Numerical Modeling of Annular Two-phase Flows in Vertical Ducts

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ABSTRACT

This work presents a new solution algorithm for predicting hydrodynamic parameters for developing and equilibrium annular, two-phase flow. The mass and momentum transport differential equations for both the core and the liquid film are solved across their entire domains, including the wall film. The axial diffusive fluxes are neglected in the conservation equations and the resulting parabolic equations are solved, capturing velocity and shear stress distributions from the tube center to the wall, together with the average film thickness and the pressure gradient, making no use of empirical closure relations nor assuming any known velocity profile to solve the so called triangular relationship within the liquid film. The parabolic approach allows the resolution of the velocity field within the liquid film, with fair computational effort even for very thin films. Mass and momentum conservation equations were discretized using the Finite Volume Method and an iterative procedure is proposed to solve all flow variables for given phase superficial velocities. In addition, a differential turbulence model is proposed and implemented in the context of this algorithm, based on the low-Reynolds k-ε model, specifying interfacial conditions for k and ε to account for the effects of the interface in the turbulence structure.

INTRODUCTION

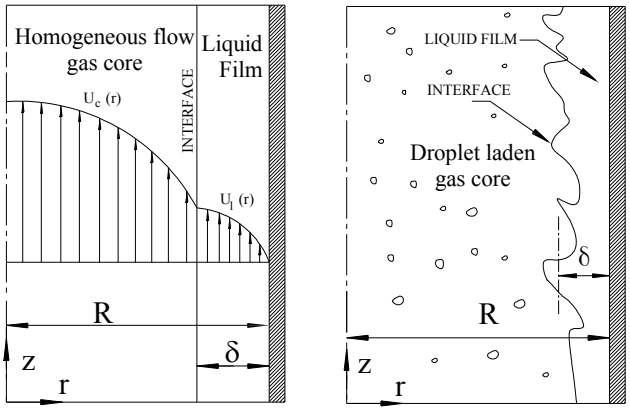
Two-phase gas-liquid annular flow is one of the most common patterns encountered in internal flows in nuclear and oil industries processes. This pattern occurs at moderate to high gas superficial velocities and it is characterized by the existence of a liquid film adjacent to the wall and a gas core flowing in the center of the duct. A wavy interface exists between both phases and its morphology depends on the gas and liquid mass flow rates.

Most common approaches for gas-liquid turbulent flow assume that the velocity profile within the liquid film is obtained from logarithmic wall law relations, similar to those for single phase turbulent flow. Some researchers ([3]; [4]) presented more sophisticated models considering the liquid film as to be divided into two sub-layers: a continuous liquid layer next to the wall which responds to the law-of-wall, and a wavy one next to the liquid-gas interface which makes use of a correlation for friction factor in rough tubes to obtain the interfacial shear stress.

Unlike previous approaches, the present model makes no use of empirical closure correlations to relate shear stresses, pressure gradient and film thickness nor assumes any velocity profile for the liquid film. It provides an accurate, simple and complete numerical computation of all the hydrodynamic parameters, requiring just gas and liquid mass flow rates as input data to solve mass and momentum conservation equations. As the modeling is based on the solution of conservation equations with its boundary conditions and interfacial constraints, its application is broad, allowing for instance, the solution of developing annular flow for non-equilibrium case. Its extension for heat transfer computations is also straightforward. The success of the algorithm relies on two main features. First, the coupled solution of the liquid film and gas core velocity fields, which inherently satisfies the continuity of the velocity and shear stress fields at the interface, and, second, the pressure gradient calculation through an iterative procedure based on the fulfillment of the global mass conservation of the gas core. Then, no use is made of the global force balance that relates pressure gradient and wall shear stress. Instead, this balance is used in the model validation stage for verification of the global force equilibrium in fully developed conditions.

