^{n+1} - \rho_F^n s_F^n)_p \frac{\phi_p \Delta V_p}{\Delta t} + \sum_i (\rho_F^n \vec{\mathbf{v}}_F^n \cdot \Delta \vec{\mathbf{S}})_i = 0 \quad (6)$$

where the sum involves all integration points over the control volume boundary. After combination with the other model equations, one obtains two partially decoupled equations, one for pressure and other for saturation, which can be written, respectively, as:

$$\sum_i b_{p,i}^P (\vec{\nabla} P_D^n \cdot \Delta \vec{\mathbf{S}})_i + c_p^P (P_D^n)_p$$
$$= \sum_i d_{p,i}^P (\vec{\nabla} P_C^n \cdot \Delta \vec{\mathbf{S}})_i + \sum_i e_{p,i}^P (\vec{\mathbf{g}} \cdot \Delta \vec{\mathbf{S}})_i + f_p^P \quad (7)$$

$$\sum_i b_i^s (\vec{\nabla} s_I^{n+1} \cdot \Delta \vec{\mathbf{S}})_i + c_p^s (s_I^{n+1})_p$$
$$= \sum_i d_i^s (\vec{\mathbf{v}}_T^n \cdot \Delta \vec{\mathbf{S}})_i + \sum_i e_i^s (\vec{\mathbf{g}} \cdot \Delta \vec{\mathbf{S}})_i + f_p^s \quad (8)$$
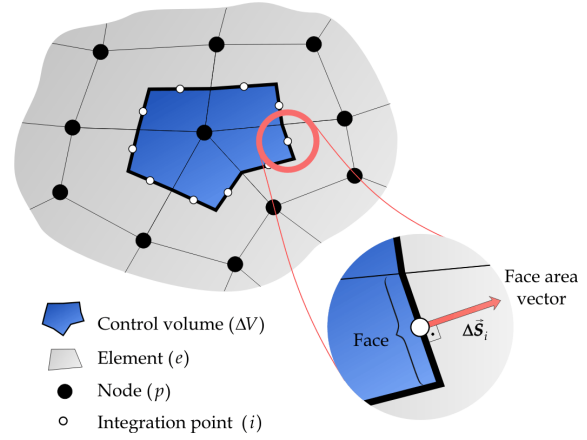

Figure 1
Main geometrical entities in EbFVM.

The coefficients in those equations are related to flow and medium properties, and their definition can be found elsewhere [1]. Equation (8) has been put in the Buckley-Leverett form, using the total velocity (i.e. the sum of the two phase velocities) as coupling variable between that equation and the pressure equation. Moreover, the capillary pressure term has been approximated in a semi-implicit way, in order to reduce the timestep stability restriction. The gradients of pressure and saturation are approximated assuming a bilinear variation inside any element.

In order to systematize the calculations of integration point contributions on the control volume equations, they are performed element by element, storing partial coefficients and independent terms in local matrices and column vectors. For instance, the local matrix and column vector for the pressure equation are:

$$\left[\sum_i b_{p,i}^P (\vec{\nabla} P_D^n \cdot \Delta \vec{\mathbf{S}})_i \right]_e =$$
$$\begin{bmatrix} b_{1,1}^P [B]_1 - b_{1,2}^P [B]_2 \\ b_{2,2}^P [B]_2 - b_{2,3}^P [B]_3 \\ b_{3,3}^P [B]_3 - b_{3,4}^P [B]_4 \\ b_{4,4}^P [B]_4 - b_{4,1}^P [B]_1 \end{bmatrix} \begin{bmatrix} P_{D1}^n \\ P_{D2}^n \\ P_{D3}^n \\ P_{D4}^n \end{bmatrix} \equiv [A]_e^P [P_D^n]_e \quad (8)$$

$$\left[\sum_i d_{p,i}^P (\vec{\nabla} P_C^n \cdot \Delta \vec{\mathbf{S}})_i + \sum_i e_{p,i}^P (\vec{\mathbf{g}} \cdot \Delta \vec{\mathbf{S}})_i \right]_e =$$
$$\begin{bmatrix} d_{1,1}^P [B]_1 - d_{1,2}^P [B]_2 \\ d_{2,2}^P [B]_2 - d_{2,3}^P [B]_3 \\ d_{3,3}^P [B]_3 - d_{3,4}^P [B]_4 \\ d_{4,4}^P [B]_4 - d_{4,1}^P [B]_1 \end{bmatrix} \begin{bmatrix} P_{C1}^n \\ P_{C2}^n \\ P_{C3}^n \\ P_{C4}^n \end{bmatrix} + \begin{bmatrix} e_{1,1}^P G_1 - e_{1,2}^P G_2 \\ e_{2,2}^P G_2 - e_{2,3}^P G_3 \\ e_{3,3}^P G_3 - e_{3,4}^P G_4 \\ e_{4,4}^P G_4 - e_{4,1}^P G_1 \end{bmatrix} \equiv [F]_e^P \quad (9)$$

