IBP08604

DEVELOPMENT OF A LABORATORY DATA INTERPRETATION SOFTWARE FOR DETERMINATION OF RELATIVE PERMEABILITY CURVES BY A PARAMETER ESTIMATION METHOD

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Abstract

It is unquestionable the significant role that reservoir simulation has gained in the petroleum industry today. Nevertheless, the accuracy of this prediction tool is frequently degraded not only by inherent uncertainty in the reservoir characterization, but also by usually deficient estimation of relative permeability curves, which are the key elements for macroscopic description of multiphase flow in porous media. In practice, these flow functions should be estimated from data collected in displacement experiments. Although numerous methods for estimating relative permeabilities from these experimental data have been developed over the years, their practical application have been suffered from many problems mainly arisen on the oversimplified mathematical models on which those methods were based. In recent years, parameter estimation techniques are being increasingly applied to estimate relative permeabilities, mostly because they allow employing a flow model as accurate as necessary to represent all influencing factors on the fluid displacement. The present work describes the main characteristics and potentialities of an application software developed as a supporting tool for the task of estimation of reliable relative permeability curves by a parameter estimation method. Special emphasis was given to the implementation of a numerical flow model including relevant physical factors, such as rock heterogeneity, capillary pressure, gravity effects, and fluid compressibility.

Resumo

É indiscutível o importante papel que a simulação de reservatórios tem adquirido na indústria de petróleo, na atualidade. Contudo, a precisão desta ferramenta de predição é freqüentemente prejudicada não apenas pela incerteza inerente à caracterização do reservatório, mas também pela deficiente estimação das curvas de permeabilidade relativa, as quais são elementos-chave na descrição macroscópica do escoamento multifásico em meios porosos. Na prática, estas curvas devem ser estimadas a partir de dados obtidos em testes de deslocamento realizados em laboratório. Embora numerosos métodos de estimação de curvas de permeabilidade relativa têm sido desenvolvidos, a aplicação prática da maioria deles sofre de muitos inconvenientes cuja origem encontra-se principalmente no modelo matemático extremamente simplificado sobre o qual estão baseados. Em anos recentes, os métodos denominados de estimação de parâmetros estão sendo cada vez mais aplicados à estimação de curvas de permeabilidade relativa, principalmente porque permitem empregar um modelo de escoamento tão detalhado quanto necessário para representar todos os fatores relevantes no deslocamento de fluidos. O presente trabalho descreve as principais características e as potencialidades de um pacote computacional desenvolvido para ser uma ferramenta de apoio na tarefa de estimar curvas de permeabilidade empregando o método de estimação de parâmetros. Especial ênfase é dada à implementação de um modelo numérico que inclua fatores físicos relevantes tais como heterogeneidade do meio, pressão capilar, gravidade e compressibilidade dos fluidos.

1. Introduction

Relative permeabilities are essential properties of multiphase flow because they describe the dynamical interaction between the fluids and the porous medium. Consequently, reliable prediction of oil production processes requires of accurate representations of these flow functions. But, as usually happens with most of the petrophysical information concerning to a reservoir, in general, only it is possible to obtain estimates of them from laboratory experiments performed on small core samples extracted from the reservoir. It is not possible either to measure directly

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relative permeabilities, so they should be inferred from analysis of measurable data collected in those experiments. Numerous methods have been developed over the years for that purpose, most of them based on the Buckley-Leverett analytical solution for the two-phase immiscible displacement, which ignores capillary and gravity effects, and presumes that the core is perfectly homogeneous. To this category belongs the JBN method (Johnson, Bossler and Naumann, 1959), one of the most popular methods in the petroleum industry in earlier years. However, as pointed out by various authors (Urkedahl et al., 2000; Sylte et al., 1998; Mejia et al., 1994) hardly ever core samples are completely homogeneous, and, when reservoir low-velocity conditions are attempted to be replicated in experiments, capillary effects inevitably will influence the behavior of the flow.

