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ADVANCED NUMERICAL TECHNIQUES FOR IMPROVING RESERVOIR SIMULATION

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Abstract

Recent developments in numerical techniques for enhancing reservoir simulation are reported in this paper. These techniques, developed in our research group, are mainly related to the use of unstructured grids for representing complex reservoirs. In order to discretize the differential equations modeling the flow, the element-based finite volume method (EbFVM) is considered. This is a strictly conservative method able to deal with unstructured grids formed by elements of any shape, which can be used for representing the reservoir and all its geological objects in an accurate and efficient way. In addition, multidimensional upwind interpolation schemes for approximating advection terms are introduced aiming the reduction of the grid orientation effect, which is a numerical artifact present in customary numerical methodologies used in reservoir simulation. As showed in this paper, exceptional results with almost no grid orientation effects were obtained using this approach. Moreover, since discretization methods on unstructured grids are more computationally demanding than traditional methods on regular grids, the suitability of using algebraic multigrid methods was studied in order to reduce the computation time solving linear systems. Preliminary results using those methods are presented showing that computational effort can be scaled almost linearly with the size of the grid, which is the ideal performance for a numerical method.

Resumo

Neste artigo são descritos recentes desenvolvimentos nas técnicas numéricas destinadas à simulação de reservatórios de petróleo. Tais técnicas, desenvolvidas no nosso grupo de pesquisa, estão relacionadas principalmente ao uso de malhas não-estruturadas para a representação de reservatórios complexos. Para discretizar as equações diferenciais descrevendo o escoamento é considerado o método de volumes finitos baseado em elementos (EbFVM). Este é um método estritamente conservativo com o qual é possível utilizar malhas não-estruturadas formadas por elementos de qualquer forma. Com este tipo de malhas é possível representar reservatórios com características geológicas complexas em uma forma precisa e eficiente. Além disso, esquemas de interpolação *upwind* multidimensionais são utilizados com o intuito de reduzir o efeito de orientação de malha, o qual é uma anomalia numérica presente nas metodologias numéricas usadas na simulação de reservatórios. Conforme é mostrado neste artigo, excelentes resultados sem quase nenhum efeito de orientação de malha foram obtidos usando esta abordagem. Uma vez que os métodos de discretização para malhas não-estruturadas demandam maior esforço computacional que os métodos tradicionais para malhas estruturadas, como uma estratégia para reduzir o tempo de computação resolvendo sistemas lineares, foram utilizados métodos *multigrid* algébricos. Resultados preliminares usando esses métodos são apresentados, mostrando que o esforço computacional aumenta em forma aproximadamente linear com o tamanho da malha, o qual é o desempenho ideal para um método numérico.

1. Introduction

One of the major current challenges for reservoir simulation is to incorporate the high level of detailing of geological reservoir models into the flow models. Thanks to recent improvements in geosciences techniques, nowadays it is possible to obtain accurate reservoir static models including very detailed description of all geological objects in the reservoir. Unfortunately, most of the discretization methods commonly used in reservoir simulation, mainly based on structured grids, are not capable to represent the detailed geometry of such geological objects or other complicated entities such as horizontal wells. As recognized for several authors (Fung et al., 1992; Verma & Aziz, 1997; Heinemann & Heinemann, 2001) the key solution for that question is the use of unstructured grids for representing reservoir geometry in the flow models.

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The present paper summarizes some advanced numerical methods and techniques for reservoir simulation that are being developed in our research group. The main objective of this research is the establishment of a numerical formulation using general unstructured grids, capable to solving the most complex reservoir flow problems in an efficient and accurate way. Initially, we will briefly describe the element-based finite volume method, which is the heart of the numerical formulation being developed. Subsequently, some promising results obtained with our formulation are presented, showing that the use of improved interpolation schemes can prevent the so-called grid orientation effect. Finally, we present some results regarding the use of a multigrid method for improving the performance of the solution process in terms of computation time. The review presented in this paper does not pretend to be exhaustive but illustrative. Further details can be found in several works referenced along this paper.

2. The Element-based Finite Volume Method

The so-called control-volume finite element method (CVFEM), developed at first for solving the Navier-Stokes equations, seems to be the best alternative for discretizing conservation equations arising in reservoir flow models. Unstructured element grids can be used to represent arbitrarily complex geometries without regarding on the heterogeneity or anisotropy of the medium. In reservoir simulation, such discretization method has been applied mainly with triangular grids (Forsyth, 1990; Gottardi & Dall'olio, 1992; Fung et al., 1992; Fung et al., 1994). Numerical approximations used with this type of grids permit arranging the discretized equations in a form similar to those arising from conventional finite difference methods. Although this characteristic is advantageous at first, because it facilitates the implementation of CVFEM formulations into existing reservoir simulators, several drawbacks arises from that practice. As discussed by Cordazzo et al. (2004), some of the approximations considered in those formulations are questionable for multiphase flow and lead to erroneous interpretations of the coefficients on the discretization equations. As a result, numerical simulations can exhibit non-physical behavior in several situations.

