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NUMERICAL SOLUTION OF THE MULTIPHASE FLOW OF OIL, WATER AND GAS IN HORIZONTAL WELLS IN NATURAL PETROLEUM RESERVOIRS

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Abstract. The estimation of oil production in natural petroleum reservoirs requires the solution of the coupled flow between the reservoir and the wellbore. The oil, water and gas flow in the well is the result of the lateral flow coming from the reservoir, obeying different completions, aiming to have an oil production with almost constant rate along the horizontal well. The multiphase flow is governed by the Navier-Stokes equations and mass conservation for each phase. Due to the large difference in spatial scales between the wellbore and the petroleum reservoir domains, the flow in the wellbore can be considered as one dimensional, taking into account, through friction models, the pressure loss caused by the lateral mass income of oil, gas and water.

In this paper, the homogeneous flow model is adopted, and a slip between phases is specified according to some algebraic correlation available in the literature. The conservation equations are approximated using a finite volume method employing a staggered grid for the variable arrangement, what renders to the method robustness and guarantees a strong coupling between pressure and velocity in the wellbore. Pressures and void fractions are located at the center of the mass conservation control volume, while velocities are at the surfaces. The block-matrix is solved using a Newton-like method, with the Jacobian matrix constructed using precise numerical derivatives. Comparisons with available numerical solutions for two-phase flow are performed, showing that the model performs well and allows the use of larger time steps than the ones reported in the literature. Examples demonstrating the ability of the method for solving three-phase flows are also presented. As an overall outcome, it can be said that the developed method is suitable for taking part in a more general application for estimating oil production in petroleum reservoirs.

1 INTRODUCTION

The use of deviated wells has become an attractive alternative, since they can provide a considerable increase in production. In contrast, the high cost associated with drilling and completion of wells requires a detailed study in order to find the optimal parameters for their construction. In deviated wells, the drilling is initially vertical and it becomes more deviated from the vertical as the depth increases until it reaches the desired final position, usually horizontal. The knowledge of flow characteristics in these wells is essential to predict the fluid dynamic behavior and define the design parameters for their construction (eg. its optimum length, diameter, type of completion, etc.) and for ensuring maximum production over the life of the well at a lower cost.

The communication between reservoir and well is given by the well completion holes, which allow the passage of oil from the reservoir to the wells. The completion of horizontal wells is a key issue, since there is an optimum lenght for a specified pressure at the well heel.The pressure field established along the well serves as a boundary condition for the flow solution in the reservoir, modeled by *Darcy* equations. The reservoir and well domains are quite different, exhibiting different geometric and physical scales.

In general, the well model is to be coupled with the reservoir model. Hence, it should represent the multiphase phenomenon inside the well physically consistently and need to be robust. From the technological point of view, solving the flow with broader hypotheses, as in the case of multiphase flows, has become a fundamental need for the design of these wells. If the coupled multiphase flow is solved in the well and reservoir, it is possible to study the interaction between the well and the reservoir through numerical solutions and therefore achieve a better design of the production wells.

The solution of the multiphase flows can be achieved by solving the momentum conservation equations for each phase. In these models, the so-called multi-fluid models (Ishii and Hibiki, 2006), it is required to specify boundary conditions for each phase interface. Since this is an impossible task, at least with the nowadays computer resources and methods, one needs to specify the interfacial mass, momentum and energy transfer, giving rise to the drag, inertial, virtual, lift and lubrication forces for closing the mathematical model. This approach, however, is too expensive when the goal is to couple the solution of the flow in the wellbore with the solution in the petroleum reservoir. In this case one should seek for a robust model with reasonable computing effort.

There have been several approaches for developing a multiphase model for oil wells. Ouyang (1998) presents a very detailed review of available models and methods for dealing with each flow pattern in two-phase flow. Shi et al. (2003) proposes an approach for dealing with three-phase flow in wellbores with the usage of a drift-flux model. A compositional model has also being proposed by Shirdel and Sepehrnoori (2009) and Livescu et al. (2009) proposed a black-oil thermal model.

