A BOUNDARY FITTED METHOD FOR THE SOLUTION OF COMPRESSIBLE AND INCOMPRESSIBLE FLOWS USING SIMULTANEOUS VELOCITY AND DENSITY CORRECTIONS

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ABSTRACT

The available algorithms for the numerical solution of the Navier-Stokes equations are, in general, suitable for a specified flow regime. Methods which are designed to solve supersonic flows are built taking advantages of the flow characteristics, like the hyperbolic nature of the equations, making it not applicable, for example, for a low Mach number flow. The reversal is also true, that is, methodologies developed to solve low Mach number flows can not be used for the solution of supersonic flow problems. The key question behind this is the way the density and velocity are treated in the continuity equation. If mass conservation is enforced through density corrections only, as usual in the numerical algorithms for supersonic flows, the resulting density fields are meaningless, since only velocity field should be corrected in incompresible flows. In the other hand, if mass is enforced through velocity corrections only, of course, the methodology is not suitable for high Mach number flows.

It seems that for having numerical models with ability of solving problems in a wide range of Mach numbers that both, density

and velocity should be corrected in order to enforce mass conservation. The idea was employed in slight different manners in [1][2][3] but practically no efforts has been dedicated to improve the robustness of such a methods. In a recent work [4] the idea was again explored but, at this time, performing a complete analysis of the role of the velocity and density corrections in compreesible and incompressible flows. The method was employed to solve fluid flow problems, compressible and incompressible, in a Cartesian coordinate framework.

In this work the idea is used in a boundary-fitted coordinate framework in order to increase the applicability and generality of the model. In extending a numerical model from the Cartesian coordinates to a general curvilinear coordinates several important aspects need to be considered. One of them is the storage of the dependent variables in the computational cell. Due to the need of having both Cartesian components of the velocity vector calculated at the same point for performing mass balances there are many alternatives for this storage. In this research work the staggered grid is adopted where the u-component is stored at the west and east faces and the v-component at the south and north faces, while pressure, density and temperature are located in the center of the elemental control volume. The u and v components needed at places where they are not stored are obtained through average of the neighbouring velocities. This storage eliminates the need of solving for twice as many velocity points.

The coupling between pressure and velocity is handled using the SIMPLEC method and the resulting linear system of equations are solved employing the Modified Strongly Implicit procedure.

The method is applied in the solution of viscous supersonic flow past a cylinder for a wide range of Mach numbers, employing two different boundary fitted discretization. The results are in excellent agreement with the experimental results, wich encourages to devote further efforts to extend the methodology for axi-symmetric and 3-D flows. Studies are also carried out to test the incompressible limit of the methodology.

REFERENCES

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