More recently, parameter estimation techniques are being increasingly applied to the relative permeabilities determination task, mostly because they permit to employ a flow model as accurate as would be needed to represent all relevant influencing factors on the fluid displacement. In these methodologies, functional representations of relative permeability curves depending on a finite number of parameters are chosen, and then values of those parameters are estimated minimizing the differences between data collected during the displacement experiment and numerical results obtained from a theoretical model (Kerig and Watson, 1987; Watson et al., 1988; Watson et al., 1994; Mejia et al., 1995). Since the basic structure of this approach is independent of the mathematical model employed to describe the flow, it can be improved to better represent the specific characteristics of a given displacement experiment without any need to modify the parameter estimation algorithm. Moreover, the representation of relative permeability curves can be made as flexible as necessary to characterize actual displacement processes in the entire saturation range.

The present work describes the main characteristics and potentialities of an applicative software developed as a supporting tool for the task of estimation of reliable relative permeability curves by a parameter estimation method. In order to provide enough generality for a wide spectrum of experimental scenarios, special emphasis was given to the implementation of an accurate numerical flow model including the most relevant physical factors, such as rock heterogeneity, capillary pressure, gravity effects, and fluid compressibility.

2. The Displacement Experiment

Relative permeability curves normally are obtained from dynamic fluid displacement experiments on reservoir core samples. Typically, before one of these experiments, a given initial distribution of the fluids is created inside the core sample, generally attempting to reproduce the actual initial conditions on the reservoir. After reaching equilibrium under this initial condition, the displacing fluid is injected through the inlet end, whereas the displaced fluid is collected at the outlet end of the core. During the injection process, the transient variation of different measurable physical quantities related to the flow are measured and recorded, until the flow reaches steady-state conditions, that is, when no more displaced fluid is contained in the outlet effluent. As depicted in Fig. 1, a two-phase separator is commonly used for separating produced fluids, as well as for measuring and recording the cumulative volume of one or both fluids. Other typical recorded quantity is the pressure drop between the inlet and the outlet of the core, which is usually

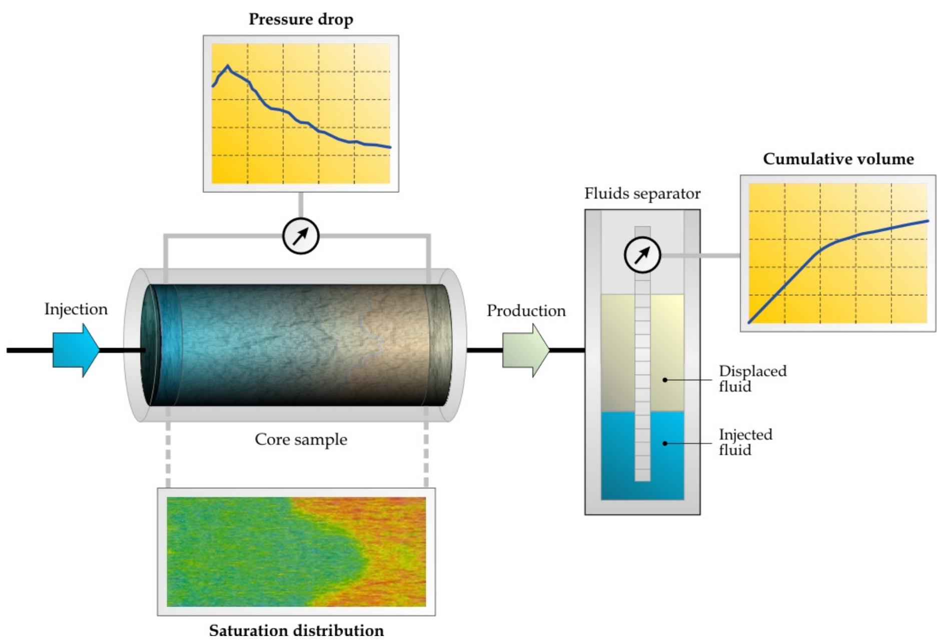


Figure 1. Schematic representation of a displacement experiment.

measured by a high resolution differential pressure transmitter. When injection is performed at constant pressure, as usually happens in gas-oil displacement experiments, injection flow rate is measured instead of pressure drop, which become a constant parameter in such conditions.

