ON THE FACTORS INFLUENCING THE GRID ORIENTATION EFFECT IN RESERVOIR SIMULATION

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Abstract. *Grid orientation effect is one of the unresolved issues in reservoir simulation. This anomalous phenomenon is observed when computational grid is rotated and substantially different numerical solutions are obtained for a same problem. Although numerous works have been published during last three decades dealing with this topic, currently there is no universal solution for that problem. The objective of the present work is to perform a comprehensive examination of the factors that cause the grid orientation effect. Several results of simulations are compared in order to analyze the behavior of this phenomenon when mobility ratio is increased, grids are refined, and different interpolation schemes are considered. Those comparisons suggest that for a given mobility ratio greater than unity, the interpolation scheme is the most important source of grid orientation effect for coarser grids. With those grids, interpolation schemes that take into account flow direction can generate numerical solutions without noticeable grid orientation effect, even for very high mobility ratios. However, as already pointed out by some authors, visible manifestations of grid orientation effect appear when grids are refined, no matter of which interpolation scheme is used.*

Keywords: *Reservoir simulation, grid orientation effect, interpolation schemes, finite volume method.*

1. INTRODUCTION

The grid orientation effect is an anomalous dependence of a numerical solution on the computational grid employed for the discretization of the problem spatial domain. This phenomenon is particularly severe in reservoir simulation and in certain cases it can increase substantially the intrinsic uncertainty of the numerical predictions. For certain unfavorable conditions, completely different numerical solutions can be obtained considering grids with different orientation in relation to the main direction of the flow, even for problems with simple geometries. The numerical prediction of the advance of the front separating two fluid phases can be adversely influenced by the grid, causing progressive strong distortions in the front shape. This can produce absolute meaningless simulations of multiphase flow processes in petroleum reservoirs. All these related phenomena are known in the literature with the generic name of grid orientation effects.

Since Todd et al. (1972) described initially the negative aspects of grid orientation effects in reservoir simulation, many works have been published both analyzing the causes and proposing practical remedies for this phenomenon. Among the most significant works in the first group we can mention the papers of Fanchi (1983), Potempa (1985b), Bajor and Cormack (1989), Shubin and Bell (1989), and Brand et al. (1991). There is a vast literature devoted to suggest diverse techniques for reducing grid orientation effect. Among the most recognized are the works of Yanosik and McCracken (1979), Ko and Au (1979), Vinsome and Au (1979), Shubin and Bell (1995), Frauenthal et al. (1985), Potempa (1985a), Chen et al., (1991), and Shin and Merchant (1994). Although until now no universal solution to grid orientation effect has been found, there exists a consensus about some facts related to this numerical issue. So, it is recognized that its origin is mainly the anisotropic distribution of the truncation error associated to the numerical approximation of advection terms in flow equations. Such different amount of truncation error depending on the local direction of velocity field triggers a nonlinear process of error growth when the displacing phase has a larger mobility than the displaced phase in the porous medium. In cases when such adverse condition does not occur, no appreciable grid orientation effect is noticed in the numerical solutions. It is observed also that displacement flows with sharper fronts are more prone to grid orientation effects than smoother ones.

Despite the large amount of papers addressing grid orientation effect, there is a lack of a systematic study showing the form in which diverse factors influence that numerical problem, through an extensive comparison of numerical results. A common feature of most of the related works is the poverty of results, thus an overall picture of the phenomenon is hardly attained. The main objective of the present work is to perform such a systematic analysis of numerical experiments concerning grid orientation effect. In this paper we perform several comparisons of solutions of the so-called radial problem. Since such problem has a simple analytical solution, a more clear quantification of the strength of grid orientation effects on each case can be done. In some way, this paper follows the guidelines of the work of Bajor and Cormack (1989). Our results are based in a more general discretization method, though, which is more amenable to the implementation of different interpolation schemes for approximating advection terms. Thus, one our goals is to determine if there exists an interpolation scheme able to prevent the development of grid orientation effects.

This paper is organized as follows. Section 2 gives a brief description of the basis formulation considered for obtaining the numerical results. Next, in Section 3 is described the radial flow problem, which will be used as basis test for studying the grid orientation effect. Several results for different grids, mobility ratios, and interpolation schemes are presented and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. THE NUMERICAL FORMULATION

We consider a two-dimensional two-phase immiscible flow model to carry out our study. This is the simplest model in which manifestations of the grid orientation effect are noticeable. For the sake of simplicity and clarity, we consider incompressible phases, homogeneous and isotropic media and absence of capillary pressure effects.