Differently from the classical finite element approach, local and global mass conservation can be directly enforced in the CVFEM approach. This is possible because discretized equations are derived as finite volume equations, namely as mass balances over polygonal control volumes formed by element contributions. We prefer to designate the family of methods sharing this characteristic as element-based finite volume methods (EbFVM), since elements are used only as supporting geometric entities and no mathematical foundation of the finite element method is actually considered for discretizing the differential equations (Maliska, 2004).

Because of its capability of representing complex solution domains and its strong physical appealing, the EbFVM approach is ideal for developing an advanced reservoir simulator. In recent years, our research group has been concentrating efforts for establishing a basic numerical formulation for reservoir simulation applying EbFVM. Differently from existent formulations on triangular unstructured grids, any attempt of adapting discretized equations to conventional forms is discarded. Thus, for example, the concept of transmissibility is completely abandoned. Many ideas suggested originally by Raw (1985) for the solution of the Navier-Stokes equations was adapted to the discretization of the differential equations arising in reservoir flow models.

For the application of the EbFVM to the discretization of differential equations of the flow, the solution domain must be broken up into much smaller sub-domains, called *elements*. These entities are used for defining the discretized geometry of the domain as well as for defining the spatial variation of medium physical properties. The unknowns of the problem are calculated at the *nodes*, located at every element corner. Around every node is built a *control volume*,

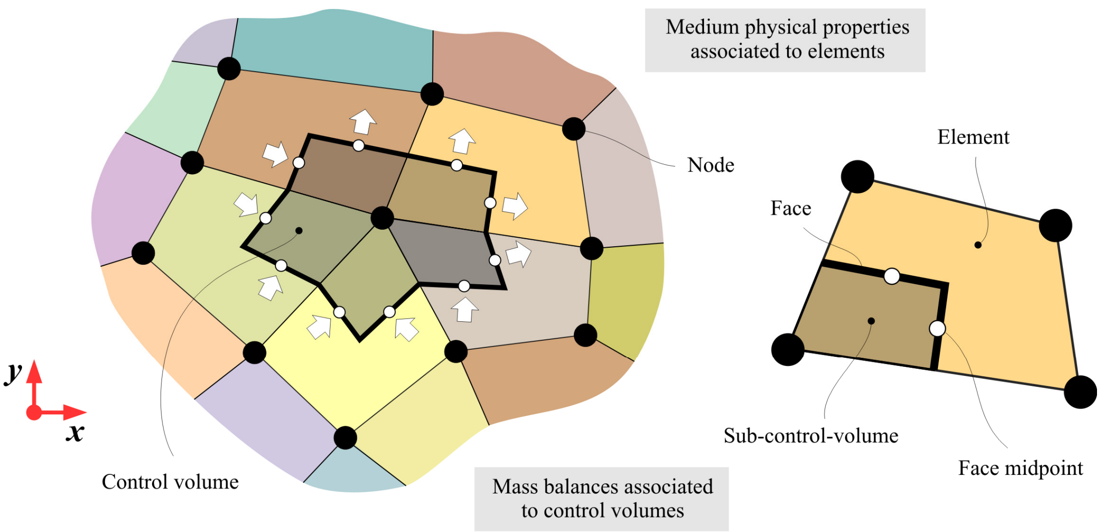

Figure 1. Main geometrical entities on the element-based finite volume method.

formed by portions of the elements, the *sub-control-volumes*, sharing a common node. Every control volume is delimited by a certain number of *faces.* For the two-dimensional case, those faces are obtained joining the center of every neighboring element with the midpoint of one side sharing the node around which the control volume is built. All these geometrical entities are depicted in Fig. 1 for a quadrilateral grid. For triangular or mixed quadrilateral/triangular grids all previous definitions are still valid.

As in any finite volume methodology, the conservation of physical quantities over every control volume is the essential premise of the EbFVM. However, since the shape of control volumes constructed following the described procedure can become extremely complex, a special strategy is required for dealing with the increased geometrical complexity. The strategy employed in the EbFVM, borrowed from the finite element technique, is the definition of a local coordinate system inside every element. Therefore, all needed calculations can be easily made based upon the geometry of isolated transformed elements, and then conservation equations of every control volume can be simply assembled using the contributions coming from all neighboring elements. The coordinate transformation for a quadrilateral element is depicted schematically in Fig. 2. The reader is referred to Hurtado (2005), Hurtado et al. (2005) and Cordazzo (2006) for the mathematical details and a complete description of the numerical formulation.