In recent years considerably progress has been achieved in visualization and measurement of multiphase flow in porous media by imaging methods (Majors and Peters, 1995; Watson and Chang, 1997; Watson, 1999), especially by nuclear magnetic resonance (NMR). In order to improve the estimation of relative permeability curves by parameter estimation procedures, various authors have been included information about the instantaneous distribution of saturation within the core obtained by those imaging methods (Chardaire et al., 1993; Mejia et al., 1995; Kulkarni et al., 1996). In this way experimental data set becomes richer, improving the accuracy of the relative permeabilities estimation. Imaging methods assist also in the measurement of the intrinsic properties of the porous media, namely porosity and absolute permeability (Watson and Chang, 1997; Zuluaga et al., 2002). Since accurate spatial distribution of these properties is needed for numerical simulation of the flow when the core sample is heterogeneous, the use of those measurement methods is essential in that case.

3. The Parameter Estimation Procedure

The whole process of estimating relative permeability curves by a parameter estimation method can be better understood by means of the schematic flowchart depicted in Fig. 2. The first stage is the *laboratory work* described in the preceding section, where all experimental data that will be used in the process are gathered. Then, a specific functional form for the relative permeability curves must be chosen, employing a certain number of unknown parameters. An initial estimate of parameter values must be provided and passed, along with all experimental data, to

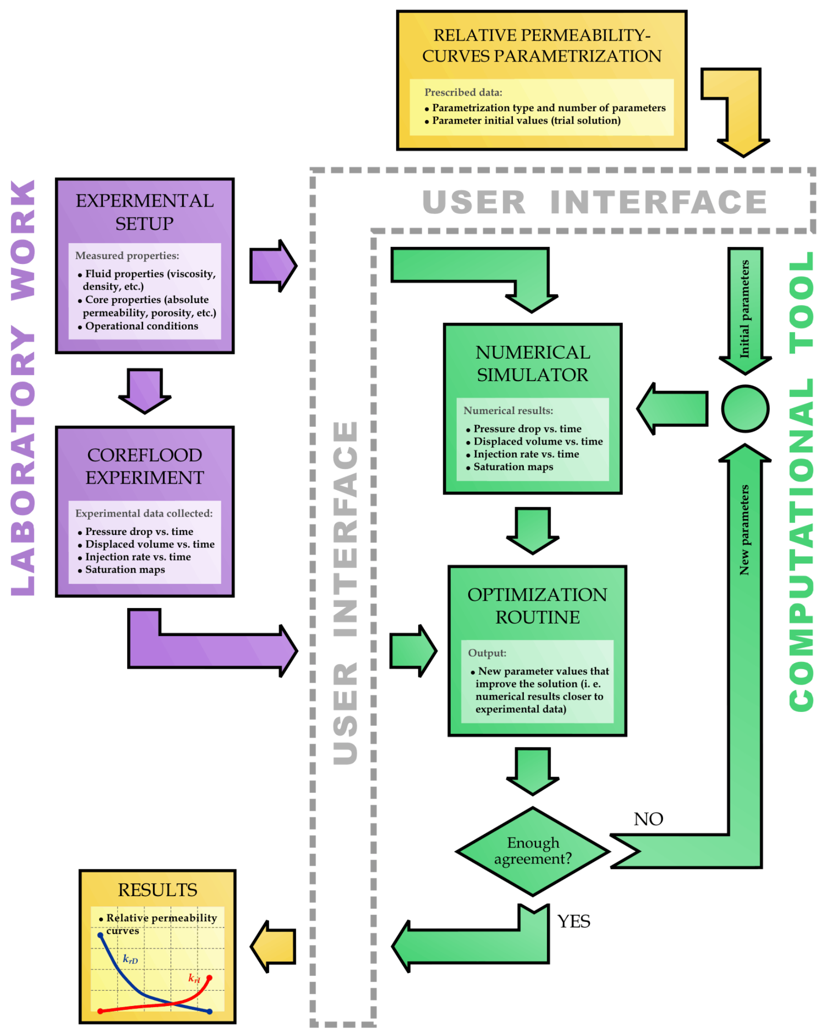


Figure 2. Flowchart illustrating the process of determination of relative permeability curves by a parameter estimation procedure.