2 MODEL FORMULATION

The physical problem consists in the solution of the one dimensional isothermal flow for each phase (gas, oil and water) on a pipe with lateral mass inflow. Therefore, pressure and velocity fields need to be solved for each phase. The Mass Conservation Equation for a phase p is given by

$$\frac{\partial\left(\alpha_p \rho_p\right)}{\partial t} + \frac{\partial\left(\alpha_p \rho_p u_p\right)}{\partial x} = q_p, \tag{1}$$

where q_p is a mass flux per unit volume that is associated with a possible lateral mass inflow that comes from an oil reservoir. The volumetric fraction of the phase (α_p) is defined by

$$\alpha_p = \frac{V_p}{V}, \tag{2}$$

where V_p is the volume occupied by the phase and V the total volume of a control volume of interest. Additionally, by the definition of α_p, we should have

$$\sum \alpha_p = 1. \tag{3}$$

In order to couple pressure and velocities of each system, it is necessary to solve the momentum equations. Assuming the same pressure for all phases and noting that, for an advective dominant flow, diffusive flux in x direction can be neglected and the Momentum Conservation Equation for phase p becomes

$$\frac{\partial\left(\alpha_p\rho_p u_p\right)}{\partial t} + \frac{\partial\left(\alpha_p\rho_p u_p u_p\right)}{\partial x} = -\frac{\partial(\alpha_p P)}{\partial x} - \alpha_p\rho_p g_x + F_{fp}, \tag{4}$$

where F_{fp} is associated with interphase and wall forces that could affect the momentum of the system. Bonizzi and Issa (2003) presents a formulation that solves Eq. (4) with different models for F_{fp} depending on the multiphase flow pattern.

It is important to observe that, for a gas/oil/water system, if we assume that $\rho_p = \rho_p(P)$, there are 7 unknowns: α_g, α_o, α_w, u_g, u_o, u_w and P. Since Eq. (1), Eq. (3) and Eq. (4) produce 7 equations, the problem is well posed. However, if one is interested in a faster solution of the problem, this procedure would result on a rather large non-linear system to be solved, with the extra effort of modeling complicated interphase forces in each momentum equation. Therefore, it is mandatory to reduce the number of equations, and this can de done by the use of a drift-flux model (Hibiki and Ishii, 2003). This model uses the sum of Eq. (4) for all phases and introduces a *mixture velocity*, defined as

$$u_m = \frac{\alpha_g\rho_g u_g + \alpha_o\rho_o u_o + \alpha_w\rho_w u_w}{\rho_m}, \tag{5}$$

where ρ_m is the *mixture density* given by

$$\rho_m = \alpha_g\rho_g + \alpha_o\rho_o + \alpha_w\rho_w. \tag{6}$$

Because the drift-flux model is a two-phase model, it is necessary to somehow extend this procedure for a three-phase system. The procedure is the same as the one shown by Soprano et al. (2010), i.e., splitting the three-phase system in two sub-systems: a gas/liquid and a oil/water system, where the liquid phase is the sum of the oil and water phases.

Now, it is possible to assume that the velocity of each phase in the gas/liquid system is a function of u_m, given by

$$\begin{aligned} u_g &= u_m + \frac{\rho_l}{\rho_m} u_d^{gl}, \\ u_l &= u_m - \frac{\alpha_g}{\alpha_l}\frac{\rho_g}{\rho_m} u_d^{gl}, \end{aligned} \tag{7}$$

where the second parcel represents a relative velocity between the phase velocity and the mixture velocity. For the oil/water system, phase velocities are given as functions of u_l, through the

following

$$u_o = u_l + \frac{\rho_w}{\rho_l} u_d^{ow},$$
$$u_w = u_l - \frac{\alpha_o}{\alpha_w}\frac{\rho_o}{\rho_l} u_d^{ow}. \tag{8}$$

The liquid velocity is defined in a similar way as Eq. (5):

$$u_l = \frac{\alpha_o \rho_o u_o + \alpha_w \rho_w u_w}{\rho_l}, \tag{9}$$

and

$$\rho_l = \alpha_o \rho_o + \alpha_w \rho_w. \tag{10}$$

The term u_d^{gl} is a function of u_m and it depends on a few extra parameters. Its formula for the gas/liquid system is given by

$$u_d^{gl} = \frac{V_d^{gl} + (C_0 - 1)\, u_m}{\left[1 - (C_0 - 1)\, \alpha_g \frac{(\rho_l - \rho_g)}{\rho_m}\right]}, \tag{11}$$

and for the oil/water system,

$$u_d^{ow} = \frac{V_d^{ow} + (C'_0 - 1)\, u_l}{\left[1 - (C'_0 - 1)\, \alpha_o \frac{(\rho_o - \rho_w)}{\rho_l}\right]}. \tag{12}$$

Here, for the gas/liquid system, C_0 is known as the *profile parameter* and is associated with the distribution of the phase along the cross-sectional area of the pipe. The term V_d^{gl} is known as *drift velocity* and is associated with the slip between phase and the mixture. The same concepts apply for C'_0 and V_d^{ow}. Those parameters can be modeled and they depend on flow pattern, geometry and fluid properties.