MODEL DESCRIPTION

The proposed model divides the flow domain in two distinct regions shown in Figure 1, the liquid film (*f*) and the gas core (*c*); the former is treated as a continuous, fully turbulent layer having no entrained bubbles, with a mean film thickness δ. The mean film thickness represents the time averaged value for a given axial position. The wavy nature of the interface and its effect on the interfacial shear stress is taken into account through the turbulence model as will be described later. In addition, as stated before, one of the main hypotheses considered is that the axial diffusive fluxes can be neglected in both, gas core and liquid film. Thus, the resulting approximate equations can be solved through a marching process along the axial coordinate of the duct. For axis-symmetric flow, as it is the case, this means that, instead of solving a 2D model with r-z coordinates, a sequence of 1D profiles along radial direction is solved, marching along z direction. In terms of CPU requirements this approach is much less demanding allowing for the use of fine enough meshes in the liquid film to fully resolve velocity and shear stress profiles within it, even for very thin films.



Model Representation FlowSquematic Representation

Figure 1: Model parameters

The gas core is considered to have entrained liquid droplets, which are assumed to displace at the same velocity of the gas phase. Govan's correlation ([1]) was used to compute the entrained fraction. A systematic comparison among the main entrainment correlations was developed in [2] and this correlation was found to perform better, at least, for the conditions range for which the model was validated (see [3]). Having the entrained fraction, the gas core properties are calculated through the homogeneous flow equations.

The radial velocities are neglected within the liquid film even for the developing flow. Using the continuity equation, it can be shown that the radial velocity within the liquid film is of order of U_I times $d\delta/dx$, where $d\delta/dx$ is known to be of order of 1e-5. Then, radial and axial velocities are computed in the gas core and only axial velocities in the liquid film. This does not mean that velocities within the liquid film are invariant along the axial direction, as these are coupled though the interface with gas core and, additionally, the film thickness varies along the axial direction. In addition, it is considered that pressure gradient is constant along the transverse area of the duct and shared by liquid film and gas core.

In summary, modeling is carried out under the following assumptions:

1. Steady-state, co-current, vertical, annular flow.
2. Axis-symmetric parabolic flow (axial diffusive fluxes are neglected)
3. The radial velocity is neglected within the liquid film
4. There is no slip between the gas and the entrained liquid droplets.
5. Fluid properties are assumed to be constant, including gas density, i.e., all phases are considered incompressible.
6. Pressure is only function of the axial coordinate of the duct

Governing Equations

Under assumptions discussed in the precedent section, the time-averaged linear momentum equations for the flow in the gas core and the liquid film are given by,

$$\frac{\partial}{\partial z}\left(\rho_c u_c u_c\right)+\frac{\partial}{\partial r}\left(\rho_c v_c u_c\right)= -\frac{dp}{dz}+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu_c^{eff}\, r\, \frac{\partial u_c}{\partial r}\right)-\rho_c g \qquad 0<r<R-\delta \qquad (1)$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(\mu_f^{eff}\, r\, \frac{\partial u_f}{\partial r}\right)=-\frac{dp}{dz}-\rho_l g \qquad R-\delta<r<R \qquad (2)$$

where, μ_f^{eff} and μ_c^{eff} are the liquid film and gas core effective viscosities, respectively, u_f and u_c are the liquid film and gas core velocities and dp/dz is the pressure gradient which is shared by both phases and is considered to be constant across the duct cross-sectional area. The differential mass conservation equations for the gas core is given by,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho_c v_c r\right)+\frac{\partial}{\partial z}\left(\rho_c u_c\right)=0 \qquad 0<r<R-\delta \qquad (3)$$

Under the assumption of parabolic flow, the radial velocity component in the gas core is calculated explicitly from the mass conservation equation (Eq. (3)).