the *computational tool,* which performs all needed calculations. In order to execute the task of estimate relative permeability curves by a parameter estimation method, that computational application must include two main modules: a *numerical simulator* and an *optimization routine*. The numerical simulator should be able to solve the equations of the flow mathematical model and, for a given set of parameter values, provide numerical results for the physical quantities used in the estimation process, e.g. cumulative produced volume, pressure drop, saturation distribution, etc. The role of the optimization routine is to improve the values of the parameter set through the minimization of an objective function that quantifies in a given way the differences between the experimental data and the numerical results supplied by the simulator. Since the objective function is always highly non-linear, the parameter estimation process is in fact an iterative one, in which the parameter set values are improved until a suitably small value of the objective function is achieved, that is, numerical results are closer enough to experimental data. If the process finishes in such successful way, then the founded parameter set defines the better pair of relative permeability curves in the family of curves defined by the parameterization scheme used. In the following sections will be described the main characteristics of the numerical simulator and the optimization algorithm implemented in the computational application described here.

4. The Numerical Simulator

The numerical simulator employs a mathematical model based on the standard macroscopic description of the flow of multiple immiscible fluid phases through porous media. Although the comparative significance of some physical factors over the flow can be different at reservoir-scale and at core-scale, the model equations for both scales are essentially the same when a macroscopic approach is employed.

The main components of the model are the mass-conservation differential equations for the two phases and the multiphase extension of Darcy's law. The former can be written as

$$\phi\,\partial_t(\rho_F s_F) + \vec{\nabla}\cdot(\rho_F \vec{\mathbf{v}}_F) = 0\,; \qquad F = I,\,D \tag{1}$$

Here ρ_F , s_F , and $\vec{\mathbf{v}}_F$ are the density, saturation, and mean velocity vector of the given phase, respectively; ϕ is the porosity of the medium. For the sake of generality, the fluid phases are designated as injected phase (I) and displaced phase (D), respectively. The mean velocity of each phase is given by Darcy's law, which is usually extended to multiphase flow in the following way:

$$\vec{\mathbf{v}}_F = -\frac{K\,k_{rF}}{\mu_F}\,(\vec{\nabla}P_F - \rho_F\,\vec{\mathbf{g}})\,; \qquad F = I,\,D \tag{2}$$

in which K is the absolute permeability of the porous medium, $\vec{\mathbf{g}}$ is the acceleration due to gravity, P_F is the phase pressure, μ_F is the phase viscosity, and k_{rF} is the phase relative permeability. In this model, relative permeabilities can be interpreted as the parameters that account for the decrease on the capacity of one phase to flow due to reciprocal interference between the fluid phases.

The coupling between the preceding equations results from the volumetric restriction equation

$$s_I + s_D = 1 \tag{3}$$

and the definition of capillary pressure

$$P_C = P_D - P_I \tag{4}$$

The heterogeneity of the medium is addressed considering porosity and absolute permeability as functions of the position. Moreover, densities of both phases are considered in general as functions of the phase pressures. Following the standard macroscopic modeling of multiphase flow in porous media, relative permeabilities and capillary pressure are considered as only saturation functions.

Existent commercial packages for relative permeability estimation usually employ a flow simulator based on the one-dimensional version of the above model. In order to arrive at a compromise between accuracy and computational cost, it was decided to implement a simulator based on a two-dimensional model. This choice is justified by the fact that such numerical model would be able to capture most of the heterogeneity-related phenomena, without increasing excessively the computational effort needed for solving the discretized equations, as certainly a three-dimensional model would do. Therefore, the vertical middle section of the core sample is taken as solution domain for the description of the displacement process, as depicted in Fig. 3.