With well defined relations for calculating phase velocities and noting that they all depend on the mixture velocity (u_m), the resulting problem has actually 5 unknowns: α_g, α_o, α_w, P and u_m. Now it is possible to reduce the number of equations to be solved. This reduction is done through the summation of Eq. (4) over all phases. The advantage of this procedure is that interphase forces terms are canceled and only wall friction forces are left. The resulting system of equations is

$$\frac{\partial (\alpha_g \rho_g)}{\partial t} + \frac{\partial (\alpha_g \rho_g u_g)}{\partial x} = q_g, \tag{13}$$

$$\frac{\partial (\alpha_o \rho_o)}{\partial t} + \frac{\partial (\alpha_o \rho_o u_o)}{\partial x} = q_o, \tag{14}$$

$$\frac{\partial (\sum \alpha_p \rho_p)}{\partial t} + \frac{\partial (\sum \alpha_p \rho_p u_p)}{\partial x} = \sum q_p, \tag{15}$$

$$\frac{\partial (\sum \alpha_p \rho_p u_p)}{\partial t} + \frac{\partial (\sum \alpha_p \rho_p u_p u_p)}{\partial x} = -\frac{\partial P}{\partial x} - \rho_m g_x - \frac{f}{2D} \rho_m u_m |u_m|. \tag{16}$$

Because Eq. (3) is an algebraic equation, one of the volumetric fractions can be directly calculated. Hence, the non-linear system is composed by Eqs. (13)-(16). For convenience, the sum of the mass balance equations was used instead of the water mass balance equation.

3 NUMERICAL PROCEDURE

Equations were discretized using the Finite Volume Method. The pressure-velocity coupling was resolved through a staggered grid where pressure and volume fractions were stored in one grid (mass conservation control volume) and phase and mixture velocities were located on another grid (Maliska, 2004). A scheme of this procedure is shown in Fig. 1. As well recognized, this approach renders to the method a strong coupling between pressure and velocities, since all variables are calculated where they are needed for the fluxes calculation required by the equation integration.

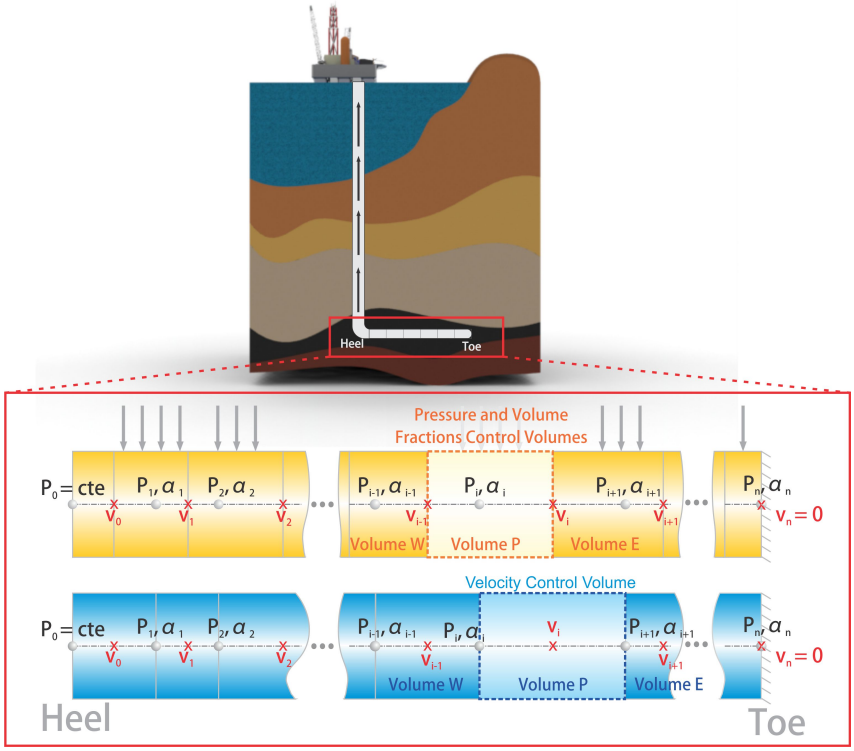


Figure 1: Staggered grid used in the discretization of the problem.

Residual equations are obtained by an integration of Eqs. (13)-(16) over its control volumes. Mass balance equations were integrated over the yellow control volumes indicated in Fig. 1 and the mixture momentum equation was integrated over the control volume indicated by the blue color.

Due to strong nonlinearities associated to the system of PDEs, the Newton Method is applied in order to obtain the coupled solution of the set of equations. Hence, it becomes necessary to construct a Jacobian matrix. The residual function used for Newton iterations for a control volume P has the following form

$$R_P = \begin{bmatrix} R_P^g & R_P^o & R_P^m & R_P^u \end{bmatrix}^T . \tag{17}$$

The jacobian matrix is the built by deriving Eq. (17) with respect to α_g, α_o, P and u_m. The derivatives of the residual functions were calculated approximately through finite difference formulas. Given a residual function R, its derivative is calculated by

$$\frac{\partial R}{\partial x} = \frac{R(x + \Delta x) - R(x)}{\Delta x}, \tag{18}$$

with the increment Δx being carefully chosen to result in good derivative approximations.