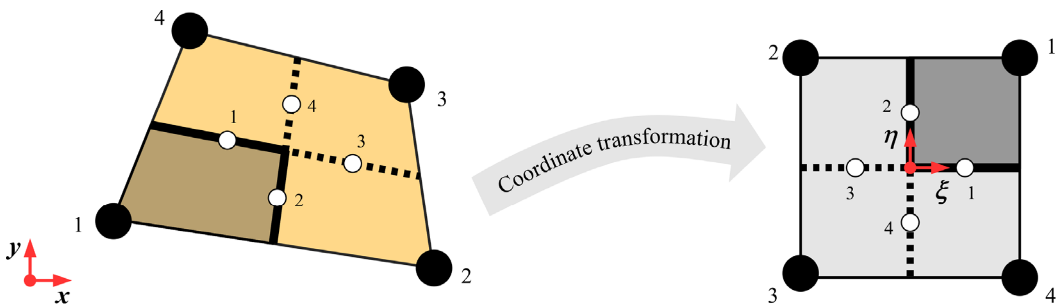

Figure 2. Element represented in global and local coordinate systems.

Although all calculations regarding the discretization process are carried out at element level, after an assembling process, a set of discretized equations representing mass balances over all control volumes is obtained. The solution of this equations permits to determine a numerical approximation of the time evolution of relevant model variables. In order to obtain such solution any solution scheme can be utilized, for instance, an IMPES (Implicit Pressure, Explicit Saturation) or a fully implicit scheme (both described in Mattax & Dalton, 1990). There is no restriction either regarding the heterogeneity or anisotropy of the porous medium. For example, a full permeability tensor can be associated to grid elements for representing a heterogeneous and anisotropic medium.

In order to demonstrate the potential of EbFVM applied to reservoir simulation, a two-dimensional simulation of a waterflood on a faulted reservoir is presented next. The quadrilateral unstructured grid used is shown in Fig. 3. Local refinement is considered in regions around wells (one injection and two production wells) since usually more accurate solutions are required in those regions. This is one of the main advantages of using unstructured grids, because small elements can be concentrated only in localized interesting areas without increasing excessively the size of the discrete problem. Moreover, with unstructured grids, the transition between refined and coarse regions can be made smoothly, in order to avoid introducing further discretization errors associated to element sizes varying abruptly. A geological fault present into the reservoir was modeled as an internal impervious boundary. The grid was enforced to conforming to the domain boundary, as well as the internal fault. The heterogeneous but isotropic absolute permeability distribution

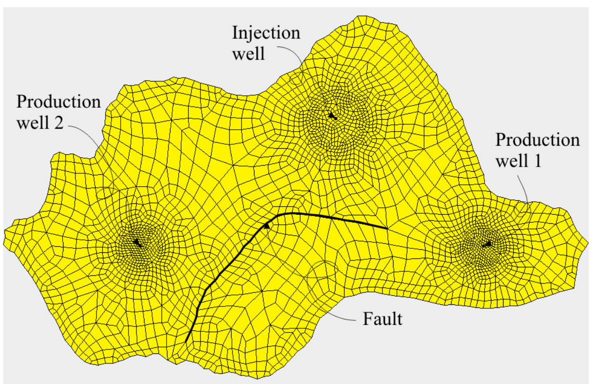

Figure 3. Unstructured grid for a reservoir considered in the example.

considered is depicted in Fig. 4. This distribution was generated randomly. Figure 5 shows the time evolution of water saturation in the reservoir, predicted using the EbFVM formulation.

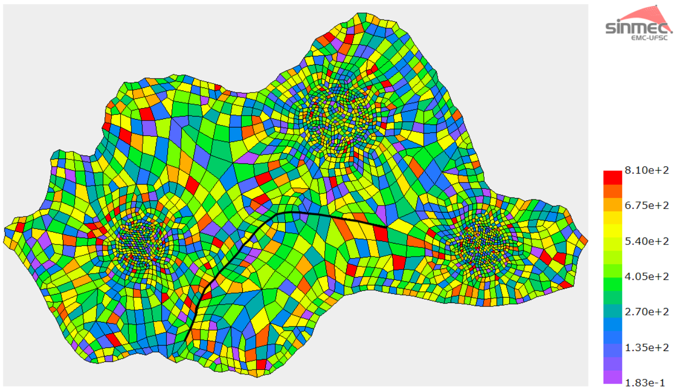

Figure 4. Heterogeneous absolute permeability distribution for the example.

