NUMERICAL SIMULATION OF THE SUPERSONIC FLOW AROUND ARBITRARY SHAPES USING THE SHOCK-CAPTURING TECHNIQUE

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ABSTRACT

The three-dimensional supersonic flow around complex configurations like a launch vehicle is determined using a shock-capturing technique embodied in a finite-difference approach in generalized coordinates. The equations are written in conservation-law form and integrated from an initial data plane downstream over the body. Existing shock waves are captured automatically. Results are compared with experimental data to demonstrate the ability of the model to accurately predict the inviscid flows around a space vehicle.

INTRODUCTION

In the design of a supersonic launch vehicle there are many effects that are of utmost importance to be determined. Among them are the aerodynamic loads necessary for the prediction of the vehicle trajectory. The ability of computing numerically the load distribution around launch vehicles is of considerable importance, mainly because the high costs of wind-tunnel tests. Careful and well chosen wind tunel experiments, in conjunction with powerful numerical simulations are the less costly approach for designing a space vehicle. By its turn, the numerical solution of the three-dimensional flow field equations is not an easy task, with added complexity, when the solution domain is irregular.

In the present work a three-dimensional numerical model, using the second order non-centered finite-difference scheme of MacCormack in boundary-fitted coordinate, is employed to solve the inviscid flow field equations cast in conservation-law form. The shock-capturing technique is used which is capable of numerically predict the location and intensity of all predominant shock waves without the explicit use of any shock-fitting procedure.

GOVERNING FLOW EQUATIONS

The equations of motion can be written in conservation-law form, using vector notation in the Cartesian coordinate system as [1]

$$\frac{\partial \bar{E}}{\partial z} + \frac{\partial \bar{F}}{\partial x} + \frac{\partial \bar{G}}{\partial y} = 0 \qquad (1)$$

where $\bar{E}$, $\bar{F}$ and $\bar{G}$ are four-dimensional vectors defined by

$$\bar{E} = \begin{vmatrix} \rho u \\ kp + \rho u^2 \\ \rho uv \\ \rho uw \end{vmatrix} \quad ; \quad \bar{F} = \begin{vmatrix} \rho v \\ \rho uv \\ kp + \rho v^2 \\ \rho vw \end{vmatrix} \quad ;$$

$$\bar{G} = \begin{vmatrix} \rho w \\ \rho uw \\ \rho vw \\ kp + \rho w^2 \end{vmatrix} \qquad (2)$$

The Eq.(1), which represent the mass conservation equation and three momentum equations, comprise a complete set when coupled with the energy equation in the following form

$$p = (1 - q^2) \qquad (3)$$

where

$$q = \sqrt{u^2 + v^2 + w^2} \qquad (4)$$

Due to the irregular shape of the solution domain it is not convenient to use the conservation equations written in the Cartesian coordinate system, because of the boundary conditions application and code generality. A more general model can be obtained if the conservation equations are transformed to a new coordinate system, coincident with the calculation domain. The suitable transformation for the type of problem analized here is

$$\tau = z \quad ; \quad \xi = \xi(x,y,z) \quad ; \quad \eta = \eta(x,y,z) \qquad (5)$$

Fig.1 shows two cross-planes and the corresponding transformed domains for the transformation given by Eq.(5). It is seen that with this transformation any calculation domain laid out over blunt bodies, even axially non-symmetric, can be transformed onto a parallelepiped in the computational plane.

The conservation-law form of the equations of motion, Eq.(1), can be retained in the new coordinate system as

$$\frac{\partial E}{\partial \tau} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} = 0 \qquad (6)$$

where the new variables are defined as follows

$$F = (\bar{E}\xi_z + \bar{F}\xi_x + \bar{G}\xi_y) \;/\; \bar{J}$$

$$G = (\bar{E}\eta_z + \bar{F}\eta_x + \bar{G}\eta_y) \;/\; \bar{J} \qquad (7)$$

$$E = \bar{E}\,\tau_z \;/\; \bar{J}$$

The integration of Eq.(4) is performed with respect to τ, since the equation is hyperbolic with respect to that coordinate. The flow variables p, ρ , u, v and w are, therefore, determined from the components of the conservative variable E.