The discretization of the model differential equations is performed employing the Element-based Finite Volume Method (EbFVM). This approach follows the basic guidelines of the conventional finite volume method, namely the integration of differential equations over control volumes in a way that conservation is automatically enforced. However, a significant improvement in flexibility is introduced through the concept of element as the geometrical entity for discretization of the solution domain. As depicted in Fig. 4, four basic geometrical entities are considered in

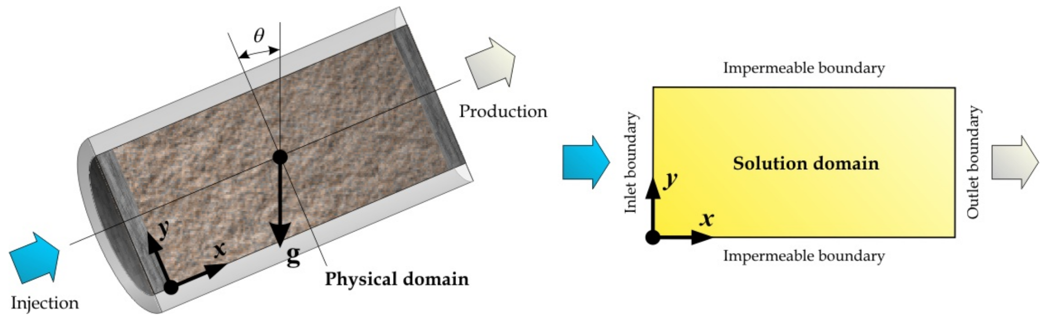


Figure 3. Two-dimensional domain considered in the model.

EbFVM: *elements*, which cover entirely the solution domain without overlapping; *control volumes*, formed by portions of neighboring elements; *nodes*, located at element vertices and where state variables are computed and stored; and *integration points*, located at control volume interfaces and where mass-fluxes are calculated. Further details on the numerical formulation employed can be found in Hurtado et al. (2004), whereas the fundamentals of EbFVM are described in Maliska (2004).

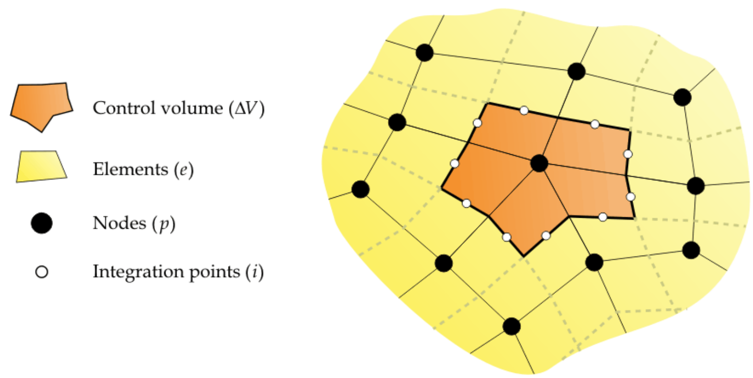


Figure 4. Fundamental geometrical entities for domain discretization.

For solving the discretized equations was chosen a sequential solution algorithm similar to the standard IMPES (Implicit Pressure, Explicit Saturation) procedure, in which some improving modification were introduced in order to accelerate its performance. After obtaining the transient evolution of pressure and saturation discrete fields by solving numerically the model equations, all flow-related quantities employed in the parameter estimation procedure are computed based on that solution.

5. The Optimization Procedure

Since the aim of a parameter estimation process is to find the set of unknown parameters, which will be denoted here as vector $\vec{\alpha}$, that produces the best agreement between predicted results and experimental data, one suitable way of quantify that agreement is defining a residual vector, whose components are given by

$$\varepsilon_i(\vec{\alpha}) = w_i \left[\theta_i^N(\vec{\alpha}) - \theta_i^E \right] \tag{5}$$

Here θ_i^E and θ_i^N are the experimental and predicted values, respectively, which can be values of cumulative production volume, pressure drop, or any other quantity employed in the estimation process. Moreover, w_i are weighting factors, which usually are taken as equal to the inverse of the variance of the experimental measurements (Kerig and Watson, 1987; Mejia et al., 1995). The dependence of the residual vector on the parameter vector comes from the fact that at any stage the predicted values are computed by numerical simulation with the current values of the vector $\vec{\alpha}$. Subsequently, the objective function can be defined as

$$\chi(\vec{\alpha}) = \|\vec{\varepsilon}(\vec{\alpha})\|_2^2 = \sum_i [\varepsilon_i(\vec{\alpha})]^2 \tag{6}$$