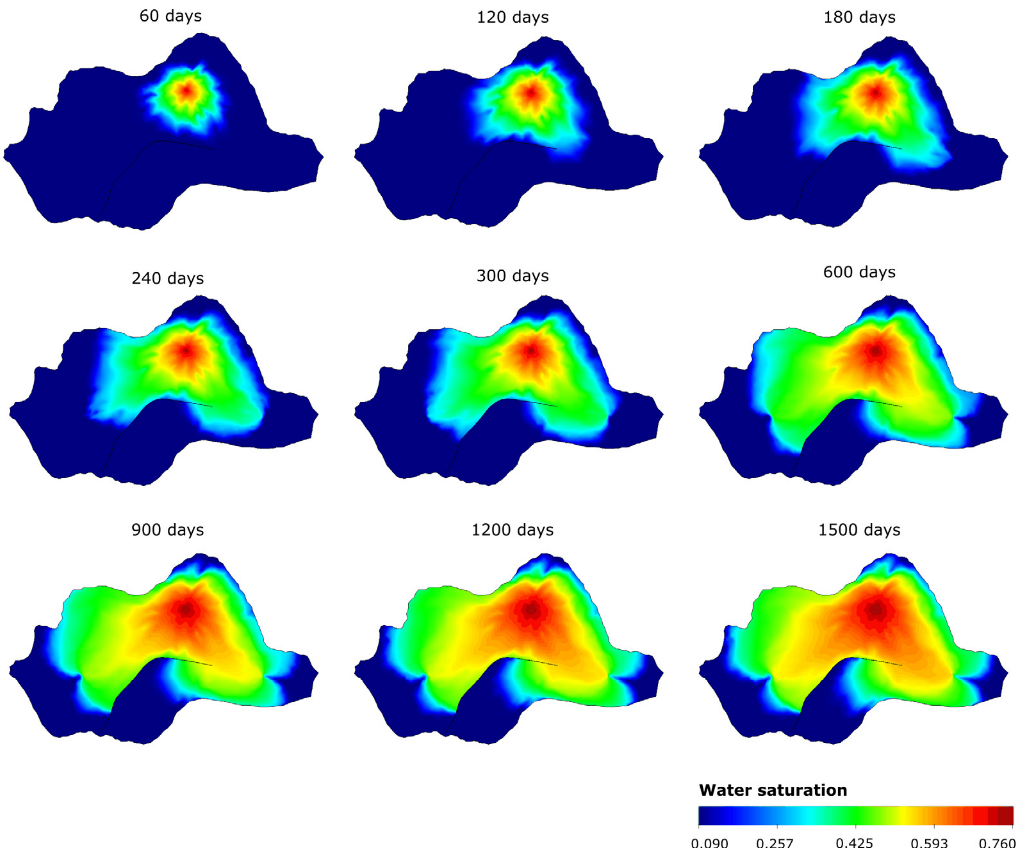

Figure 5. Predicted time evolution of water saturation in the faulted reservoir.

3. Preventing the Grid Orientation Effect

Grid orientation effect is one of the unresolved problems in reservoir simulation. This anomalous phenomenon is observed when computational grid is rotated and substantially different numerical solutions are obtained for a same problem (Aziz & Settari, 1979). It was recognized first by Todd et al. in 1972, and since then numerous works have been published both attempting to explain its causes and proposing practical solutions to suppressing it (Brand et al., 1991; Bajor & Cormack, 1989; Yanosik & McCracken, 1979; Potempa, 1985, among others). However, currently there is no universal solution for grid orientation effect.

All authors agree that grid orientation effect is more evident when saturation front is steep and the mobility ratio is unfavorable. When the displacing phase has greater mobility than the displaced phase, the usual situation when water or steam displaces highly viscous oils, it is said that the mobility ratio is unfavorable (Aziz & Settari, 1979). In conventional numerical formulations, mobilities are interpolated according to one-dimensional upwind schemes along grid lines. This practice seems to be one of the main sources of grid orientation effect, because numerical simulations show a faster advance of saturation front along grid lines as the main manifestation of that phenomenon.

Taking advantage of the increased geometric flexibility provided by the EbFVM discretization approach, we used an interpolation scheme that takes into account the multidimensional nature of the flow, the flow-weighted upwind scheme (FWUS). The original form of the interpolation scheme considered herein was proposed for approximating the advection terms in the Navier-Stokes equations (Schneider & Raw, 1986). It has two fundamental features: the absolute preservation of the positivity of the coefficients on the discretized equations and the consideration of the local direction of the flow. The former feature guarantees that numerical solutions will be free of spurious spatial oscillations and unbounded values. The fact that interpolation takes into account the direction of the flow and not anymore the direction of grid lines greatly reduces grid orientation effect, as will be shown below.

Figure 6 shows schematically how the flow-weighted upwind scheme works. In order to express the discretized equations in a closed form, mobility values at face midpoints should be expressed as weighted combinations of nodal values and possibly other face midpoint values. The weighting factor for that combination is a function of the flow ratio ω_i between the flow-rate across the face considered and the flow-rate across a face located upwind. As shown in Fig. 6, the value of this ratio depends on the direction of a local streamline. In order to guarantee strict positivity of the coefficients on the discretized equations, the weighting factor is limited to the interval $[0,1]$. A complete description of the flow-weighted upwind interpolation scheme can be found elsewhere (Hurtado, 2005; Hurtado et al. 2005).