The mathematical model for that kind of flow can be reduced two a pair of differential equations (Peaceman, 1977; Aziz & Settari, 1979), the so-called Buckley-Leverett form of the saturation equation,

$$\phi\,\partial_t s + \vec{\nabla}\cdot(F\,\vec{\mathbf{v}}_T) = 0 \qquad (1)$$

and the pressure differential equation,

$$\vec{\nabla}\cdot\left(\lambda_T K \vec{\nabla} P\right) = 0 \qquad (2)$$

Here, ϕ and K are the porosity and the absolute permeability of the medium, respectively. The main variables on each equation are the saturation of water s and the pressure P. Furthermore, $\lambda_T = \lambda_w + \lambda_o$ is the total mobility (w stands for water phase and o stands for oil phase), and $F = \lambda_w/\lambda_T$ is known as fractional flux function, all of them being functions of the saturation. Finally, $\vec{\mathbf{v}}_T = \vec{\mathbf{v}}_w + \vec{\mathbf{v}}_o$ is the total velocity, a variable that couples the pressure and saturation equations, since it can be expressed as

$$\vec{\mathbf{v}}_T = -\lambda_T K \vec{\nabla} P \qquad (3)$$

In order to discretize the system of differential equations in space, the element-based finite volume method (EbFVM) will be considered. This approach follows the basic guidelines of the conventional finite volume method, namely the integration of differential equations over control volumes in a way that conservation is automatically enforced. However, a significant improvement in flexibility is introduced through the concept of element as the basic geometrical entity for discretization of the solution domain, since in this way the use of unstructured grids is readily affordable. Although only Cartesian grids are considered for the tests presented afterwards, the EbFVM has been chosen because of the easiness of incorporating different interpolation schemes into the numerical formulation.

Here we only give a very brief introduction on the application of EbFVM, more details can be found elsewhere (Maliska, 2004; Hurtado, 2005; Hurtado et al., 2007). The main geometric entities considered in EbFVM are shown in Fig. 1. The grid is formed by *elements*, which are quadrilateral in this work. These entities are used for defining the discretized geometry of the solution domain as well as for defining the spatial variation of physical properties of the medium, if necessary. The unknowns of the problem are calculated at the *nodes*, located at every element corner. Around every node is built a *control volume*, formed by portions of the elements sharing a common node. Every control volume is delimited by a certain number of *faces*, obtained joining the center of every neighboring element with the midpoint of its two sides sharing the node around which the control volume is built. Normally, a discretized equation

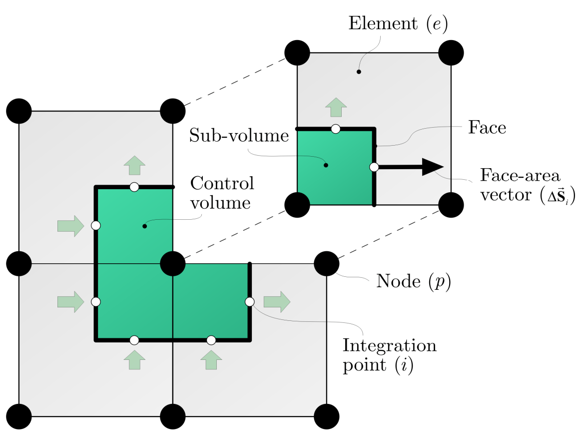

Figure 1. Main geometrical entities on the element-based finite volume method.

for a control volume represents some kind of balance, thus, it becomes necessary to compute fluxes across the faces. As surface integrals defining fluxes are usually approximated by the midpoint rule, the face center points are also important entities in EbFVM and they are commonly known as *integration points*.