The boundary conditions for these equations are,

$$\frac{\partial u_c}{\partial r}=\frac{\partial u_f}{\partial r}=0 \quad ;\ v=0 \quad at \quad r=0 \qquad (4)$$

$$u=v=0 \quad at \quad r=R \qquad (5)$$

The continuity of velocities and shear stresses at interface are respectively given by Eqs. (6) and (7). Once the radial velocities are neglected in the liquid film, this component is considered to be zero at the interface for the gas core,

$$u_c(R-\delta)=u_f(R-\delta)=u_I \ ;\ \ v_f(R-\delta)=v_c(R-\delta)=0 \qquad (6)$$

$$\tau_c\big|_{R-\delta}=\tau_f\big|_{R-\delta}=\tau_I \qquad (7)$$

In these equations, τ_l and τ_c are the liquid film and gas core shear stresses, respectively, and u_I and τ_I are the interfacial velocity and shear stress.

The mass flows within gas core and liquid film are calculated by the integration of the velocity profiles as,

$$\dot{m}_c=2\pi\rho_c\int_0^{R-\delta} u_c(r)\ r\,dr \qquad (8)$$

$$\dot{m}_f=2\pi\rho_f\int_{R-\delta}^{R} u_f(r)\ r\,dr \qquad (9)$$

where ρ_l and ρ_c are the liquid film and gas core densities, respectively. These relations represent the global mass conservation for the gas core and liquid film and are used in the solution algorithm, together with momentum conservation, for pressure gradient and film thickness calculation.

Numerical Solution

The momentum equation for each region is integrated in its corresponding domain using the Finite Volume technique. An independent mesh with a fixed number of volumes for each region allows refinement within the liquid film. The gas-liquid interface is positioned between the last gas-core volume (*Nc*) and the first liquid film volume (*Nc+1*), as shown in Fig. 2 .

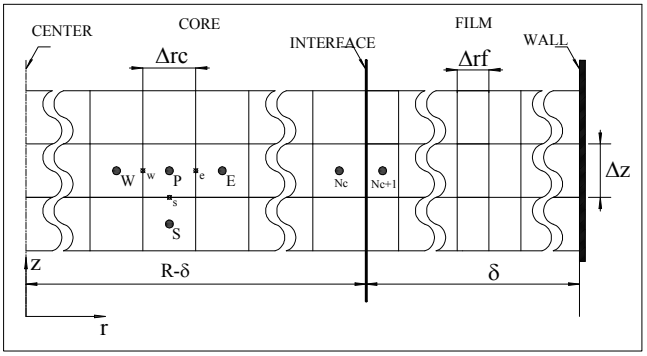


Figure 2: Control volumes for finite volume integration

Along the iterative procedure, the film thickness varies each time it is corrected, so Δr_l and Δr_c are adjusted since the number of volumes within liquid film and gas core remains constant along the calculations.

Integration of Eq. (1) in volume P (Fig. 2) gives,

$$r_P \Delta r \left(\rho u_c u_c \big|_n - \rho u_c u_c \big|_s \right) + \Delta z \left(\rho v_c u_c r \big|_e - \rho v_c u_c r \big|_w \right) = r_e \underbrace{\mu_c^{eff} \left. \frac{\partial u_c}{\partial r} \right|_e}_{\tau_e} - r_w \underbrace{\mu_c^{eff} \left. \frac{\partial u_c}{\partial r} \right|_w}_{\tau_w} + \left(-\frac{dp}{dz} - \rho g \right) \frac{(r_e^2 - r_w^2)}{2} \quad (10)$$

where r_e and r_w are the radial positions of volume faces.

A central differencing scheme (CDS) for interpolation of the velocity gradients which appear in the viscous terms and upwind differencing scheme (UDS) for the convective terms was used. After collecting terms, this results in the following algebraic equation for the discrete momentum conservation within volume P,

$$A_p U_P = A_e U_E + A_w U_W + A_S U_S + B \quad (11)$$

The corresponding for the liquid film volumes can be obtained by straightforward analogy but considering that convective momentum transport in radial direction is zero. The use of upwind scheme in the axial momentum transport is justified once the axial diffusive fluxes have been neglected and only convection is present in the axial direction. On the other hand, in the radial direction both convection and diffusion are present, but as already stated, radial velocities are small and thus diffusion dominates the radial momentum transport. In addition, very fine meshes are used in the radial direction, and the use of first order scheme for the convective terms is also appropriate.