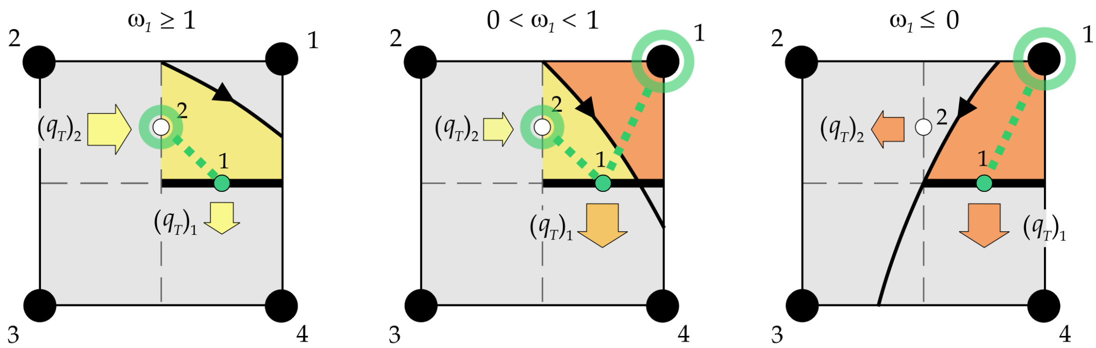

Figure 6. Three possible situations considered in the flow-weighted upwind mobility interpolation.

The five-spot problem is a well-known testing problem for evaluating the grid orientation effect. It consists in the simulation of a water-oil displacement in a periodic arrangement of injection and production wells, as illustrated in Fig. 7. Due to symmetry, two Cartesian grids in simple square domains can be used to solve the flow, the so-called diagonal and parallel grids. Ideally, the numerical solution in any grid should be the same, or at least nearly the same, but this is not the case when conventional formulations are used in adverse cases. The most adverse situation arises when the displacement is of piston-type and the mobility ratio is high. The ability of the numerical formulation using FWUS for preventing grid orientation effect is illustrated by Fig. 8. This figure shows a comparison of results obtained for a piston-type water-oil displacement in a quarter of the five-spot pattern, for a mobility ratio of 10. The numerical results showed

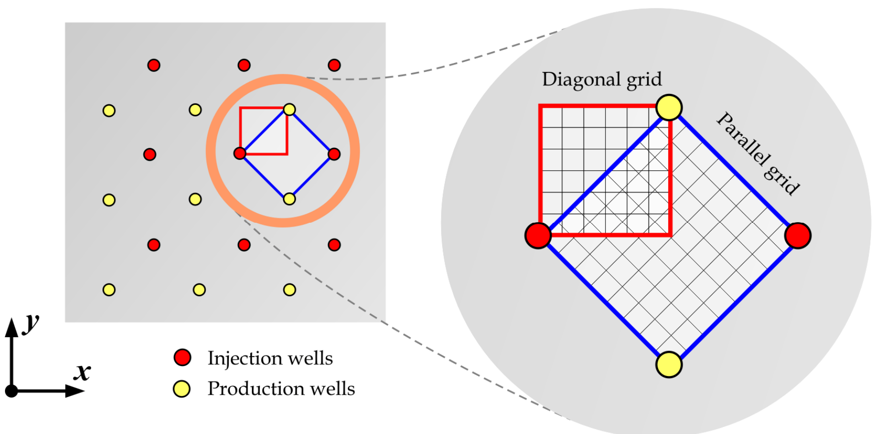

Figure 7. Schematic diagram of five-spot well arrangement.

in Fig. 8(a) were obtained using the conventional one-dimensional upwind scheme in two Cartesian grids: a 20×20-element diagonal and a 28×28-element parallel grid. In the case of Fig. 8(b), the results were obtained using FWUS in the same Cartesian grids as the previous case. While the simulations with the conventional interpolation show strong manifestations of grid orientation effect, the simulations with FWUS in the diagonal and parallel grids are practically identical, as should be. In order to demonstrate the behavior of FWUS with unstructured grids, the previous problem was solved using a 440-element 'diagonal' grid and a 790-element 'parallel' grid. The numerical solutions obtained in those grids are shown in Fig. 8(c). Again no significantly differences among solutions are perceptible, although the shape of the saturation front is not as smooth as in the Cartesian grids.

