2º CONGRESSO BRASILEIRO DE P&D EM PETRÓLEO & GÁS

STUDY OF THE TRANSMISSIBILITIES BY PHYSICAL SIMILARITY WITH HEAT TRANSFER PROBLEMS

Rafael Mendes, Rodrigo F. A. F. da Silva, Jonas Cordazzo,
Antonio F. Carvalho da Silva, Clovis R. Maliska

SINMEC – Computational Fluid Dynamics Laboratory
Mechanical Engineering Department – Federal University of Santa Catarina – UFSC
88040-900 – Florianópolis – SC – Brazil
www.sinmec.ufsc.br

Resumo – Este trabalho apresenta um estudo das transmissibilidades, considerando apenas a influência da geometria do reservatório de petróleo, através da similaridade física existente entre os fenômenos de transferência de massa em meios porosos e a transferência de calor. Inicialmente, a partir das equações de transporte, a transmissibilidade é definida, sendo na seqüência, apresentada a analogia entre as equações de transferência de calor e massa. Um problema bidimensional de transferência de calor, com solução analítica conhecida, é resolvido utilizando-se quatro modelos diferentes de determinação de transmissibilidades. Pela comparação dos resultados, percebe-se que o modelo atualmente utilizado pelos simuladores comerciais, embora seja exato para problemas unidimensionais, não é o mais apropriado para problemas bidimensionais. Nestes casos recomenda-se a utilização do método que interpreta a malha como sendo a de Voronói.

Palavras-Chave: transmissibilidade; simulação de reservatório; refino localizado

Abstract – This work analyses the transmissibility calculation, considering taking into account only the geometric influences, using the physical similarity between the mass transfer in porous media and heat transfer in solids. Initially, the transmissibility is defined from the transport equations. In the sequence, the analogy between the heat and mass transfer equations is presented. Finally, a two-dimensional heat transfer problem, that has analytical solution, is solved by four different models for the transmissibility. In this paper it is shown that the conventional model used in the commercial simulators, even though it is exact for one-dimensional problems, is not the most recommended for two-dimensional problems. For these cases, the best results occur when the simulator interprets the grid as a Voronoi one.

Keywords: transmissibility, reservoir simulation, local refinement

1. Introduction

The Cartesian is the grid type most commonly used in petroleum reservoir simulation. Even though, using variable grid spacing, it does not supply a good description of some domain areas. In order to improve the domain representation, saving computer time, only some small parts (blocks) of the grid can be refined. This is called "local grid refinement" (Heinemann and Brand, 1989).

Generally, those grids are constructed such that modifications of the discretization formulas are needed only at the interfaces between fine and coarse blocks.

This paper will discuss some models used to represent these blocks connections. The approach is done by the similarity with heat transfer problems. Initially, the concept of transmissibility and the correspondence between mass and heat transfer problems are presented. After, four models for the transmissibility calculation are compared and some concluding remarks are made.

2. Transmissibility Definition

In mass transfer problems in petroleum engineering the equation for one of the components in a multiphase flow is given by

$$\nabla \cdot (k\lambda \, \nabla P) = \frac{\partial}{\partial t}\left(\varphi \frac{S}{B}\right) + \; \overline{q} \tag{1}$$

where P is the pressure, λ is the mobility, ϕ is the porosity, S is the saturation, B is the formation volume factor and $\overline{q}$ is the flow rate per unit of reservoir volume, at reservoir conditions. When the Equation 1 is integrated in a control volume, the term in the left side becomes a sum of mass fluxes through the surfaces of this volume.

Most of the reservoir simulators use two-point flux approximation schemes. The mass flux of a component between two adjacent grid-blocks i and j in the discrete solution of the transport equations is given by (Heinemann and Brand, 1989)

$$Q_{ij} = \sum_{p=1}^{n} (\lambda_p k)_{ij} \frac{A_{ij}}{h_{ij}} \left(P_j - P_i\right)_p \tag{2}$$

where n is the number of phases; k is the absolute permeability, A_{ij} and h_{ij} are, respectively, an area where the mass flows and an adequate length for the gradient determination in the surface. In Equation 2, the terms independent of pressure and saturation can be grouped in the form

$$Q_{ij} = T_{ij} \sum_{p=1}^{n} (\lambda_p)_{ij} \left(P_j - P_i\right)_p \tag{3}$$

where T_{ij} is called transmissibility which is, therefore, defined as $T_{ij} = k_{ij} \; A_{ij} / h_{ij}$.