Being the problem one-dimensional, the resulting jacobian matrix has a tridiagonal block structure, as shown in Fig. 2. Each block of the Jacobian matrix (Fig. 2) has the form shown in

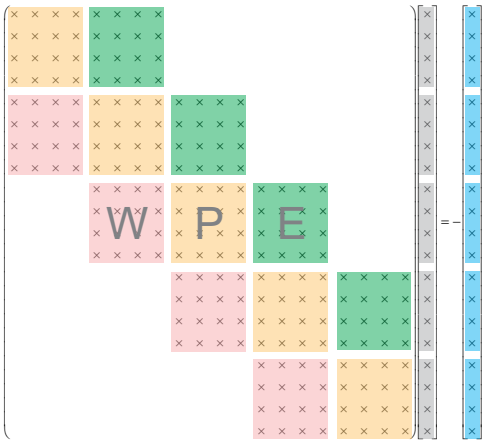


Figure 2: Structure of jacobian matrix.

Table 1.

West Block (W):	*Central Block (P):*	*East Block (E):*
$\begin{pmatrix} \frac{\partial R_g^P}{\partial \alpha_g^W} & \frac{\partial R_g^P}{\partial \alpha_o^W} & \frac{\partial R_g^P}{\partial P_W} & \frac{\partial R_g^P}{\partial u_m^W} \\ \frac{\partial R_o^P}{\partial \alpha_g^W} & \frac{\partial R_o^P}{\partial \alpha_o^W} & \frac{\partial R_o^P}{\partial P_W} & \frac{\partial R_o^P}{\partial u_m^W} \\ \frac{\partial R_m^P}{\partial \alpha_g^W} & \frac{\partial R_m^P}{\partial \alpha_o^W} & \frac{\partial R_m^P}{\partial P_W} & \frac{\partial R_m^P}{\partial u_m^W} \\ \frac{\partial R_u^P}{\partial \alpha_g^W} & \frac{\partial R_u^P}{\partial \alpha_o^W} & \frac{\partial R_u^P}{\partial P_W} & \frac{\partial R_u^P}{\partial u_m^W} \end{pmatrix}$	$\begin{pmatrix} \frac{\partial R_g^P}{\partial \alpha_g^P} & \frac{\partial R_g^P}{\partial \alpha_o^P} & \frac{\partial R_g^P}{\partial P_P} & \frac{\partial R_g^P}{\partial u_m^P} \\ \frac{\partial R_o^P}{\partial \alpha_g^P} & \frac{\partial R_o^P}{\partial \alpha_o^P} & \frac{\partial R_o^P}{\partial P_P} & \frac{\partial R_o^P}{\partial u_m^P} \\ \frac{\partial R_m^P}{\partial \alpha_g^P} & \frac{\partial R_m^P}{\partial \alpha_o^P} & \frac{\partial R_m^P}{\partial P_P} & \frac{\partial R_m^P}{\partial u_m^P} \\ \frac{\partial R_u^P}{\partial \alpha_g^P} & \frac{\partial R_u^P}{\partial \alpha_o^P} & \frac{\partial R_u^P}{\partial P_P} & \frac{\partial R_u^P}{\partial u_m^P} \end{pmatrix}$	$\begin{pmatrix} \frac{\partial R_g^P}{\partial \alpha_g^E} & \frac{\partial R_g^P}{\partial \alpha_o^E} & \frac{\partial R_g^P}{\partial P_E} & \frac{\partial R_g^P}{\partial u_m^E} \\ \frac{\partial R_o^P}{\partial \alpha_g^E} & \frac{\partial R_o^P}{\partial \alpha_o^E} & \frac{\partial R_o^P}{\partial P_E} & \frac{\partial R_o^P}{\partial u_m^E} \\ \frac{\partial R_m^P}{\partial \alpha_g^E} & \frac{\partial R_m^P}{\partial \alpha_o^E} & \frac{\partial R_m^P}{\partial P_E} & \frac{\partial R_m^P}{\partial u_m^E} \\ \frac{\partial R_u^P}{\partial \alpha_g^E} & \frac{\partial R_u^P}{\partial \alpha_o^E} & \frac{\partial R_u^P}{\partial P_E} & \frac{\partial R_u^P}{\partial u_m^E} \end{pmatrix}$

Table 1: Block components of the Jacobian Matrix.