A key point of the numerical algorithm is the interfacial coupling which is embedded into the same equation system, i.e., the momentum equation for both, gas core and liquid film are solved in one linear equation system. To integrate Eqs. (1) and (2) within volumes contiguous to the interface, *Ng* and *Ng+1* in Figure 2, the continuity of momentum flux has to be ensured by finding a single expression to evaluate the interfacial shear stress in both volumes, as

$$\underbrace{\mu_c^{eff} \left. \frac{du_c}{dr} \right|_{R-\delta}}_{\tau_{e(Ng)}} = \underbrace{\mu_f^{eff} \left. \frac{du_f}{dr} \right|_{R-\delta}}_{\tau_{w(Ng+1)}} = \tau_I \quad (12)$$

To accomplish this, a procedure described in [4] for averaging non-uniform diffusion coefficient is used. An expression for an equivalent interfacial viscosity is derived out of the continuity of shear stress, resulting in,

$$\mu_I = \left(\frac{1 - f_I}{\mu_c^{eff}} + \frac{f_I}{\mu_f^{eff}} \right) \quad ; \quad f_I = \frac{\Delta r_f}{\Delta r_f + \Delta r_c} \quad (13)$$

Then, the shear stress is obtained as,

$$\tau_I = \mu_I \left. \frac{du}{dr} \right|_I = \mu_I \frac{u_{Ng+1} - u_{Ng}}{\Delta r_I} \quad (14)$$

where

$$\Delta r_I = \frac{1}{2} \left(\Delta r_c + \Delta r_f \right) \quad (15)$$

The effective viscosity, which is a function of the radial position, is stored at the volume faces in a staggered grid. This simplifies discretization since the viscosities are required at the volume interfaces to calculate momentum fluxes. So, the viscosities can be written as,

$$\mu^{eff} = \begin{cases} \mu_c^{eff} & 0 < r < R - \delta \\ \mu_f^{eff} & R - \delta < r < R \\ \dfrac{1 - f_I}{\mu_c^{eff}} + \dfrac{f_I}{\mu_f^{eff}} & R = \delta \end{cases} \quad (16)$$

where, μ_c^{eff} and μ_f^{eff} can be obtained by any turbulence model that is adequate for annular flow. In this paper a modification of a differential turbulence model is proposed, as described below. Nevertheless, it is important to emphasize that the solution algorithm hereby proposed is independent of the turbulence model and any turbulence model based on eddy viscosity concept can be used. In this work, an algebraic turbulence model proposed by [5] was also implemented into the solution algorithm for comparison. For the case of laminar flow without entrainment the viscosities will simply be the ones of the liquid and gas. In addition the algorithm can be used for the computation of liquid-liquid annular flows, as described in [2].

Velocity fields for liquid film and gas core, for a give axial position are solved in a single matrix equation simultaneously, as schematically shown in Eq. (17), satisfying the momentum equations and boundary conditions together with velocity and shear stress continuity at the interface, for given pressure gradient and film thickness. The resulting matrix has three non-zero diagonals and can be easily solved by the Tri-Diagonal Matrix Algorithm - TDMA.

$$\begin{bmatrix} \mathbf{A}_c & \ddots \\ \ddots & \mathbf{A}_f \end{bmatrix} \begin{bmatrix} \mathbf{U}_c \\ \mathbf{U}_f \end{bmatrix} = \begin{bmatrix} \mathbf{B}_c \\ \mathbf{B}_f \end{bmatrix} \quad (17)$$

Solution Algorithm

The flow chart shown in Figure 3 summarizes the solution procedure. The input data consist of mass flow rates of each phase, pressure gradient at the entrance and initial entrained fraction. Additionally, fluid properties, tube dimensions and numerical parameters such as convergence tolerance and mesh size, have to be specified.