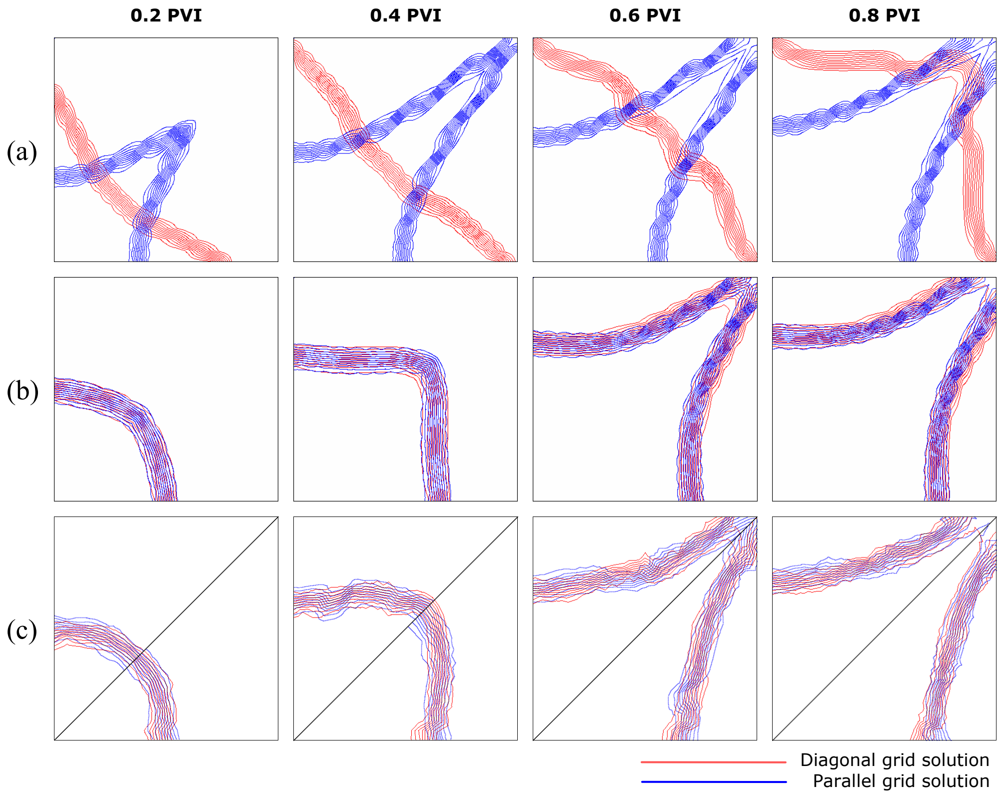

Figure 8. Comparison of diagonal-grid and parallel-grid solutions for a piston-type displacement in a quarter of a five-spot (mobility ratio = 10), using (a) conventional upwind interpolation and Cartesian grids; (b) flow-weighted upwind interpolation scheme and Cartesian grids; and (c) flow-weighted upwind interpolation scheme and unstructured grids.

4. The Additive Correction Multigrid

Discretization methods on unstructured grids are in general more demanding computationally than conventional methods on regular grids. Therefore, more efficient algorithms for solving discretized equations in unstructured grids should be used in order to apply them to large-scale reservoir problems. As the major cost in any algorithm for reservoir simulation is the solution of linear systems (as much as 80% of the total computational cost), it becomes absolutely essential to use an efficient solver able to deal with unstructured grids.

While some solution methods for linear systems can be efficient and robust for small problems, unfortunately most of them exhibit a poor performance for larger problems. For instance, popular iterative methods like Gauss-Seidel and Krylov subspace methods have performances with computational times growing exponentially with the size of the problem. In order to improve the efficiency of iterative solvers were developed multigrid methods, which form a class of methods that uses a hierarchy of grids of different coarsening level. During the iterative process, iterations on different grids of the hierarchy are performed in order to accelerate the convergence to the solution of the linear system. This is possible because in each grid is possible to remove more rapidly errors of a frequency range compatible with the coarsening level of that grid. The high robustness and monotonic convergence are the most important characteristics of multigrid methods (Raw, 1996). The aim of using multigrid methods is to attain the optimal performance, namely required computational time for convergence scaling linearly with the number of unknowns.

The multigrid methods can be classified according to the coarsening procedure in two groups: geometric and algebraic multigrid methods. The former creates the coarse grids joining cells according to the geometry of the original grid, while the last use only the information available in the coefficients of the linear system. For reservoir problems, where heterogeneities and geological objects such faults are common, algebraic multigrid appears to be the most suitable method, because it takes into account those characteristics during the coarsening process. Moreover, an algebraic multigrid solver can be considered as 'black-box' solver, since only the linear system has to be provided.

The additive correction multigrid (ACM), a method that belongs to the algebraic multigrid class, was implemented into a reservoir simulation code based on the EbFVM formulation discussed above. The ACM employs a coarsening algorithm based on the evaluation of relative strength of the coefficients. Besides, the resulting coarse grid equations can be constructed in a conservative framework (Elias, 1993), which gives an adequate physical support to this method. Fig. 9 illustrates the coarsening process of ACM, showing that the coarse cells are built joining control-volumes from the fine grid. This 'two-grid' procedure is applied recursively to construct the grid hierarchy.

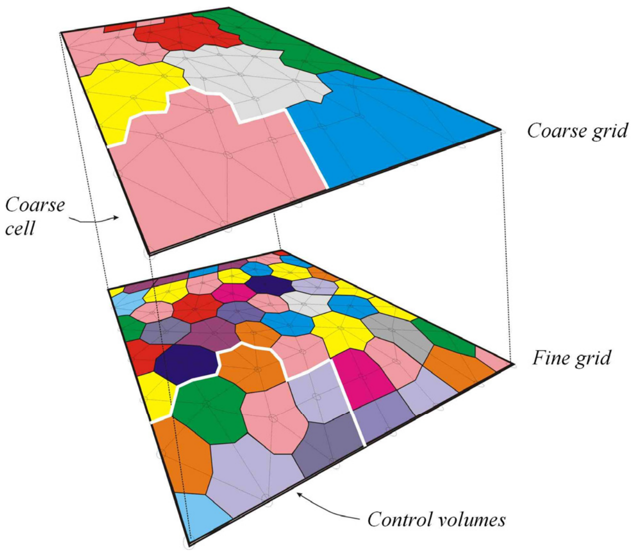

Figure 9. Coarsening process in additive correction multigrid.