After integration of differential equations (1) and (2) over a generic control volume, they can be approximated, respectively, as

$$\frac{s_p^{n+1} - s_p^n}{\Delta t^n} \phi_p \Delta V_p + \sum_e \left\{ \sum_i F_i^n (\vec{\mathbf{v}}_T)_i^n \cdot \Delta \vec{\mathbf{S}}_i \right\}_e = 0 \tag{4}$$

$$\sum_e \left\{ \sum_i (\lambda_T)_i^n K_e (\vec{\nabla} P)_i^n \cdot \Delta \vec{\mathbf{S}}_i \right\}_e = 0 \tag{5}$$

Here, subscripts specify the points where the variables are evaluated and superscripts specify the time level. A forward Euler approximation was considered for discretization in time while the midpoint rule was applied for approximating surface integrals at control volume faces. In Eqs. (4) and (5), ΔV_P is the volume of the control volume, $\Delta \vec{\mathbf{S}}_i$ is the area vector of a face limiting the control volume, and Δt^n is the timestep. The outer summations involve all elements contributing to the given control volume. On the other hand, the inner summations involve the two integration points that lies over the control volume faces inside a contributing element.

The forward Euler time approximation leads to the well-known IMPES approach (Aziz and Settari, 1979; Mattax and Dalton, 1990) for solving the discretized equations. This solution scheme is utilized in this work, since is the most straightforward way of obtaining numerical solutions from the two-phase discretized model. The time approximation considered in IMPES decouples the pressure equation from the saturation equation and because of this each equation can be solved independently for its own variable. For a given saturation field at time level n, after assembling of Eq. (5) for all control volumes in the grid, a linear system of equations for the nodal values of pressure is obtained. Solving this system, the corresponding discrete pressure field for time level n can be determined and, consequently, total velocities at integration points $(\vec{\mathbf{v}}_T)_i^n$ can be computed with Eq. (3). Finally, saturation can be advanced to the next time level solving Eq. (4) for s_P^{n+1} at each node in the grid. The whole transient solution for a two-phase displacement problem can be obtained repeating iteratively the basic steps outlined above.

At this point nothing has been stated about the spatial interpolation schemes needed for relating integration point values to nodal values in Eqs. (4) and (5). Pressure equation is an elliptic equation and a second-order scheme, like bilinear interpolation, can be used safely for expressing integration point values of pressure gradient $(\vec{\nabla} P)_i^n$ as a function of correspondent nodal values. This approximation leads to a 9-point stencil for the pressure equation in Cartesian grids, but it arises naturally from the EbFVM approach and has no connection with the well-known 9-point scheme after Yanosik and McCracken (1979). Interpolation schemes for the total mobility in pressure equation and fractional flux in saturation equation needs a more careful selection. The common approach in reservoir simulation for approximating such terms is the single-point upwind or donor cell scheme (Peaceman, 1977; Aziz and Settari, 1979). Into the EbFVM framework another interpolation schemes better suited can be easily implemented, however. These schemes are described in the following sections.

3. THE RADIAL FLOW TEST

The radial flow problem is adequate for performing numerical experiments in order to characterize the grid orientation effect, since it admits a simple analytical solution. This problem is depicted schematically in Fig. 2, where a single injection well is placed in an areal section of an initially oil-saturated reservoir. Bajor and Cormack (1989) considered a set of production wells located approximately at the same distance of the injection well, but we preferred to consider a domain with two outflow boundaries instead. As Fig. 2 shows, we discretize that domain with a regular Cartesian grid. We consider also a piston-type displacement, in which grid orientation effects are more severe. This type of displacement can be modeled, for instance, using the following flow functions

$$F(s) = s^2 \tag{6}$$

$$\frac{\lambda_T(s)}{\lambda_o^{\max}} = \frac{M}{M - (M-1)F(s)} \tag{7}$$

which coincide with those considered by Yanosik and McCracken (1979) and other authors. In Eq. (7), M is the mobility ratio, defined as

$$M = \frac{\lambda_w^{\max}}{\lambda_o^{\max}} \tag{8}$$

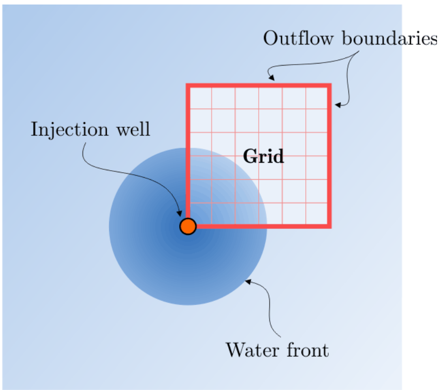

Figure 2. The radial flow problem.

where $\lambda_w^{\max}$ and $\lambda_o^{\max}$ are the maximum water mobility and oil mobility, respectively.