The computational algorithm for the coupled solution of the equations using Newton's Method is summarized as:

1. Initialize variables;
2. Calculate residuals;
3. Build the Jacobian matrix;
4. Solve the linear system $J \cdot \Delta x = -R$;
5. Update variables: $x = x^o + \Delta x$;
6. If solution has not converged go to step 2;
7. Update variables from previous time step;
8. If it has not reached final time or steady state go to step 2.

4 RESULTS

To demonstrate the applicability of the proposed numerical scheme it is solved, firstly, a gas/liquid flow for numerical validation purposes. Results are compared with two different sources in the literature, trying to point out aspects in which the method advanced here. Following, an oil, gas and water flow is solved, a realistic flow normally encountered in engineering applications.

4.1 Validation test

The gas/liquid flow solution is compared with two references (Provenzano, 2007; Evje and Fjelde, 2003), where simple correlations for the drift-flux model in a gas/liquid system is tested. The problem consists in a horizontal tube with prescribed inflow of both phases in one side and constant pressure on the other. In this case, values for the profile parameter were $C_0 = 1.2$ and $V_d^{gl} = 0.5m/s$ for the drift velocity. The inflow in the left side of the pipe is a function of time and the profiles are shown in Fig. 3. Total simulation time is 175 seconds and the timestep used was $\Delta t = 0,01s$. The full problem description, with fluid properties and geometric parameters can be found in Provenzano (2007).

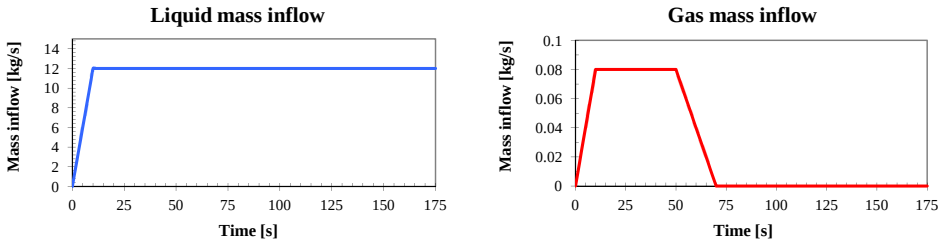


Figure 3: Influx profile for each phase as a function of time.

One of the goals with this validation test is to asses the convergence characteristics of the model when gas phase disappears. When the gas inflow is interrupted, the amount of gas moves towards the other end of the pipe as a slug, as shown in Fig. 4 and Fig. 5. Numerically, every pulse, temporal or spatial, poses challenges to the numerical model, since the pulse requires extremely fine grids in order to be well captured.

Before any comparisons are made, it is important to mention some characteristics of the three models under consideration. Since Provenzano (2007) also used a finite volume method but with a segregated solution for pressure-velocity coupling, results were very similar to the ones of the present work. The work reported in Evje and Fjelde (2003) used a different formulation, with higher order interpolation and, therefore, the results show a better capturing of the large gradients present in the pulse for the same mesh and temporal refinement. The advantage of the proposed method was the possibility of using much larger timesteps during the solution procedure, since the method solves all equations in a single linear system. Other tests carried out also demonstrated that one of the strong points of the method is its ability in dealing with larger time steps, suggesting that the implicit solution for the pressure-velocity coupling is a viable approach.

4.2 Example test

To demonstrate the ability of the model for solving three-phase flow problems, an example test is solved. The drift-flux correlations for this three-phase system can be found in Shi et al.

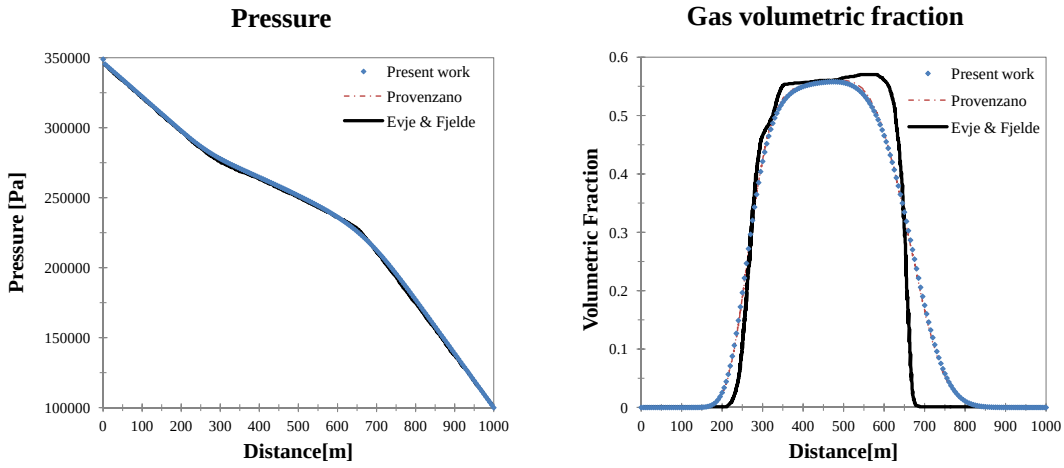


Figure 4: Results of pressure and gas volumetric fraction for $t = 175s$ and 200 control volumes.