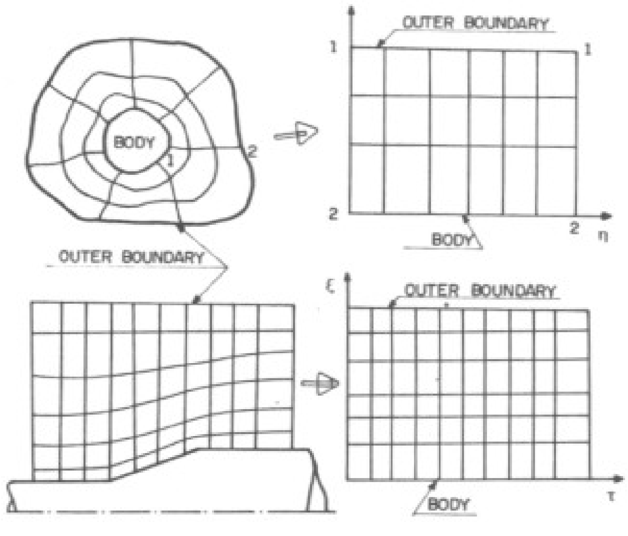

Fig. 1 - Cross-planes of the 3-D transformation

FINITE-DIFFERENCE TECHNIQUE AND SOLUTION PROCEDURE

Shock-capturing is one of the most widely used techniques for computing inviscid flow with shocks. The capability of an SCT to accurately predict the location and intensity of all shock waves, in addition to the continuous determination of the flow field, depend in part on the finite-difference scheme used. The shock waves predicted by these methods are indeed smeared over several mesh interval but the simplicity of this approach may outweigh the slight compromise in results compared to shock-fitting schemes.

The MacCormack second-order scheme [1],adopted in this work is

$$E^{(1)}_{i,j} = E^{n}_{i,j} - \alpha_1 \left\{ \Delta\tau \, (F^{n}_{i,j+1} - F^{n}_{i,j}) + \Delta\tau \, (G^{n}_{i+1,j} - G^{n}_{i,j}) \right\}$$

$$E^{n+1}_{i,j} = (E^{n}_{i,j} + E^{(1)}_{i,j}) / 2 - w_1 \left\{ \Delta\tau \, (F^{(1)}_{i,j+1} - F^{(1)}_{i,j}) + \Delta\tau \, (G^{(1)}_{i+1,j} - G^{(1)}_{i,j}) \right\} \quad (8)$$

where

$$\alpha_1 = 1.2 \quad ; \quad w_1 = 0.4 \quad ; \quad \Delta\xi = \Delta\eta = 1$$

The solution procedure is shown schematically in Fig. 2. Since the method employed here is valid only for the supersonic region, the solution in the plane AA' must be known. In the region I the flow is subsonic/supersonic and some other methodology must be employed in that region. The solution used in this work was obtained in [6], using a time-dependent methodology. The transfer of the information from region I to plane AA' is done through a grid overlapping procedure. The solution marches downstream from the z_i to z_{i+1} surface, taking advantage of the hyperbolic nature of the equation set. The compression and expansion shock-waves are fully captured with this model. The size of the marching step is variable and changes to meet the stability conditions, as discussed latter.

BOUNDARY CONDITIONS

In this work two boundary conditions procedures are used. The reflection is used in the body sublayer and in the plane of symmetry fringe, and the Abbett [3] scheme for the surface tangency condition. The pressure, density, and tangencial velocity at the sublayer point are set equal to their respective values at the first point above the body, while the normal velocity is set equal to the negative of its value.

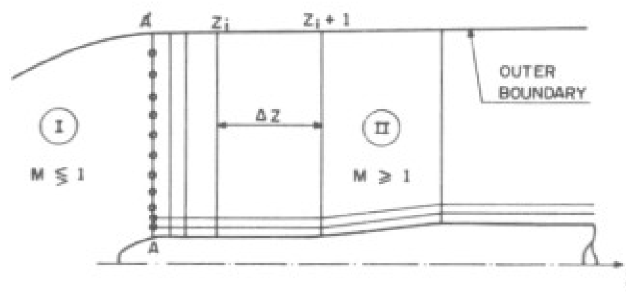

Fig. 2 - Schematic of the marching procedure

In applying the Abbett's scheme one first predicts values of the flow variables (p, ρ , u, v, w) at $\tau^{n+1} = \tau + \Delta\tau$, using the Euler predictor and then correct these quantities using simple compression or expansion waves to invoke the surface tangency condition exactly. The details for the present problem can be found in [3].