At time t, the exact solution for the front in a piston-type displacement is a circumference of radius

$$r_f = \sqrt{\frac{4Q_{inj}t}{\pi\phi}} \tag{9}$$

where Q_{inj} is the water injection flow-rate.

4. RESULTS

The interpolation scheme considered for approximating integration point values of fractional flux in Eq. (5) has a fundamental role on the grid orientation effect. The way in which that value is approximated will determine to a great extent the nature of the truncation error of the advection term in that equation. As Brand et al. (1991) mention, the more anisotropic is the distribution of that truncation error, the larger the probability of grid orientation effects arise.

Initially we consider numerical results for the radial flow problem employing the single-point upwind scheme. Unquestionably this is the most used scheme in commercial reservoir simulators because of its simplicity and robustness. Since Eq. (5) is a nonlinear hyperbolic equation, the use of a non-monotonicity preserving scheme would cause oscillations and unbounded values near shock fronts, situations which are unacceptable in reservoir simulation. The single-point upwind scheme produces always positive advection operators and therefore gives rise to stable and always monotonic solutions. But, as we will see, this not necessarily implies solutions free of grid orientation effects.

In the EbFVM context, the single-point upwind scheme is emulated as shown schematically in Fig. 3(a). The value at a given integration point is set equal to the value corresponding to the nearest node in the upstream direction, that is $F_i = F_{p-u}$ according to the notation employed in Fig 3(a). This scheme generates a 5-point stencil when applied to the discretization of the saturation equation for a given control volume, as depicted in Fig. 3(b).

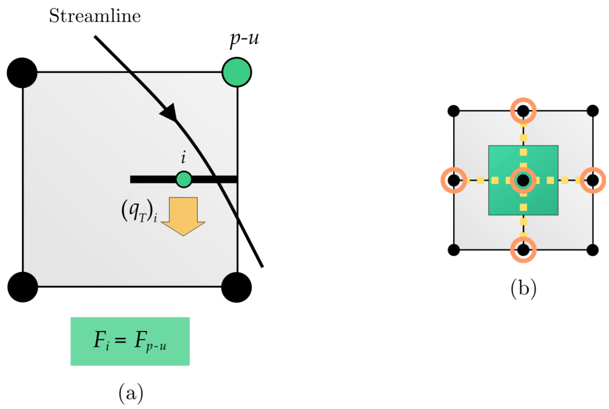

Figure 3. **(a)** Single-point upwind interpolation of an integration-point fractional flux function. **(b)** 5-point stencil resultant of that interpolation scheme.

The Fig. 4 shows results of the radial problem employing the single-point scheme for approximating fractional flux values at integration points in the saturation equation, but also for approximating total mobility values in the pressure equation. It is a common practice also in reservoir simulation to consider the same interpolation scheme in both equations, but it is not essential as we will show later. In each row of Fig. 4 are shown the results obtained on a given grid for four different mobility ratios ($M = 1, 10, 100, 1000$). A series of Cartesian uniform grids going form 5×5 elements to 80×80 elements is considered. On each graph, saturation isolines for the respective numerical solution after the injection of 0.2 pore volumes are superimposed to the analytical front at that time.

The results shown in Fig.4 confirm some of the initial assertions about the grid orientation effects and are consistent with results from other authors. First of all, only for $M > 1$ the distortion effects are evident and they worsen as M increases. The deterioration of the numerical solutions is more severe, however, when the grids are refined. The front seems to advance faster in regions where the flow is parallel to the grid lines, i.e. near the symmetry boundaries, causing a progressive and irreversible deformation. Only for $M = 1$ the numerical solutions behave as would be