where $[B]_i$ are row vectors that depend on element geometry, and G_i are geometry and gravity-dependent scalar parameters (see more details in [1]). Assembling those element contributions and then inserting

boundary conditions, a linear system of equations for the pressure can be obtained:

$$[A]^P[P_D^n] = [F]^P \qquad (12)$$

A similar linear system can be obtained for the saturation, starting from Eq. (8). A sequential solution algorithm, which can be considered an accelerated version of the IMPES algorithm, has been employed for obtaining the time evolution of pressure and saturation fields, solving sequentially the linear systems for pressure and saturation.

APPLICATION EXAMPLE

An imbibition displacement in a core sample was simulated taking as solution domain the vertical middle section. The spatial variation of absolute permeability and porosity on that section is shown in Fig. 2. Those maps were obtained by nuclear magnetic resonance imaging by Zuluaga et al. [3]. Two grids were used for simulating the flow: a 3600-element Cartesian grid and a 3582-element unstructured grid. Portions of those grids are shown in Fig. 3. In Fig. 4 the saturation distributions obtained with those grids are shown, at three selected times. As can be observed, the flow is strongly influenced by the medium heterogeneity, and regardless of the different grid topology, both solutions are almost identical. Furthermore, in the latter time is noticeable the presence of the capillary end effect as a region of high saturation gradient.

CONCLUSIONS

A numerical formulation for simulating immiscible two-phase flows in porous media has been described in this paper. The formulation is able to deal with unstructured grids and enforces mass conservation in a similar way than the conventional finite volume method.

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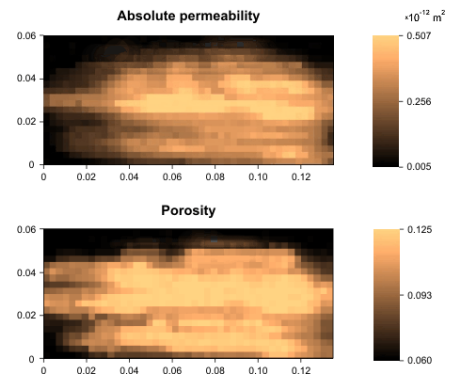1. Hurtado, F. S. V., Maliska, C. R., Silva, A. F. C.,

Figure 2
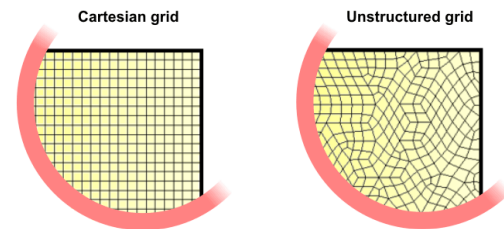Absolute permeability and porosity maps.

Figure 3
Topology of the grids considered.

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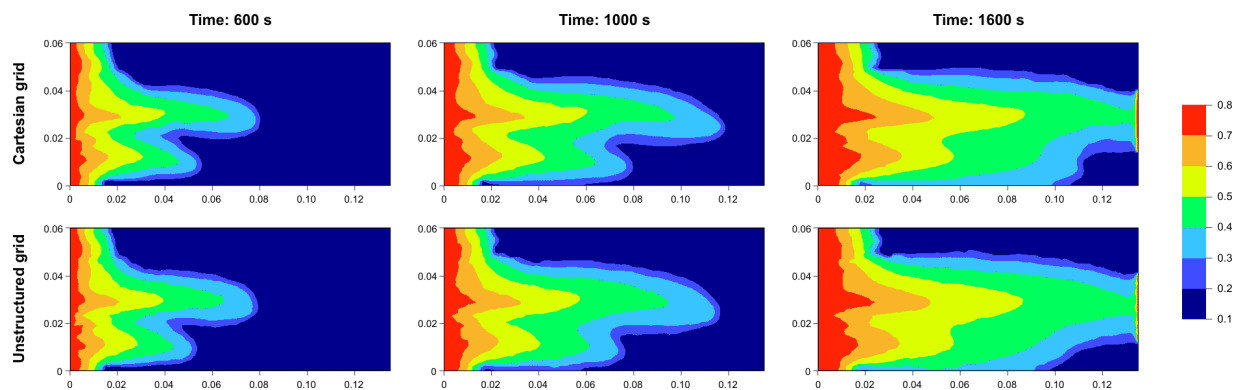

Figure 4
Comparison of saturation distributions in a Cartesian and in an unstructured grid.