X CONGRESSO BRASILEIRO DE ENGENHARIA MECÂNICA, RIO DE JANEIRO, RJ (DEZEMBRO, 1989)
10th BRAZILIAN CONGRESS OF MECHANICAL ENGINEERING

NATURAL CONVECTION IN OPEN ENDED CHANNELS - TREATMENT OF THE INLET BOUNDARY CONDITIONS

PEM
PROGRAMA DE
ENGENHARIA
MECÂNICA
COPPE/UFRJ

F. Marcondes, C. R. Maliska and A. F. C. Silva

Mechanical Engineering Department
Federal University of Santa Catarina
P.O. Box, 476
88049 - Florianópolis - SC - Brazil

ABSTRACT

When natural convection problems are solved in open ended channels, using the elliptic form of the conservation equations, the application of the boundary conditions for pressure and velocity at the inlet are of utmost importance. This paper describes a new method for applying these boundary conditions and demonstrates that applying zero inlet pressure as boundary conditions, as usually done, result in a wrong mass flow rate prediction. The new approach improves significantly the convergence rate when compared with similar methodologies.

INTRODUCTION

The analysis of the natural convection flows in vertical channels are normally undertaken under the assumption that the stream-wise diffusion of momentum and energy are negligible, ending up in a numerical formulation which allows to employ a marching procedure along the flow direction. The marching starts at the channel entrance, with a prescribed mass flow rate, and terminates when the pressure excess vanishes, determinating the channel height for that prescribed mass flow. It is clear that the use of the parabolic aproximation removes the problem of the unknown boundary conditon for velocity at the inlet. However, if the channel height is given and the mass flow is the unknown parameter, as is the case in real applications, the method results iterative.

Furthermore, for some problems, like the natural convection flow in a vertical channel with one heated wall and the other wall insulated, there is the appearance of flow reversal at the outlet, with clearly elliptic behaviour. In such a problem the parabolic procedure can not be adopted. The use of an elliptic model,in the other hand, gives rise to the need of specifying velocity and pressure at the inlet. The velocity profile, of course is not known and the pressure is a function of the unknown velocity. Kettlerborough [1], and Nakamura [2], employ a elliptic formulation but by-pass the problem of applying boundary conditions at tle inlet by extending the computational domain, and applying normal velocity gradient far from the channel inlet. This is not an attractive route to follow due to need of extra points outside the channel.Very recently, Nieckle and Azevedo [3], solved the natural convection flow problem , with flow reversal, using the elliptic equations without extending the computational domain. It is not clear, however, how pressure and velocity boundary conditions were applied at the channel inlet.

The main goal of the present paper is to advance a novel method for applying boundary conditions at the channel inlet. It is demonstrated that the inappropriate boundary conditions for pressure may preclude the observation of the flow reversal, despite the same Nusselt number which is encountered for different pressure boundary conditions. The CPU effort to obtain the solution to a specified level of convergence using the methodology advanced in this work is compared with a similar methodology described in [4].

PROBLEM FORMULATION

The governing equations of the two-dimensional laminar natural convection flow problem written in conservative form in the Cartesian system for a general scalar field ϕ can be represented by

$$\rho\frac{\partial\phi}{\partial t} + \rho u\frac{\partial\phi}{\partial x} + \rho v\frac{\partial\phi}{\partial y} = - P^{\phi} + \frac{\partial}{\partial x}(\mu\frac{\partial\phi}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial\phi}{\partial y}) + S^{\phi} \quad (1)$$

The symbols which appear in the above equation are the usually employed.In the above general equation the Boussinesq approximation is used, whereby the flow is assumed incompressible. Fig. 1 depicts the problem geometry with the boundary conditions. The boundary conditions for the inlet region are discussed in the next section.

INLET BOUNDARY CONDITIONS

One of the key question in solving the elliptic form of the conservation equations without extending the computational domain is the application of the inlet boundary conditions.