There are three main steps involved in a multigrid solver: restriction (a fine-to-coarse transfer of information); solution of the coarse grid problem; and prolongation (a coarse-to-fine interpolation of the corrections). In ACM, the first step is very simple, because it is necessary only to transfer coefficients and residuals to the coarse grids. The linear system for the coarser grids is obtained summing those coefficients for each cell and forcing the residuals to be zero (Cordazzo, 2006). The solution of this problem constitutes the second step of ACM. The final step, in turn, is the transfer of correction terms to the fine grid. In ACM, this procedure is again very simple because it consists only in the addition of correction terms to the best available estimate of the solution.

The potential of the ACM for solving reservoir problems can be seen through an example. In order to compare the performance of ACM and other iterative methods, a five-spot problem (Fig. 7) was solved employing the EbFVM and quadrilateral diagonal grids. The CPU time for solving the first time-step using the IMPES solution algorithm is shown in Fig. 10 for different problem sizes (number of nodal unknowns). The performance of Gauss-Seidel method,

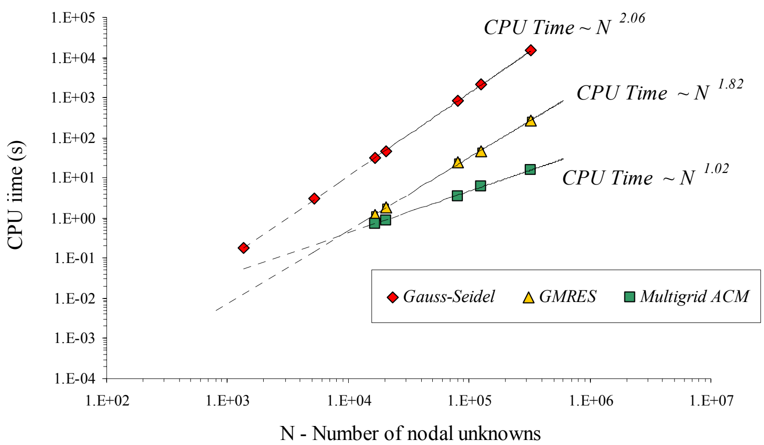

Fig. 10 – Computational time vs. number of unknowns for the first time-step in an IMPES simulation.

Generalized Minimal Residual (GMRES) method, and Additive Correction Multigrid method is compared. As can be observed, the CPU time for the multigrid method scales almost linearly with the number of nodal unknowns.

For large grids, the IMPES algorithm can become so inefficient because of the severe time-step restriction caused by the explicit treatment of saturation. For this reason, fully implicit methods are usually preferred for large scale problems, despite the larger system of equations that must be solved in each time-step. Fortunately, even in that case the ACM method present good performance, as can be seen in Fig. 11. In this figure are compared the CPU times for solving the first linear system in simulations employing ACM with IMPES and the fully implicit method.

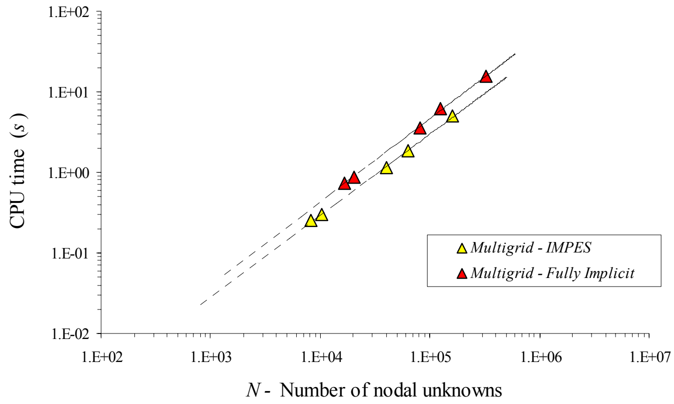

Figure 11. CPU time vs. number of unknowns for different solution algorithms using ACM.

5. Concluding remarks

Some recent developments in numerical methods for enhancing reservoir simulation were presented in this paper. These methods comprehend diverse aspects of the process of obtaining a numerical simulation of the flow in a reservoir. The use of unstructured grids in a strictly conservative framework is possible by means of the element-based finite volume method (EbFVM). By using unstructured grids, complex heterogeneous reservoirs can be discretized and thanks to the increased geometric flexibility, enhanced interpolation schemes can be used in the discretization of flow differential equations. As shown in this paper, those techniques can reduce abnormal numerical artifacts like grid orientation effect. The additive correction multigrid (ACM) method is also a powerful technique for improving the efficiency of solution methods used in reservoir simulation. Initial results showing this was presented in this paper. In general, the results obtained with all those methods are so encouraging and certainly will motivate further research works.