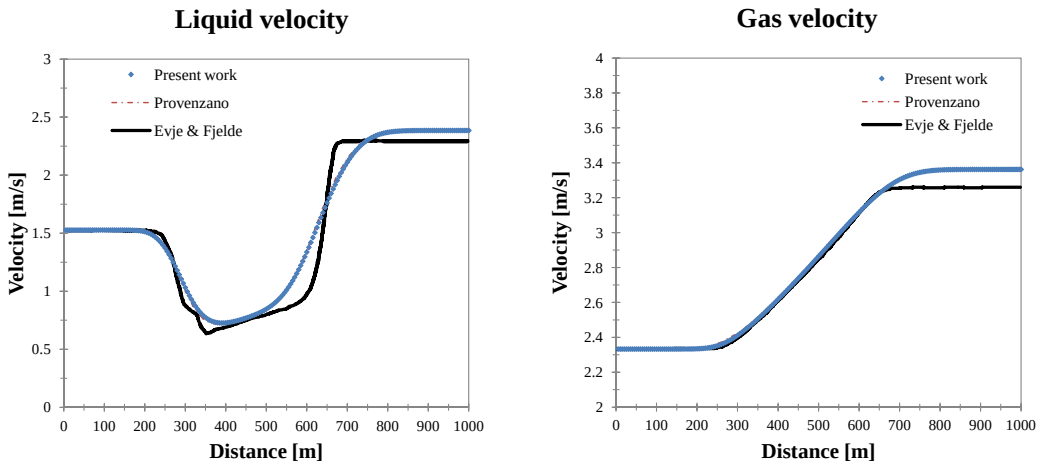


Figure 5: Results of phase velocity curves for $t = 175s$ and 200 control volumes.

(2003) and Hasan and Kabir (1998). The correlations used for calculating phase velocities are more complex and depend on a series of parameters, and can be seen in the references just mentioned. Fluid properties are given as follow:

Phase properties:	
Oil density:	$800.0\ kg/m^3$
Oil viscosity:	$1.5 \times 10^{-3}\ Pa.s$
Water density:	$1000.0 kg/m^3$
Water viscosity:	$0.5471 \times 10^{-3}\ Pa.s$
Gas sound speed:	$463.25 m/s$
Gas viscosity:	$12.09 \times 10^{-6}\ Pa.s$
Reference temperature:	$323.0\ K$

The problem is a $1000m$ well with 100 control volumes, each one with lateral mass inflow, simulating a mass flux from the natural petroleum reservoir. The well is nearly horizontal, with an inclination of $\theta = 85°$, as represented in Fig. 6. The well diameter is $D = 0.3m$ and *Churchill* friction factor was used for the friction term in Eq. (16).

Boundary conditions that simulate real conditions of a production well were employed, i.e., null velocity in the *Toe* and prescribed pressure in the *Heel* (Fig. 6) with a value of $P_h = 10^6 Pa$. To represent the inflow coming from the reservoir, a typical curve, as shown in in Fig. 7 is used. Recall that this mass flow enters as source term in the mass balance equations. The initial conditions of the well is $\alpha_o = 1$, that is, initially filled with oil.

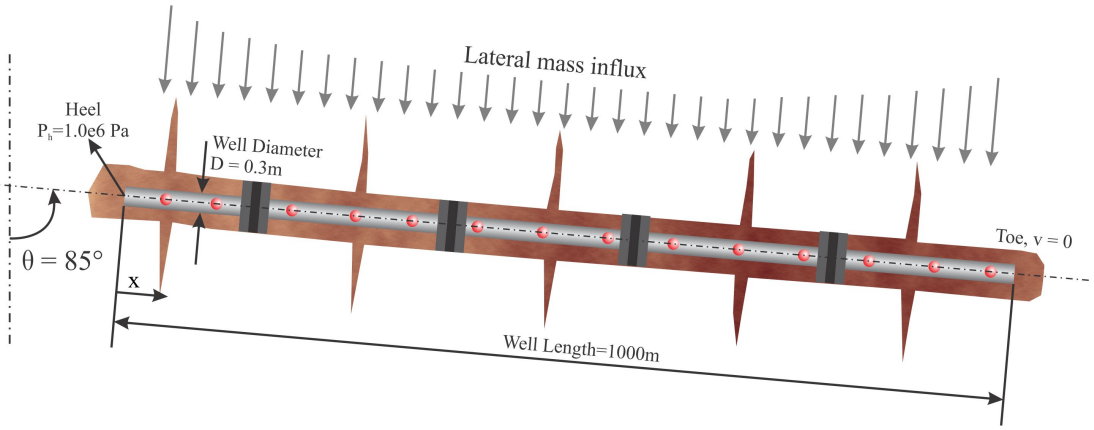


Figure 6: Well scheme.