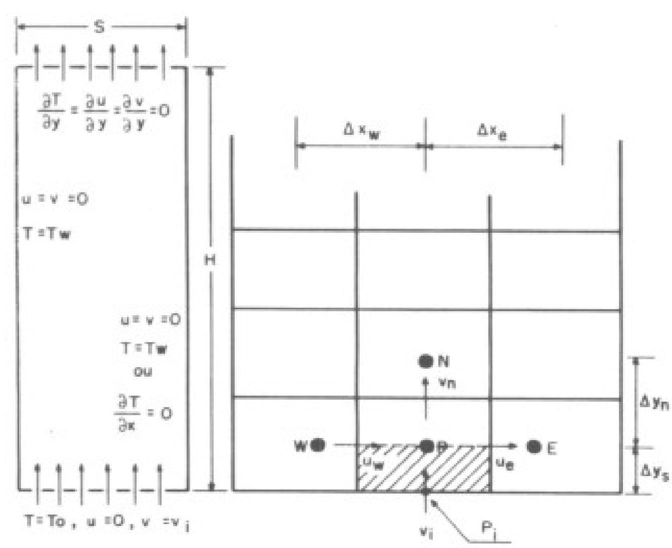

Fig. 1 - Problem geometry.

Since the velocity is not known a special equation is formed for the entrance velocity v_i corresponding to the dashed half-control volume shown in Fig. 1. It is assumed that the v_i velocity is driven by the pressure gradient given by $(P_P - P_i)/\Delta y_s$. Using the finite volume procedure the equation for v is integrated over time and over the dashed control volume, obtaining an algebraic equation for v_i. This equation has the following form

$$v_i = \hat{v}_i - \frac{P_P - P_i}{\Delta y_s} \frac{\Delta V}{A_P^{v_i}} \quad (2)$$

where the $\hat{v}_i$ term contains all terms of the momentum equation but the pressure term, and ΔV is the volume of the dashed elemental cell. Eq.(2) is used to advance the new values of v_i from the $\hat{v}_i$ value, known from the previous iteration level. The $\hat{v}_i$ value, as mentioned, is calculated using the momentum equation without the pressure term.

A similar procedure for forming an equation for v_i is described in [4]. In that work the v_i is made equal to v_i^*, where the v_i^* is obtained from the solution of the full momentum equation. The correction of the velocity using Eq. (2), adopted in the present paper, improves dramatically the convergence rate, as will be seen.

Having the inlet velocity calculated the inlet pressure can be obtained either from the momentum equation or from Bernoulli's equation, or even setting P_i equal to zero, as usually done when the parabolic procedure is used. In this paper results for the vertical channel are obtained using Bernoulli's equation and also using P equal to zero. The results will shown that the different pressure boundary conditions changes dramatically the flow field but does not affect the Nusselt number, parameter usually employed to check accuracy of numerical models for solving natural convection problems in open ended channels.

SOLUTION PROCEDURE

As seen in Fig. 1, the staggered grid arrangement is adopted. The pressure-velocity coupling is handled using the PRIME method [5] in which an equation for pressure is formed from the mass conservation equation. The solution procedure obeys the following steps: a) guess initial variables; b) compute the coefficients for the momentum equations; c) compute the $\hat{u}$ and $\hat{v}$ velocities; d) solve equation for pressure and correct the u and v velocities for mass satisfaction; e) calculate the inlet pressure; f) cycling back to step b is necessary to account for interequation coupling and nonlinearities.

NUMERICAL RESULTS

The methodology was applied in the solution of the natural convection flow problem in a vertical channel with one wall insulated and the other one with prescribed temperature. Table 1 shows the results obtained in [1],[2] and in the present work. In this table the Nusselt number is computed according Eqs.(3) and (4). These equations read that if the solution of the equation set is fully converged both Nusselt numbers must be the same. Inspecting Tab. 1 it can be seen that the present work exhibits a fully converged solution, and are similar to the results of [2] which also exhibits a reasonable converged solution.

$$Nu_m = \frac{\frac{S}{H^2}\int_0^H k \left.\frac{\partial T}{\partial x}\right|_{x=S/2} dy}{(T_w - T_0)} \quad (3)$$

$$Nu'_m = \frac{\frac{S}{H^2}\int_0^{\frac{S}{2}} \left[(\rho c_p VT)\big|_{y=H} - (\rho c_p VT)\big|_{y=0} + k\left.\frac{\partial T}{\partial y}\right|_{y=0} \right] dx}{(T_w - T_0)} \quad (4)$$