STABILITY ANALYSIS

A finite-difference approximation to a partial differential equation may be consistent but the solution will not necessarily converge to the solution of the PDE. In numerical techniques it is very important to select a step size to guarantee the stability bound, such that the computation is performed with a minimum of computer time. The method applied in this work is based on a locally linear analysis of the governing partial differential equations, coupled with a von Neumann stability analysis [1].

When the linear approach analysis is applied to Eq.(6) the manipulation of the Jacobian matrixes and the computation of the eigenvalues are very cumbersome. To perform such computations the algebraic processor REDUCE is used and, to determine the eigenvalues the Brown method is employed[4].

The amplification matrix theory, at least for two-dimensional τ, ξ space, requires that

$$\Delta\tau/\Delta\xi \le 1/(\partial\xi)_{max} \qquad \partial\xi = |\partial(\xi)|_{local\ max} \quad (9)$$

Where $\partial\xi$ is defined to be the local maximum modulus of the eigenvalues of the Jacobian matrix to ξ direction, in a given grid point of the field. A similar condition is obtained in τ, η space.

$$\Delta\tau/\Delta\eta \le 1/(\partial\eta)_{max} \qquad \Delta\eta = |\partial(\eta)|_{local\ max} \quad (10)$$

The step size is determined by a minimum $\Delta\tau$ predicted by the two relations 9 and 10. This planar analysis has been shown that the relations 9 and 10 can be replaced by

$$\Delta\tau/\Delta\xi = const/(\partial\xi)_{max}$$
$$\Delta\tau/\Delta\eta = const/(\partial\eta)_{max} \qquad (11)$$

The minimum value of relation 11 must be chosen, and const < 1 can be varied during the computation with a usually assigned value of approximately 0.9.

NUMERICAL RESULTS

The numerical model is tested solving the supersonic flow over the SCOUT vehicle for several Mach numbers. The angle of attack was taken equal to zero, for comparison purposes, so the solution is two-dimensional due to the axial symmetry of the vehicle. The problem was kept three-dimensional with a discretization with 22x41 points in the ξ, η plane and, approximately, 600 marching steps in the direction. Figs 3, 4 and 5 reports the pressure coeficcient for the SCOUT vehicle for several Mach numbers. There is some disagreement between the experimental[5] results and the numerical ones close to the compression corner. Up til now there is no explanation for such a behaviour.

The bow shock, which encompasses the supersonic marching region, although not presented here, is also well captured by this methodology