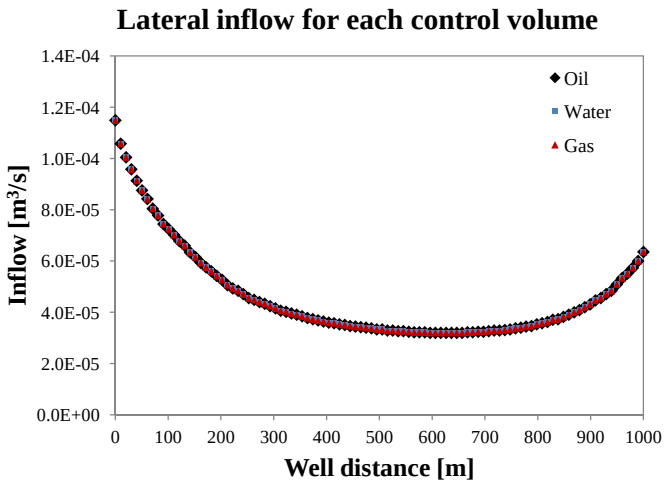


Figure 7: Lateral mass influx for each phase along the well.

The model uses an adaptive timestep, being the initial one set to $\Delta t_{\text{initial}} = 0.1s$, with the maximum time step allowed being equal to $\Delta t_{\text{max}} = 1000s$. The adaptive time step calculation is based on the maximum variation of the volumetric fractions comparing the current solution with the one from the previous time level. The new timestep is then calculated by

$$\Delta t_{\text{new}} = \min\left(\frac{\Delta\alpha_{\text{expected}}}{\Delta\alpha_{\text{max}}}\Delta t_{\text{old}}, \Delta t_{\text{max}}\right), \tag{19}$$

where $\Delta\alpha_{\text{max}}$ is the maximum volumetric fraction variation in the domain for all phases. In this test, the problem is solved until it reaches a steady state criteria of 10^{-5}. In this case, the steady state was reached for $t_{\text{final}} = 30556.3s \approx 0.35$ days.

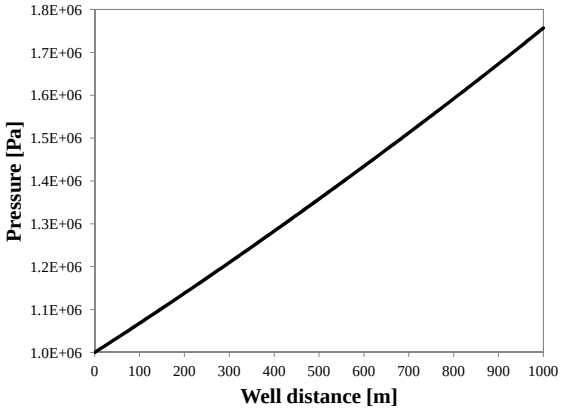


Figure 8: Pressure profile along the well in steady state with 100 control volumes.

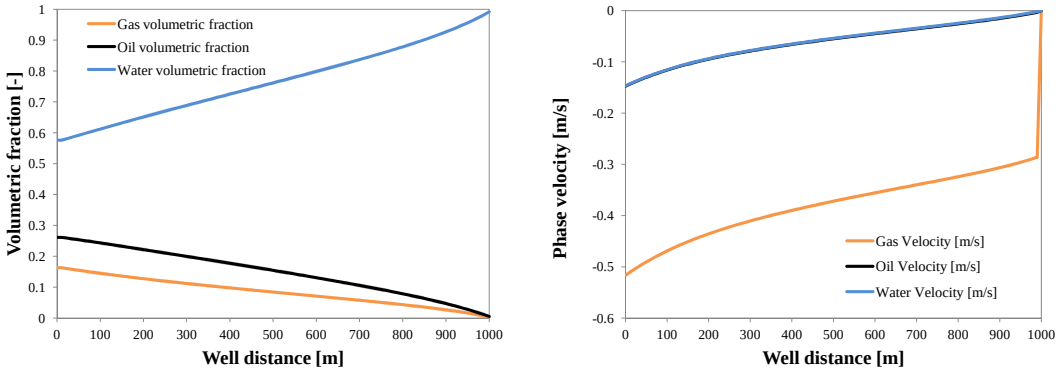
As previously mentioned, the drift-flux model solves a two-phase flow. It usually treats the two-phase system as a dispersed phase (gas phase) and a non-dispersed one (liquid phase). In our case, we also treat the liquid system as two-phase system, with the oil being the dispersed and water the non-dispersed phase. Note that according to Eq. (7), if we assume that u_d^{gl} has the same sign as u_m, the dispersed phase moves faster that the non-dispersed phase. This also applies to the oil/water system. Thus, one would expect to usually have