For a given axial position, the pressure gradient and film thickness calculated in the previous section are used as initial guess. In the case that only the fully developed (equilibrium) condition is of interest, an initial guess for these parameters must be given. An iterative procedure is used to correct the pressure gradient and film thickness in order to satisfy momentum and global mass conservation equations together with the force balance at the interface. This procedure works as follows:

For a given film thickness the external loop starts calculating the position of the volume centers and faces for the whole domain, gas core and liquid film. Then, the fraction of liquid entrained and gas core properties are calculated using Govan's correlation and considering homogeneous flow within the gas core.

Having the pressure gradient (guessed or from the previous section), the internal loop first calculates the effective

viscosity for both, gas core and liquid film, in the case of turbulent flow, using the turbulence model described in the next section. Equation system (17) is solved to obtain the velocity profiles of liquid film and gas core Having the gas core velocities, the gas core mass flow rate is obtained using Eq. (8) and the pressure gradient is adjusted to satisfy the known gas core flow rate through a procedure similar to the one proposed by [6]. To correct the pressure gradient, this algorithm uses the error between the mass flow rate calculated from the velocities obtained from momentum equations and the given value. The updated value is again used in the momentum equation to obtain new velocities. This process is repeated until the total mass flow rate within the gas core is satisfied. The total mass flow of the gas core in each axial position is given by the gas mass flow and the entrained liquid droplets.

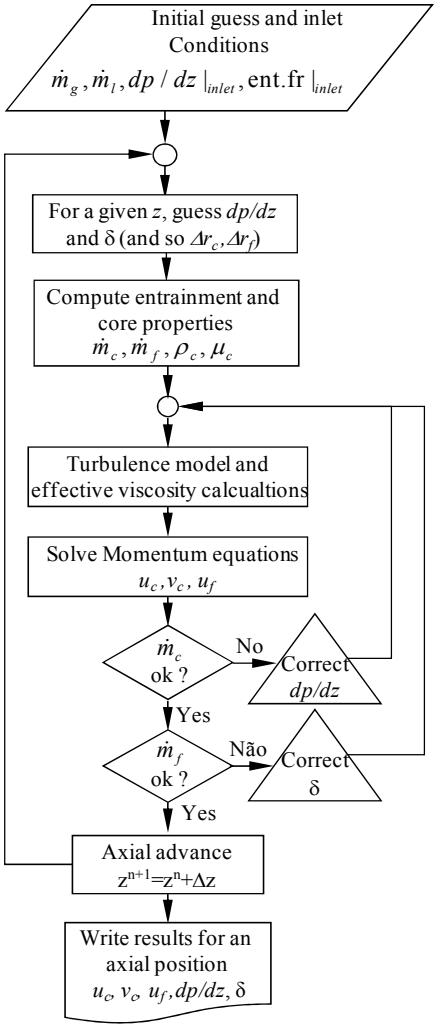


Figure 3: Solution Algorithm

Once the velocity fields which satisfy the pressure gradient for a given film thickness are calculated, the turbulence fields (k and ε) are updated. This step also requires iterations as turbulence variables depend upon velocity fields and vice-versa. After convergence of the internal loop, the velocity fields and pressure gradient satisfy momentum equation for a given film thickness.

The next step corrects the liquid film thickness to satisfy the mass flow rate of the liquid film using Eq. (9). As the number of volumes in each domain is constant, values of Δr_c and Δr_f change, as well as the positions of volume centers and faces. As the correction of δ affects the velocity field of both, gas core and liquid film, the algorithm re-enters in the internal loop, to find the pressure gradient that satisfies momentum equation for the new film thickness. This process is repeated until convergence of the whole system, obtaining velocity fields, film thickness and pressure gradient that satisfy momentum and mass conservation equations, together with interfacial velocity and shear stress continuity.