Transmissibility depends on block geometry and permeability, and its inverse is called resistivity. Using the resistivity concept, the Ohm's Law can be applied to calculate the total resistance, and in consequence the transmissibility, between connected elements.

3. Correspondence between Mass and Heat Transfer Governing Equations

The conservation equation of energy is given by

$$\nabla \cdot (k \cdot \nabla T) = \frac{\partial}{\partial t}\left(\rho c_p T\right) + S \tag{4}$$

where k represents the thermal conductivity, S is the source term and is related to a possible energy generation, ρ is the density and c_p is the specific heat. Integrating Equation 4 in a control volume, and after the application of divergence theorem, this equation yields:

$$\sum_{i=\text{interfaces}} U_{Pi} \left(T_P - T_i\right) = \frac{\partial}{\partial t}\left(\rho c_p T\right)\Delta V + S \cdot \Delta V \tag{5}$$

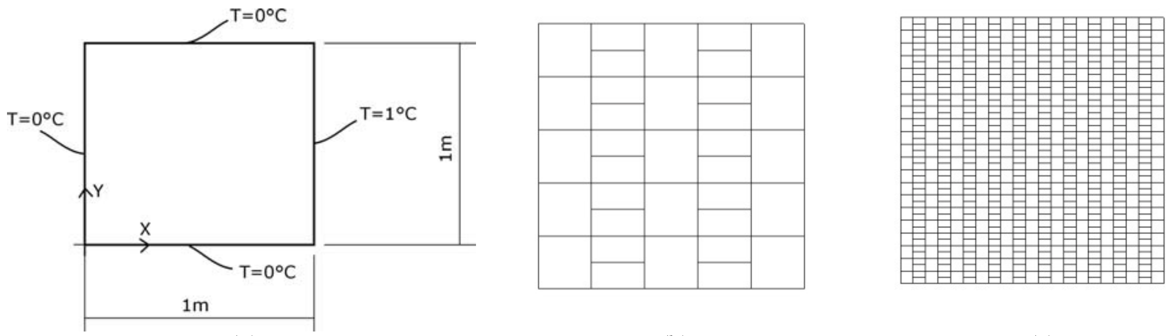
Here, we can establish an analogy between the conductance (*U*) of heat transfer problems and the transmissibility (*T*) of transport problems in porous media. Both depend only on geometric and media properties.

The great difficult to solve the problems is the exact determination of the conductance *U* between two blocks. If *U* is deduced directly from differential equation in a conservative form, there are no difficulties since the determination, in this way, is done directly from the approximate equation (Cordazzo et al. 2002).

For one-dimensional problems the concept used in most commercial softwares suffices. However, for 2D situations the transmissibility calculated using only two-grid points leads to approximation errors of the fluxes. An example of a 2D problem is discussed in the next section.

4. Transmissibility Calculation in 2D a Problem

In this section we investigate different models to calculate transmissibilities in a 2D problem. A heat transfer problem that has analytical solution is used for this purpose. This problem is depicted in Figure 1.



(a) *(b)* *(c)*

Figure 1. *(a)* 2D heat transfer problem and grids with local refinement: *(b)* with 5x5 and *(c)* with 21x21 volumes.

This problem has an analytical solution which permits an accurate evaluation of the different methods of transmissibility calculation. The analytical solution, obtained by the method of separation of variables is given by (Incropera et al. 1990)

$$T(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\,sen(n\pi(1-y))\frac{senh(n\pi x)}{senh(n\pi)} \tag{6}$$

For the numerical study the domain was discretized using Cartesian grids with local refinement, such as shown in Figure 1*(b)* and 1*(c)*. This type of local refinement originates the condition where a control volume face has contact with other two volumes. So, there are different ways to calculate the transmissibility which will be investigated through the use of a 5x5 coarse grid, Figure 1*(b)*, a fine (21x21) grid, Figure 1*(c)*. The columns are locally refined with a multiplier factor 2.