The determination of the vector $\vec{\alpha}$ that minimizes Eq. (6) is a non-linear optimization problem that can be solved employing different approaches. In the computational application developed a Levenberg-Marquardt algorithm with linear inequality constraints was employed to solve that problem. Since relative permeability curves must satisfy certain constraints, namely non-negativity, monotonicity and smoothness, the space of unknown parameters is restricted to a certain bounded region. In general, that region is defined as a set of linear inequality constraints that can be written in matrix form as

$$\mathbf{A}\vec{\alpha} \geq \vec{b} \tag{7}$$

Since the Levenberg-Marquardt method is a gradient-based method, an iteration at any stage of the estimation process will be characterized by an update relation of the form

$$\vec{\alpha}^{k+1} = \vec{\alpha}^{k} + \vec{\delta} \tag{8}$$

where $\vec{\delta}$ is a updating step computed resolving a local quadratic optimization problem. This problem results from the approximation of the objective function at the current point through the quadratic function (Gill et al. 1981):

$$\chi(\vec{\alpha} + \vec{\delta}) \approx \chi(\vec{\alpha}) + \vec{\gamma} \cdot \vec{\delta} + \tfrac{1}{2}\vec{\delta}^{T}\mathbf{H}\vec{\delta} \tag{9}$$

Here $\vec{\gamma}$ is the gradient vector and $\mathbf{H}$ is the Hessian matrix, which contains the second partial derivatives of the objective function. They can be expressed in function of the Jacobian matrix, whose elements are the partial derivatives of the residual vector components with respect to the parameters

$$J_{ij} = \frac{\partial \varepsilon_i}{\partial \alpha_j} \tag{10}$$

Employing this matrix, it can be shown that the gradient vector and the Hessian matrix can be expressed, respectively, as

$$\vec{\gamma} = \mathbf{J}^{T}\vec{\varepsilon} \tag{11}$$

$$\mathbf{H} \approx \mathbf{J}^{T}\mathbf{J} \tag{12}$$

The updating step $\vec{\delta}$ will be in a descent direction only if $\mathbf{H}$ is positive-definite. As usually this is not the case when a given point is far from the optimal point, in the Levenberg-Marquardt method this condition is enforced by means of the following modification of the Hessian matrix:

$$\mathbf{H}' = \mathbf{H} + \nu\,\mathbf{I} \tag{13}$$

where $\mathbf{I}$ is the identity matrix and ν is a non-negative scalar parameter, whose value is chosen in a way that $\vec{\delta}$ will be in a descent direction.

More details concerning to the implemented optimization procedure can be found in Ambrus et al. (2004). The fundamentals of the method are more deeply elucidated in Gill et al. (1981).

6. Parameterization of Relative Permeability Curves

Even though numerous empiric parametric functional representations for relative permeability curves have been proposed over the years, none of them has enough generality to represent actual curves accurately in most of the cases (Watson et al., 1988). On the other hand, the use of B-splines (Boor, 1993) is recommended by several authors because the high degree of flexibility that can be achieved. In this way, the relative permeabilities can be defined as

$$k_{rF} = \sum_{j=1}^{N_F} C_j^F B_j^m(s_F); \qquad F = I,\, D \tag{14}$$

where C_j^F are the unknown parameters and $B_j^m(s_F)$ are the basis functions of order m. Each one of these functions can be represented by a locally defined polynomial of degree $m-1$. In order to employ this kind of representation, the saturation range must be partitioned in a certain number of segments. It can be shown that the monotonic character and the non-negativity of the relative permeability curves is assured imposing the following constraints over the unknown parameter values

$$0 \leq C_1^F \leq C_2^F \leq \cdots \leq C_{N_F}^F \leq 1; \qquad F = I,\, D \tag{14}$$