Although this algorithm has two correction loops, it quick converges for the cases used for validation, even starting from guessed values of pressure gradient and film thickness very far from the converged ones.

It is important to highlight that the coupled solution of liquid film and gas core velocity fields (Eq. (18)) satisfies the continuity of shear stress at the interface and, together with global mass conservation for the gas core (Eq. (9)), provide a pressure gradient dependent upon the gas core mass flow rate. In turn, the liquid film velocity field, which determines the wall shear stress, is also ruled by the pressure gradient, which is shared by phases (i.e., it is constant along the cross section). Therefore, the intimate relationship between the wall shear stress and pressure gradient is fulfilled without explicitly making use of any equation to relate them within the solution process. This is one of the key points of the algorithm because the triangular relationship between the liquid film mass flow rate, the wall shear stress and the film thickness is successfully solved making no use of empirical relations for wall or interfacial friction factor nor assuming any liquid film velocity profile.

Turbulence model

A differential turbulence model is proposed herein based on a modification of the low Re k-ε model, in order to take into account the effects of the interface. This model was implemented in the framework of the algorithm described in the precedent section.

As previously stated, usual differential approaches for annular flow solve the turbulent fields (usually k and ε) within the gas core and assume a known velocity profile within the liquid film, like the logarithmic or 1/7 profile ([7]; [8]).The shear stress at the interface or, alternatively, at the wall (most models assume that these are equal) is calculated through a correlation for a rough wall, or from a force balance, knowing, a priori, the pressure drop.

An algebraic turbulence model was presented in [5] considering constant eddy viscosity along the liquid film and a linear distribution along the gas core, using experimental values of wall shear stress and film thickness to calibrate the model. In a recent paper [9] the model was extended to other fluids and evaporating flows. Nevertheless, in any case, the applicability of the model will depend on the comprehensiveness of the data base used for calibration.

The objective of the present model is to compute all annular flow parameters, which include pressure drop, film thickness and interfacial shear stress, making no use of correlations or calibration data. Then, a low Reynolds k-ε model was used in both, liquid film and gas core to compute the eddy viscosity. The transport equations for turbulence variables, k and ε and wall damping functions are not presented here for shortness. Further details can be encountered in [10].

A special treatment is proposed for the specification of k and ε at the interface, based on previous models for stratified flows, to account for the discontinuity of these variables, in

virtue of the density and viscosity difference of phases. Assuming the turbulence damping at the interface due to surface tension is small, it is proposed that,

$$k_{I,f} = \frac{\tau_I}{\rho_f \sqrt{C_\mu}} \quad \text{and} \quad k_{I,c} = \frac{\tau_I}{\rho_c \sqrt{C_\mu}} \qquad (18)$$

Considering the characteristic mixing length at the interface of order of δ,

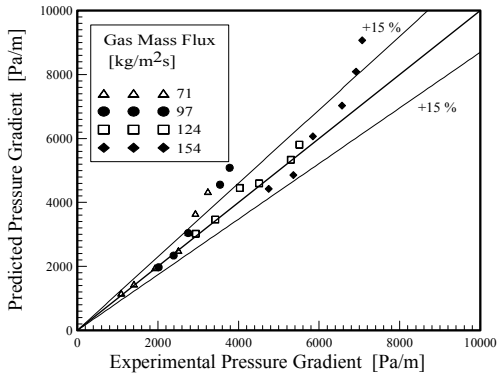
$$\varepsilon_{I,f} = \frac{k_{I,f}^{3/2}}{c_l \delta} \quad \text{and} \quad \varepsilon_{I,c} = \frac{k_{I,c}^{3/2}}{c_l \delta} \qquad (19)$$

A similar approach was proposed in [11] to compute interfacial values of k and ε for stratified flows, using the liquid height as the characteristic mixing length, instead of δ. The computation of ε at the interface given by equation (19) corresponds to the values obtained by models based on Prandtl mixing length (see, for instance, [12]). Different values have been proposed for mixing length proportionality constant, c_l. Here, the value of 0.41 was used (i.e., l_m=κ δ). The constant c_l is in fact an adjustable parameter, but also does C_μ and other turbulence model constants. This means that, although some adjustable parameters are used in differential turbulence models, these are much more physically meaningful than simply adjust the eddy viscosity or equivalent wall roughness to fit experimental values of pressure drop. Further details of the implementation of the differential turbulence model can be found in [10].