We will investigate four different models in order to determine the transmissibilities. All models can be interpreted as an electrical resistance model. However, we identify the fourth model as the electrical resistance model because it calculates one resistance for each control volume unlike the other models, which calculate only one resistance for two control volumes. An application using C++ programming language was developed in order to solve the numerical problem.

4.1 Model 2D-1: Using Sammon's equation

This model, presented by Sammon (2000) and already used by Hegre et al. (1986), uses the dashed area and the length L shown in Figure 2. It is the scheme used in the commercial simulators that calculate the flux by two points only.

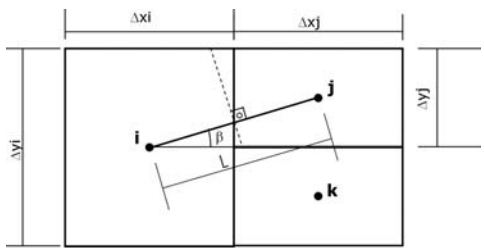


Figure 2. Dimensions used in model 2D-1

Both the length *L* and the transversal area *A,* represented by the dashed line in Figure 2, are given by

$$L = \left(\frac{\Delta x_i}{2} + \frac{\Delta x_j}{2} \right) \frac{1}{\cos \beta} \quad \text{and} \quad A = \frac{\Delta y_j}{\cos \beta} \tag{7}$$

Supposing a uniformly spaced grid ($\Delta x_i = \Delta y_j$), the conductance expression yield:

$$U_{ij} = k \cdot \frac{\Delta y_j}{\Delta x_i} \quad \text{and} \quad U_{jk} = k \cdot \frac{\Delta x_j}{\Delta y_j} \cos^2 0^o = k \cdot \frac{\Delta x_j}{\Delta y_j} \tag{8}$$

4.2 Model 2D-2: Using the correction factor cos² β

The angle β is defined here as the angle formed between the line that joins the centers of neighbor volumes and the horizontal line, as shown in Figure 3.

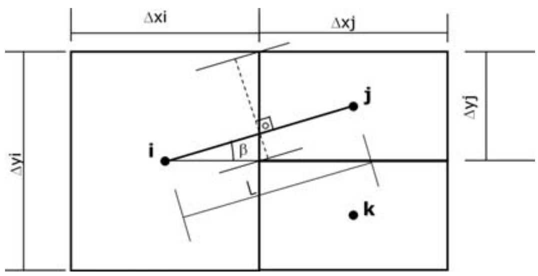


Figure 3. Dimensions used in model 2D-2

In this model, the length *L* is the same of the model 2D-1, and the transversal area, represented by the dashed line in Figure 3, is given by

$$A = \Delta y_j \cos \beta \tag{9}$$

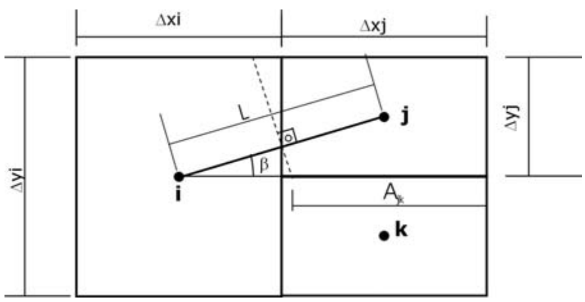
Due to the fact that the grid is uniformly spaced, we have, $\Delta x_i = \Delta y_j$ and therefore the conductance between the volumes i and j, U_{ij}, is given by

$$U_{ij} = k \cdot \frac{\Delta y_j}{\Delta x_i} \cos^2 \beta \tag{10}$$

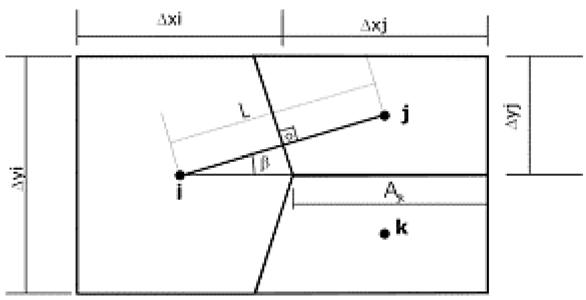
For the volumes *j* and *k*, the conductance is the same of the Model 2D-1.