Table 1 - Converged Nusselt numbers

S/H = 0.1 and Pr = 0.708

	Gr = 10^2		Gr = 10^4	
Kettleborough [1]	4.750	0.328	5.500	4.760
Nakamura [2]	0.479	0.459	3.628	3.754
Present work	0.689	0.689	3.860	3.860

Fig. 2 shows the velocity profile for $Ra(S/H) = 10^5$, where Ra is defined using S as the characteristic dimension and $(T_w - T_0)$, where T_0 is the inlet temperature, as the temperature difference. It can be seen that the velocity profile obtained using the inlet pressure equal to zero does not shown the flow recirculation at the channel exit as well as exhibit a completely different mass flow rate, when compared with the solutions obtained using the pressure calculated from Bernoulli's equation and with the solution from [3]. The Nusselt number, on the other hand, is insensitive to the inlet pressure boundary condition and to the flow recirculation at the channel outlet, as can be seen in Fig. 3. As an additional result Fig. 4 shows the penetration depth of the recirculation at the outflow boundary. When comparing the penetration depth with the experimental results of [3] the agreement is excellent.

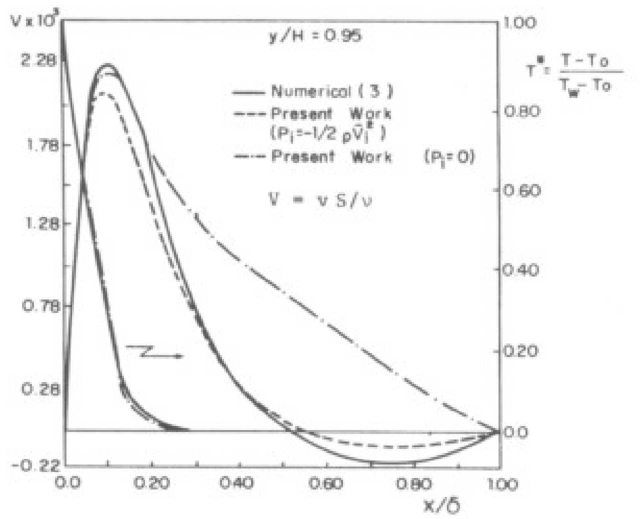

Fig. 2 - Velocity and temperature profiles

In Fig. 2 it is also plotted the temperature profiles , where it is observed that the temperature gradient is exactly the same for both inlet pressure boundary conditions. This clearly demonstrates that there are two distinct regions in the flow. In the region close to the wall the flow is dominated by the buoyance forces and hence the flow is not affected by the pressure boundary condition. In the remaining portion of the channel cross-section the flow is very sensitive to the applied pressure boundary condition. When pressure equal to zero is applied the mass flow in this region is higher than that sustained by the natural convection flow. Table 2 shows the Nusselt number and the ratio between the mass flow, as a function of the Rayleigh number, for different pressure boundary conditions at the inlet (condition 1 is for pressure equal to zero and condition 2 for pressure from Bernoulli's equation).

Table 2 - Nusselt number and mass flow ratio

S/H=0.0437 and Pr=5.0

$(S/H)*Ra_S$	10^3	5.10^3	10^4	5.10^4
Nus_1	3.645	5.449	6.473	9.964
Nus_2	3.595	5.342	6.332	9.789
$\dot{m}_1/\dot{m}_2$	1.063	1.156	1.245	1.697

As a final result Table 3 depicts the comparison between the convergence rate using the methodology developed in this work and the procedure described in [4]. It is demonstrated that the CPU effort is dramatically reduced when the velocity at the entrance in corrected using the procedure described here. The pressure boundary condition used for both methodologies was obtained from Bernoulli's equation.

Table 3 - Units of CPU effort

$(S/H)*Ra_S$	Methodoloy [4]	Present work
10^3	2.97	1.35
5.10^3	2.42	1.42
10^4	2.54	1.08
5.10^4	2.55	1.71

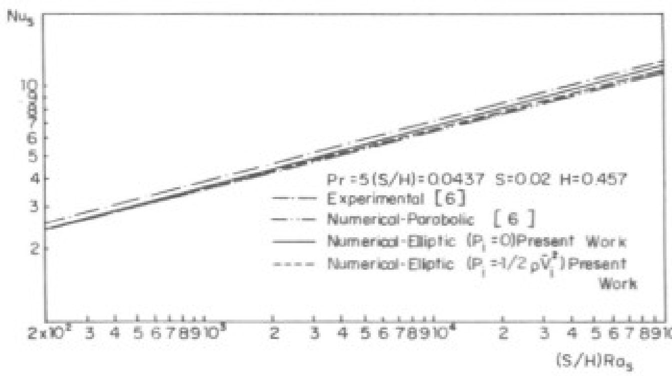

Fig. 3 - Nusselt number using different boundary conditions for P.