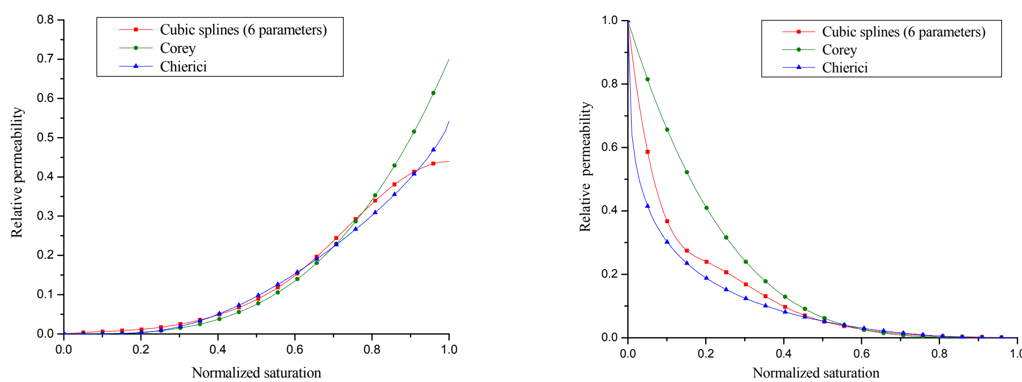
These constraints can be easily transformed to the general form of Eq. (7).

7. Application Examples

At present, the application software for relative permeability curve estimation has been implemented with all characteristics described here. Although the flow model implemented is a two-dimensional one, most of the validation tests performed until now were on situations where no data, concerning to spatial variation of core sample properties nor saturation distributions were available. Therefore, in such conditions only one-dimensional numerical simulation of the flow is possible. As an example of application of the estimation of relative permeability curves will be presented the results corresponding to a gas-oil displacement experiment whose operating conditions are detailed in Tab. 1. In the experiment, a core sample initially saturated with oil is flooded with a gaseous phase at constant pressure. For the parameter estimation process, cumulative volume of oil and injected flow rate are employed. Moreover, an ideal gas state equation is employed in the numerical flow model for the gaseous phase. In Fig. 5 are shown the curves obtained employing two empirical correlations, and the cubic B-splines representation. Furthermore, in Fig. 6 is shown a comparison among the experimental data and the predicted behavior of oil production and gas injection rate obtained with the different relative permeability curves estimated. It is evident the superiority of results obtained with the B-splines representation. Further details are described in Ambrus et al. (2004).

Table 1. Core properties and operating conditions for a gas-oil displacement.

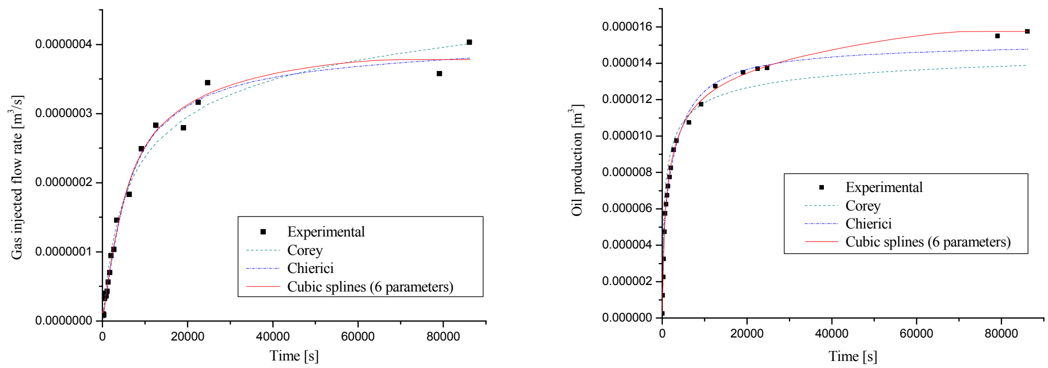
Core sample properties	
Length [m]	0.1242
Diameter [m]	0.0376
Absolute permeability [m^2]	$9.06 \cdot 10^{-14}$
Porosity	0.179
Operating conditions	
Gas viscosity [Pa·s]	$1.772 \cdot 10^{-5}$
Oil viscosity [Pa·s]	0.00135
Initial oil saturation	1
Residual oil saturation	0.362
Gas injection pressure [kPa]	121.8



(a) (b)