4.3 Model 2D-3: Using Voronoi Grids

In the Voronoi grid (Maliska, 1995), the transversal area used is located in the middle point of the cells' centre-to-centre line. For this grid, the conductance between the volumes *i* and *j* is the same that was calculated by model 2D-1 since the areas utilized in these models are identical, as shown in Figure 4. The construction is done in such a way that the line joining two grid-points is normal to the control volume's surface.



(a)



(b)

Figure 4. *(a)* Dimensions used in model 2D-3 and *(b)* the effective grid where the conductace are determined.

The only, but important difference, between this model and the model 2D-1 is the conductance between the j and k, where the area utilized is A_{jk} in Figure 4*(a)*, which is less than the area utilized in previous models, and is given by

$$A_{jk} = \Delta x_j \left(1 - \frac{tg^2 \beta}{2}\right) \tag{11}$$

Therefore, the conductance values for this case are

$$U_{ij} = k \cdot \frac{\Delta y_j}{\Delta x_i} \quad \text{and} \quad U_{jk} = k \cdot \frac{\Delta x_j}{\Delta y_j}\left(1 - \frac{tg^2 \beta}{2}\right) \tag{12}$$

The modification of the area between the volumes j and k resulted in a modification of the grid as shown in Figure 4*(b)*, which became a Voronoi grid. It is important to notice that the alteration of the normal flux surface for conductivities U_{ij} and U_{jk} identifies the two-dimensional character of the problem.

4.4 Model 2D-4: Using the electrical resistance

In this model the resistances are calculated for each volume and the total resistance is the sum of these resistances in series. The areas are the same for two volumes, and the lengths go from the centre of the volume to the center of the contact surface.

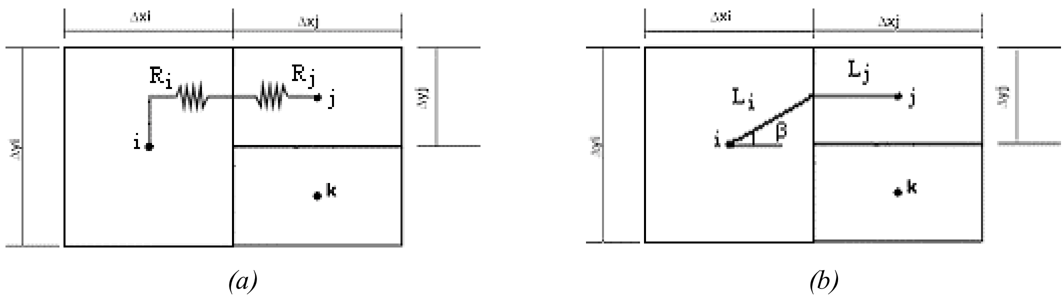


Figure 5. *(a)* Model 2D-4, based on electrical resistances, and *(b)* its dimensions.

Therefore, the equivalent resistance R_{ij} is given by

$$R_{ij} = R_i + R_j = \frac{\Delta x_i}{2 \cdot k \cdot \Delta y_i} + \frac{\Delta x_j / \cos(\beta)}{2 \cdot k \cdot \Delta y_i} \tag{13}$$