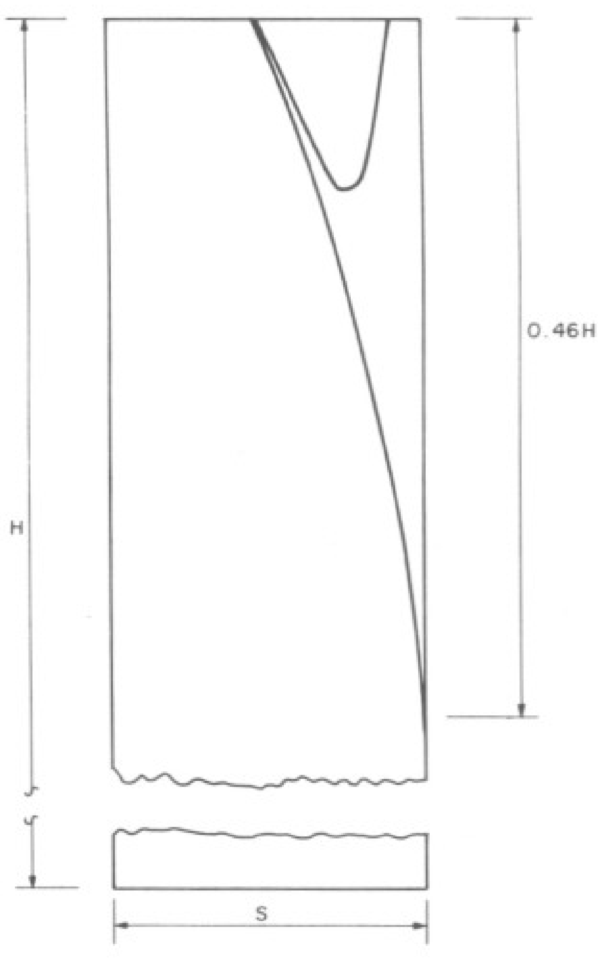

Fig.4 - Depth of the recirculation

CONCLUDING REMARKS

This paper reports a new method for calculating the velocity at the channel entrance, necessary for the boundary conditions application when the elliptic equations are solved. It was demonstrated that the inappropriate boundary condition for pressure at the inlet gives rise to a wrong mass flow rate, despite the same Nusselt number encountered, and precludes the observation of the flow recirculation at the channel outlet. The approach advanced for computing the velocity at the inlet renders to the method good convergence characteristics.

REFERENCES

[1] Kettleborough, C. F., Transient Laminar Free Convection Between Heated Vertical Plates Including Entrance Effetcts, Int. J. Heat and Mass Transfer, Vol. 15, pp.883-896, 1972.

[2] Nakamura, H. et al , Heat Transfer by Free Convection Between Two Parellel Flat Plates, Num. Heat Transfer, Vol. 5, pp.95-106, 1982.

[3] Nieckle, A. and Azevedo, L.F.A., Reverse Flow in One-Sided Heated Vertical Channel in Natural Convection, ASME Winter Annual Meeting, HTD Vol. 82, Boston, 1987.

[4] Lage, J.L. and Mendes, P.R.S., Laminar Forced Convection Between Parallel Flat Plates, I Simpósio Brasileiro de Transferência de Calor e Massa, pp.22-34, July, 1987.

[5] Maliska, C.R. and Raithby, G.D., Calculating Three Dimensional Flows Using Nonorthogonal Grids, Num. Meth. in Laminar and Turbulent Flow, pp. 656-666, Pineridge Press, 1983.

[6] Sparrow, E.M., Chrysler, G.M., and Azevedo, L.F., Observed Flow Reversals and Measured-Predicted Nusselt Numbers for Natural Convection in a One-Sided Heated Vertical Channel, Journal of Heat Transfer, Vol. 106, pp. 325-332, 1984.