$$u_g > u_o > u_w$$

in the case where gravity forces act against the flow. The opposite would occur for a downstream flow, e.g.,

$$u_g < u_o < u_w.$$

The results presented in Fig. 9 show the first situation. However, because gas density is much smaller than oil and water densities, the difference between oil and water velocities are hardly noticeable. Moreover, the oil/water drift-flux model takes into account the amount of gas in the system, which also contributes for the reduction of the slip between oil and water (Shi et al., 2005). This is the reason why, in steady state, water is accumulated in well toe, since it has the smaller velocity, its volumetric fraction is greater along the well.



(a) Volumetric fraction profile.

(b) Phase velocity profile.

Figure 9: Results for velocities and volumetric fraction profiles along the well in steady state with 100 control volumes.

5 CONCLUSIONS

This work advanced a numerical model for solving the multiphase flow of oil, gas and water in oils wells using the drift-flux approach, handling the three-phase flow considering a two-phase flow of gas and liquid and then applying the same strategy for the oil and water phases. The main goal was to have the model able for being coupled to a petroleum reservoir simulator, especially in designing horizontal wells with special completion to optimize the oil production. Therefore, lateral income flow was considered using correlations for taking into account the additional friction. The numerical strategy of solving the pressure-velocity coupling implicitly rendered to the method the use of larger time steps than the ones of similar works. Computations were performed in order to validate the numerical scheme, followed by the solution of a three-phase flow similar to the ones encountered in real applications.

REFERENCES

Bonizzi M. and Issa R.I. On the simulation of three-phase slug flow in nearly horizontal pipes using the multi-fluid model. *International Journal of Multiphase Flow*, 29:1719–1747, 2003.

Evje S. and Fjelde K.K. On a rough ausm scheme for a one-dimensional two-phase model. *Computer & Fluids*, 32, 2003.

Hasan A.R. and Kabir C.S. A simplified model for oil-water flow in vertical and deviated wellbores. *SPE - Society of Petroleum Engineers*, (SPE 49163), 1998.

Hibiki T. and Ishii M. One-dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes. *International Journal of Heat and Mass Transfer*, 2003.

Ishii M. and Hibiki T. *Thermo-Fluid Dynamics of Two-Phase Flow*. Springer Science, New York, NY, 2006.

Livescu S., Durlofsky L.J., Aziz K., and Ginestra J.C. A fully-coupled thermal multiphase wellbore flowmodel for use in reservoir simulation. *Journal of Petroleum Science and Engineering*, 2009.

Maliska C.R. *Transferência de Calor e Mecânica dos Fluidos Computacional*. LTC - Livros Técnicos e Científicos Editora, Rio de Janeiro, 2ł edition, 2004.

Ouyang L.B. *Single Phase and Multiphase Fluid Flow in Horizontal Wells*. Ph.D. thesis, Stanford University, 1998.

Provenzano C.E.C. *Previsão Numérica de Escoamento Bifásico em Tubulações Utilizando o Modelo de Deslizamento*. Master's Thesis, PUC - Rio de Janeiro, 2007.

Shi H., Holmes J., Diaz L., Durlofsky L., and Aziz K. Drift-flux parameters for three-phase steady-state flow in wellbores. *SPE - Society of Petroleum Engineers*, 2005.

Shi H., Holmes J., Durlofsky L., Aziz K., Diaz L., Alkaya B., and Oddie G. Drift-flux modeling of multiphase flow in wellbores. *SPE - Society of Petroleum Engineers*, (SPE 84228), 2003.

Shirdel M. and Sepehrnoori K. Development of a coupled compositional wellbore/reservoir simulator for modeling pressure and temperature distribution in horizontal wells. *SPE - Society of Petroleum Engineers*, (124806), 2009.

Soprano A.B., Ribeiro G.G., da Silva A.F.C., and Maliska C.R. Solution of a one-dimensional three-phase flow in horizontal wells using a drift-flux model. *Mecánica Computacional*, XXIX:8767–8779, 2010.