and the conductance between the volumes i and j, U_{ij}, yields

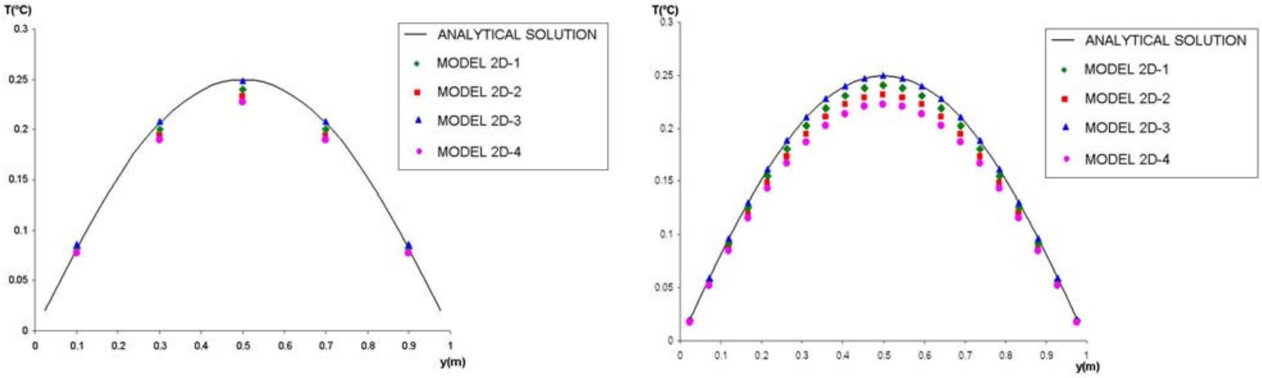
$$U_{ij} = \frac{1}{R_{ij}} = \frac{2 \cdot k \cdot \Delta y_i}{\Delta x_i + \Delta x_j / \cos(\beta)} \tag{14}$$

The conductance between the volumes j and k is the same as Model 2D-1.

4.5 Comparison between the models

Following, we present the results of the heat transfer problem defined in Figure 1*(a)* for the four different models described above. The differences verified in the coarse and fine grids depicted in Figure 1*(b)* and 1*(c)* are also shown.

In Figure 6 are plotted the results for the volumes contained in the vertical line located in x = 0.5. It is compared the solution of four models with the analytical solution. Note that the Voronoi model (Model 2D-3) shows the best solution, while the largest errors are found through the resistance model (Model 2D-4). Note also that the grid refinement did not contribute to increase the accuracy of the results.



(a) *(b)*

Figure 6. Comparison between the temperatures obtained through different models and the analytical solution for (*a)* coarse and *(b)* fine grid

5. Conclusion

In this paper it was discussed some aspects related to the transmissibility calculation between grid blocks with and without local grid refinement. The transmissibility concept was presented and its correspondence with the conductance in heat transfer problems was established.

Four transmissibility calculation models were analyzed using a two-dimensional heat transfer problem. Comparisons between the models results in fine and coarse grids and the analytical solution of the problem were made.

According to the results, the model most commonly used in commercial softwares, even though it is exact for one-dimensional problems, carries approximation errors, not vanishing when the grid is refined. The error arises because only two grid points are used in a situation which does not show local orthogonality. For these cases, the best results occur when the grid is a Voronoi one. In this case there is local orthogonality, a feature of a Voronoi grid, and therefore, two-grid points approximation for transmissibility can be used.

The methodology presented in this paper can be extended to grids with other local refinement patterns not considered here. Besides that other problems with different boundary conditions can be tested, improving our understanding.

6. References

CORDAZZO, J.. PhD Qualifying, in Portuguese, Department of Mechanical Engineering, Federal University of Santa Catarina, Florianopolis, SC, Brazil, 2002.

CORDAZZO, J., MALISKA, C. R., da SILVA, A. F. C. Interblock Transmissibility Calculation Analisys for Petroleum Reservir Simulation. 2nd Meeting on Reservoir Simulation, Buenos Aires, November, 2002.

HEGRE, T. M., DALEN, V. & HENRIQUEZ, A., Generalizes Transmissibilities for Distorted Grids in Reservoir Simulation. 61st Annual Tech. Conf. Exh. of SPE, 1986.

HEINEMANN, Z. E., BRAND, C. W.. Gridding Techniques in Reservoir Simulation. 1st/2nd Stanford Univ. & Leoben Mining Univ. Reservoir Simulation Inf. Forum, pp. 339-425 (Alpbach, Austria, Sept/1998-Sept/1989).

INCROPERA, F. P., DE WITT, D. P., Fundamentals of Heat and Mass Transfer, 3th edition, John Wiley & Sons, Inc., 1990.

MALISKA, C.R.. *Heat Transfer and Computational Fluid Dynamics – Fundamentals and Boundary Fitted*, 1st edition, in Portuguese, Livros Técnicos e Científicos Editora, 1995.

SAMMON, P. H., Calculation of Convective and Dispersive Flows for Complex Corner Point Grids, SPE 62929, Computer Modelling Group, Ltd., 2000.