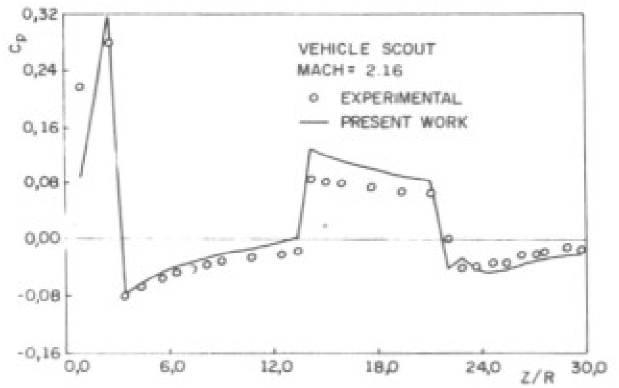

Fig. 3 - Pressure coefficients for Mach = 2.16

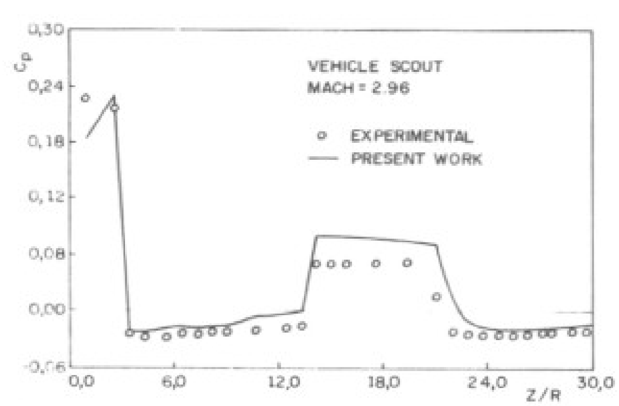

Fig. 4 - Pressure coefficients for Mach = 2.96

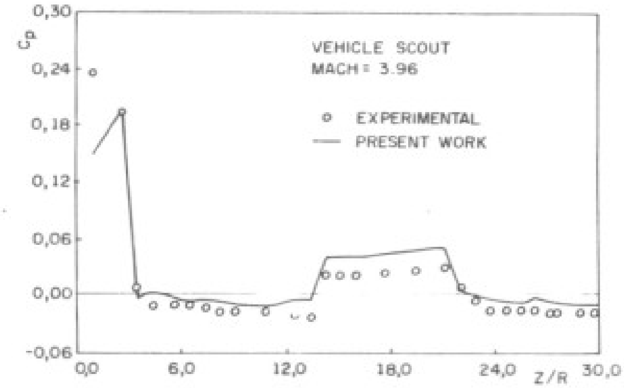

Fig. 5 - Pressure coefficient for Mach = 3.96

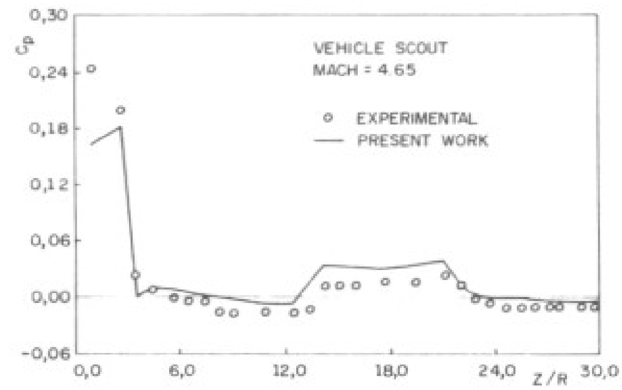

Fig. 6 - Pressure coefficient for MAch = 4.65

CONCLUSIONS

The calculations have shown that this procedure is capable of capturing weak shocks in the presence of strong ones, like the bow shock, with both shocks being well defined. Despite the discrepancies between the numerical and experimental results in the compression corner, the method predicted well the load distribution over the entire region analyzed.

This procedure can be applied to space vehicles of many shapes in supersonic regme. The strong wedges must be avoided because of the possibility of flow separation and upstream influence of pressure, causing the so called departure solutions. Smooth shapes with small wedge angles yield best results.

REFERENCES

[1]. Kutler, P., Warming, R. F, Lomax, H., Computational of Space Shuttle Flowfields using Noncentered Finite- Difference Schemes. AIAA J. vol. 11, n- 2, Feb 1973.

[2]. Maliska, C. R., A Solution Method for Three-dimensional Parabolic Fluid Flow in Nonorthogonal coordinates, Ph.D. Thesis, Waterloo, Ontario, Canada, 1981.

[3]. Abbett, M. J., Boundary Condition Computational Procedures for Inviscid Steady Field Calculations, Final Rept. 71-41, 1971, Aerotherm Corp., Mt. View, Calif.

[4]. Lobão, D. C., Um Critério de Estabilidade Aplicado as Equações da Dinâmica dos Gases, 10º

CNMAC, Congresso Nacional de Matemática Aplicada e Computacional, set 1987, Gramado, RS.

[5]. Jornell, L. S., Aerodynamic Loading Characteristics of The Three-Stage Scout Vehicle at Mach Number From 1.57 to 4.65, NASA TND-1930, Washington, USA, 1963.

[6]. Nascimento, D.P.and Prakash, S.,Análise do problema do corpo rombudo em escoamentos supersônicos/hipersônicos, RT 004 - ETP-AED/85, IAE/CTA, São José dos Campos -SP, 1985.