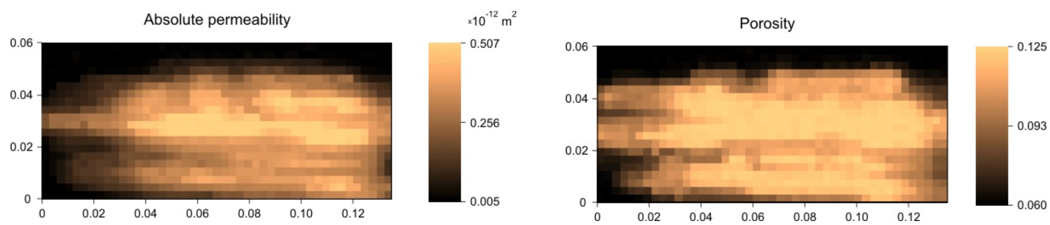
Figure 5. (a) Gas and (b) oil relative permeability curves obtained with different parameterization schemes.

In order to test the two-dimensional numerical model, it was obtained the numerical simulation of a water-oil displacement process in a heterogeneous sandstone core. The porosity and absolute permeability distributions employed were obtained by nuclear magnetic resonance (NMR) by Zuluaga et al. (2002), and are reproduced in Fig. 7. Since in this case was not performed a parameter estimation process because no further experimental data were available, a given set of relative permeability and capillary pressure curves were used for simulating an imbibition process with constant injection rate in a 3600-element Cartesian grid. Figure 8 shows the obtained saturation distribution for three selected times, in which can be clearly observed the marked influence of medium heterogeneity over the flow. In the later time, after the arrival of the water front at the outlet face, it is clearly observed the presence of a high saturation gradient adjacent to that face. As in this case water is the wetting phase, it can only pass through the outlet boundary if saturation attains the value that leads to a zero capillary pressure. Hence, this phase accumulates near the outlet face until that condition be satisfied, generating the so called end effect phenomenon.



(a) (b)

Figure 6. (a) Gas injection rate and (b) cumulative oil volume predicted and experimental values.



(a) (b)

Figure 7. (a) Absolute permeability and (b) porosity distribution maps.

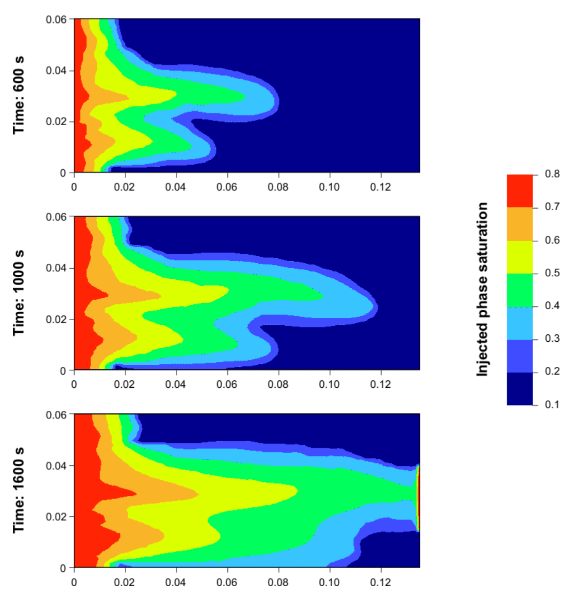


Figure 8. Simulated saturation distributions at three times.

8. Conclusions

In this paper were described the main characteristics of an application software developed as a supporting tool for the task of estimation of reliable relative permeability curves by a parameter estimation method. The fundamentals of this methodology were elucidated whereas its essential equations were outlined. In order to provide enough generality for a wide spectrum of experimental scenarios, during the developing of the application special emphasis was given to the implementation of a numerical flow model including relevant physical factors, such as rock heterogeneity, capillary pressure, gravity effects, and fluid compressibility. Some application examples were presented which permit to show the potentialities of the application at the current stage of development. In the near future it is expected to perform further tests of the parameter estimation of relative permeabilities employing saturation data obtained by means of imaging methods, in order to evaluate the full potentialities of our application software.

9. Acknowledgements

This work was supported by Agência Nacional do Petroleo (ANP), through PRH 09 - Programa de Formação de Recursos Humanos em Petróleo e Gás. We gratefully acknowledge also financial support and cooperation of Petróleo Brasileiro S. A. (Petrobras).

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