The algebraic model presented in by Cioncolini [5] was also implemented in the context of this algorithm, in order to validate the numerical solution and for results comparison with the proposed turbulence model.

RESULTS AND MODEL VALIDATION

The numerical algorithm hereby presented was already extensively validated against an analytical solution for laminar annular flow (see[10], [13]). Thus, the validation here will be focused on the application of the algorithm for turbulent annular flow. The model is validated with results for developing and equilibrium flow presented by Wolf et all. ([3]). Fist, the predicted pressure drop and film thickness for fully developed conditions for various gas and liquid mass fluxes are compared with experimental data. As already stated, Cioncollini's ([5]) algebraic turbulence model was also implemented in the context of the presented algorithm for comparison. Figure 4 presents a comparison of the predicted and experimental pressure gradient for developed flow, using the low Reynolds k-ε, with the proposed treatment at the interface and Cioncollini's algebraic model. In Figure 5 the predicted and experimental values for film thickness are compared.



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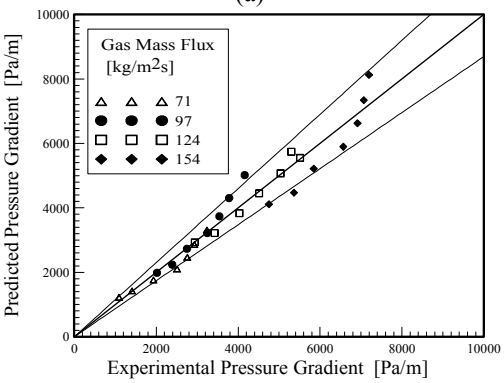

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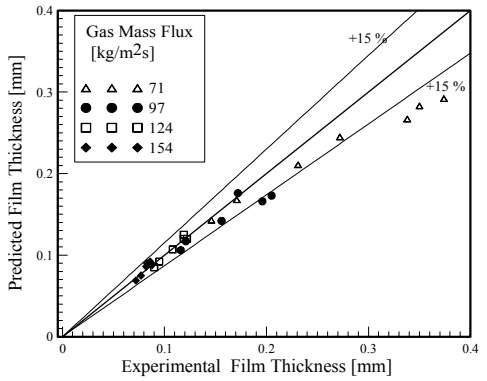
Figure 4: Predicted vs. Experimental pressure gradient. (a) k-ε model with interfacial treatment. (b) Cioncollini algebraic model



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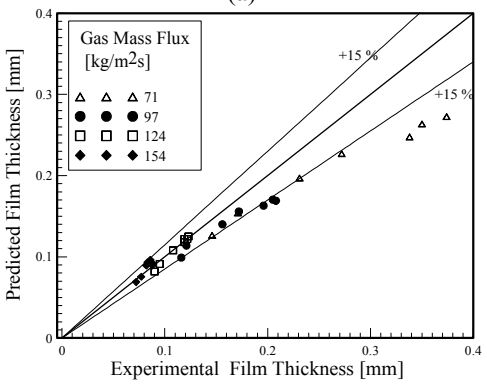

(b)

Figure 5: Predicted vs. Experimental liquid film thickness. (a) k-ε model with interfacial treatment. (b) Cioncollini algebraic model

Results using Cioncollini turbulence model better predicts pressure gradient and the differential model proposed in this work better predicts the film thickness. This could be due to the fact that turbulent viscosity in Cioncollini model eddy viscosity is fitted using experimental data for pressure gradient. On the other side, velocity profiles in Cioncollini

model within the liquid film are assumed to be linear, once the turbulent viscosity is assumed constant, while, in the model proposed herein, the resulting profile is similar to the more realistic log-law profile, which is used through Eq. (9) to predict the film thickness.