6. Acknowledgements

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7. References

AZIZ, K. AND SETTARI, A. *Petroleum reservoir simulation*, Applied Science Publishers Ltd., London, England, 1979.

BAJOR, O. AND CORMACK, D. E. A new method for characterizing the grid orientation phenomenon. *Paper SPE 19353*, Society of Petroleum Engineers, 1989.

BRAND, C. W., HEINEMANN, J. E., AZIZ, K. The grid orientation effect in reservoir simulation. Paper SPE 21228, *11th SPE Symposium on Reservoir Simulation*, Anaheim, USA, 1991.

CORDAZZO, J. *Petroleum reservoir simulation using EbFVM and algebraic multigrid*. Doctoral Thesis (in Portuguese), Federal University of Santa Catarina, Florianopolis, Brasil, 2006.

CORDAZZO, J., MALISKA, C. R., SILVA, A. F. C., HURTADO, F. S. V. The negative transmissibility issue when using CVFEM in petroleum reservoir simulation - 1. Theory. *Proceedings of the 10th Brazilian Congress of Thermal Sciences and Engineering*, November 30-December 3, Rio de Janeiro, Brazil, 2004.

ELIAS, S. R. *Enhancements to additive correction multigrid*, Master Thesis, University of Waterloo, Canada, 1993.

FORSYTH, P. A. A control-volume finite element method for local mesh refinement in thermal reservoir simulation. *SPE Reservoir Engineering*, Vol. November 1990, pp. 561-566., 1990.

FUNG, L. S., BUCHANAN L., SHARMA, R. Hybrid-CVFE method for flexible-grid reservoir simulation. *SPE Reservoir Engineering*, Vol. August 1994, pp. 188-194, 1994.

FUNG, L. S., HIEBERT, A. D., NGHIEM L. Reservoir simulation with a control-volume finite-element method. *SPE Reservoir Engineering*, Vol. August 1992, pp. 349-357, 1992.

GOTTARDI, G. & DALL'OLIO, D. A control-volume finite-element model for simulating oil-water reservoirs. *Journal of Petroleum Science and Engineering*, Vol. 8, pp. 29-41, 1992.

HEINEMANN, G. F. AND HEINEMANN, Z. E. Gridding concept for the third generation of reservoir simulators. *Sixth International Forum on Reservoir Simulation*, September, 3-7, Salzburg, Austria, 2001.

HURTADO, F. S. V. *An element-based finite volume formulation for the simulation of immiscible two-phase displacement in porous media*. Master Dissertation (in Portuguese), Federal University of Santa Catarina, Florianopolis, Brasil, 2005.

HURTADO, F. S. V., MALISKA, C. R., SILVA, A. F. C., CORDAZZO, J. An element-based finite volume formulation for reservoir simulation. *Proceedings of the XXVI Iberian Latin-American Congress on Computational Methods in Engineering – CILAMCE 2005*, October 19-21, Guarapari / ES, Brazil, 2005.

MALISKA, C. R. *Computational heat transfer and fluid mechanics*, 2nd Edition (in Portuguese), LTC Editora, Rio de Janeiro, 2004.

MATTAX, C. C. AND DALTON, R. L. *Reservoir simulation*. SPE Monograph Series, Volume 13, Society of Petroleum Engineers, 1990.

POTEMPA, T. Mobility weighting in numerical reservoir simulation. *Society of Petroleum Engineers Journal*, August 1985, pp. 565-572, 1985.

RAW, M. J. *A new control-volume-based finite element procedure for the numerical solution of the fluid flow and scalar transport equations*. Ph.D. Thesis, University of Waterloo, Canada, 1985.

RAW, M. Robustness of coupled algebraic multigrid for the Navier-Stokes equations. AIAA 96-0297, *34th Aerospace and Sciences Meeting & Exhibit*, January 15-18, Reno, USA, 1996.

SCHNEIDER, G. E. AND RAW, M. J. A skewed, positive influence coefficient upwinding procedure for control-volume-based finite-element convection-diffusion computation. *Numerical Heat Transfer*, Vol. 9, pp. 1-26, 1986.

TODD, M. R., O'DELL, P. M., HIRASAKI, G. J. Methods for increased accuracy in numerical simulation, *Society of Petroleum Engineers Journal*, Vol. December 1972, pp. 515-530, 1972.

VERMA, S. AND AZIZ, K. A control-volume scheme for flexible grids in reservoir simulation. SPE Paper 37999, presented at the *1997 Reservoir Simulation Symposium*, June 8-11, Dallas, USA, 1997.

YANOSIK, J. L. AND MCCRACKEN, T. A. A nine-point finite difference reservoir simulator for realistic prediction of adverse mobility ratio displacements, *Society of Petroleum Engineers Journal*, Vol. August 1979, pp. 253-262, 1979.