Figures 6 and 7 present the values of local pressure gradient, for developing annular flow, compared with Wolf ([3]) experimental data

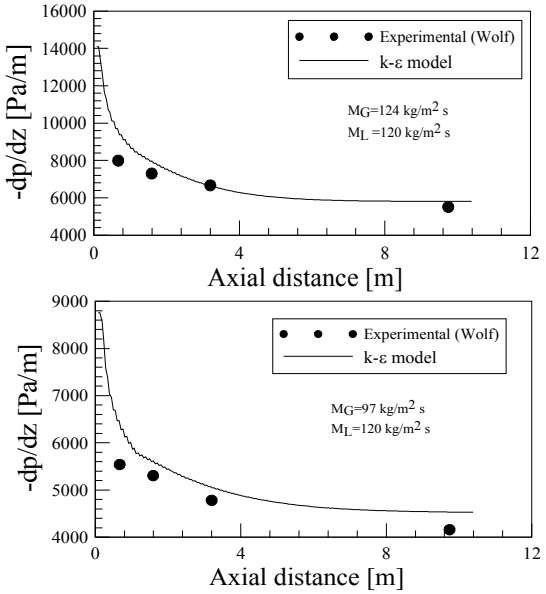

Figure 6: Evolution of the pressure gradient for two flow conditions

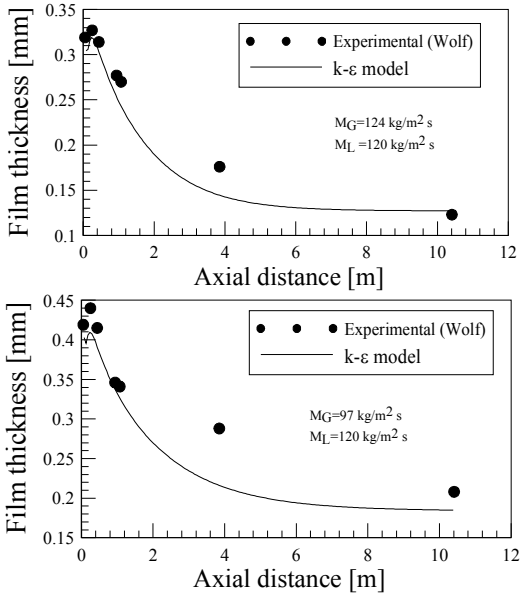

Figure 7: Evolution of the film thickness for two flow conditions.

SUMMARY AND CONCLUSIONS

A solution algorithm for the calculation of two-phase turbulent annular flow was successfully implemented. It solves the triangular relationship between the liquid film mass flow, wall shear stress and film thickness through an iterative procedure, taking advantage of the coupled solution of gas core and liquid film velocity profiles. In this way, no empirical closure correlation is used, and no velocity profile is assumed for the liquid film. Among the advantages of this algorithm, the solution of the velocity profile within the liquid film, instead of assuming a known shape profile, allows for the integration of differential turbulence models, which are able to incorporate the complex physics of near wall flow within the film, which could be different from single phase flows, from which the classical low-law profile is usually borrowed for annular flows.

A differential two-equation turbulence model was incorporated into the algorithm, based on a low-Re k-ε model, setting k and ε values at the interface, based on the continuity of turbulent stress at the interface and considering the damping of turbulent fluctuations due to density jump at the interface.

The model compares fairly well against experimental data and, although some parameters are better predicted by models based on experimental data fitting, as Cinoncolini's algebraic model ([5]), its applicability is much broader, once no global adjustable parameters are used, other than turbulence model coefficients.

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