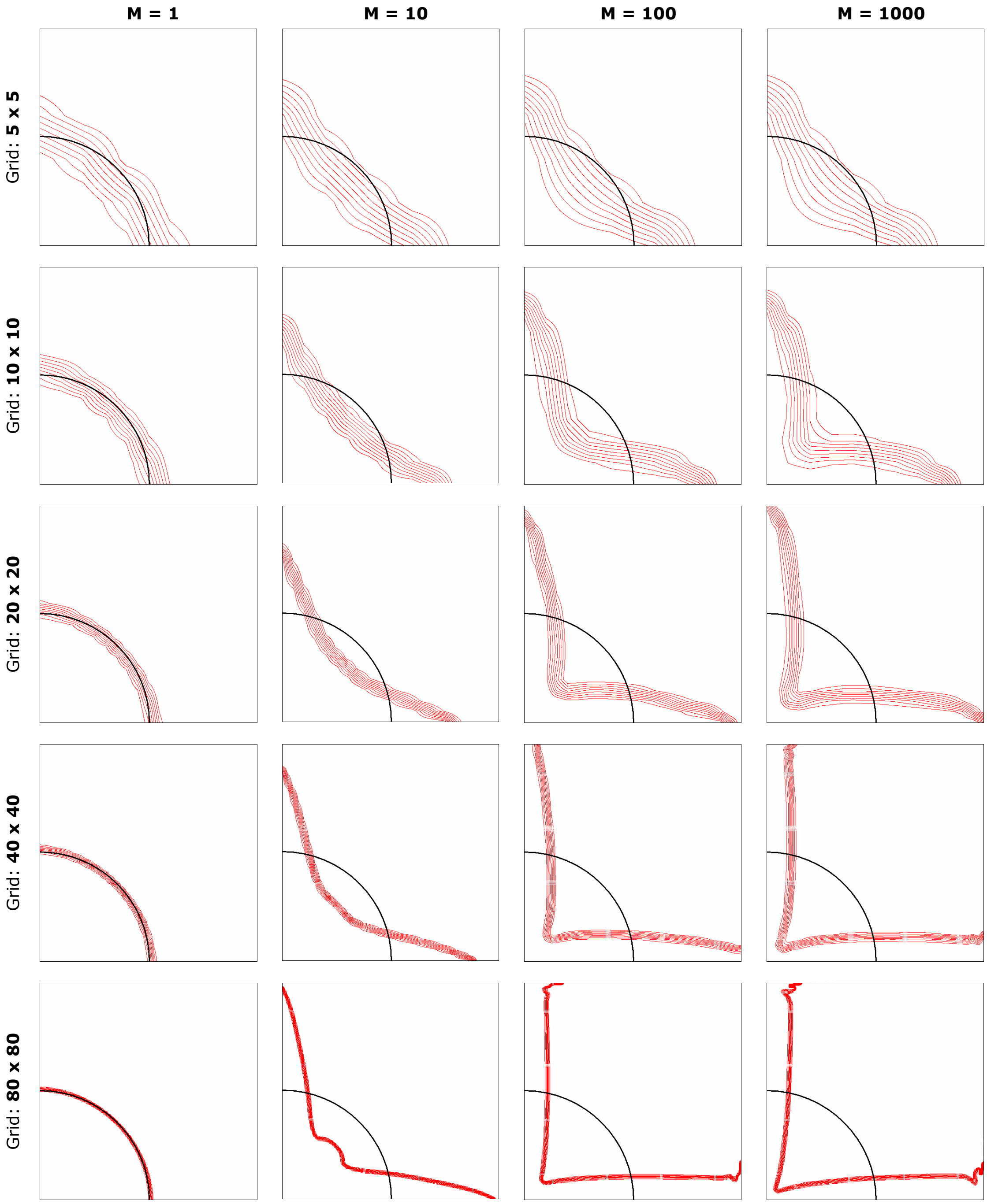

Figure 4. Comparison of numerical saturation fields with the analytical solution after injection of 0.2 pore volumes, for different mobility ratios and different grids. The single-point upwind scheme was used for both the total mobility in pressure equation and the fractional flux function in saturation equation.

expected, converging to the exact solution as the grid is refined. This behavior is observed also when $M < 1$, although these results are not presented herein. Summarizing, Fig. 4 shows how severe grid orientation effects can be when the mobility ratio is high and the grid is refined.

It is not mandatory to employ upwind-type interpolation for approximating total mobility in pressure equation, due to its elliptic nature. This fact had been already pointed out by Vinsome and Au (1979) and Potempa (1985a), among others. We tested the use of bilinear interpolation for that approximation, since this scheme is formally second-order (Raw, 1985). The results obtained using this approach are shown if Fig. 5, for the same cases considered before. As can be seen, there is a moderate improvement in the numerical solutions in relation to the ones obtained with the single-point scheme in both equations, particularly for the coarser-grid solutions.

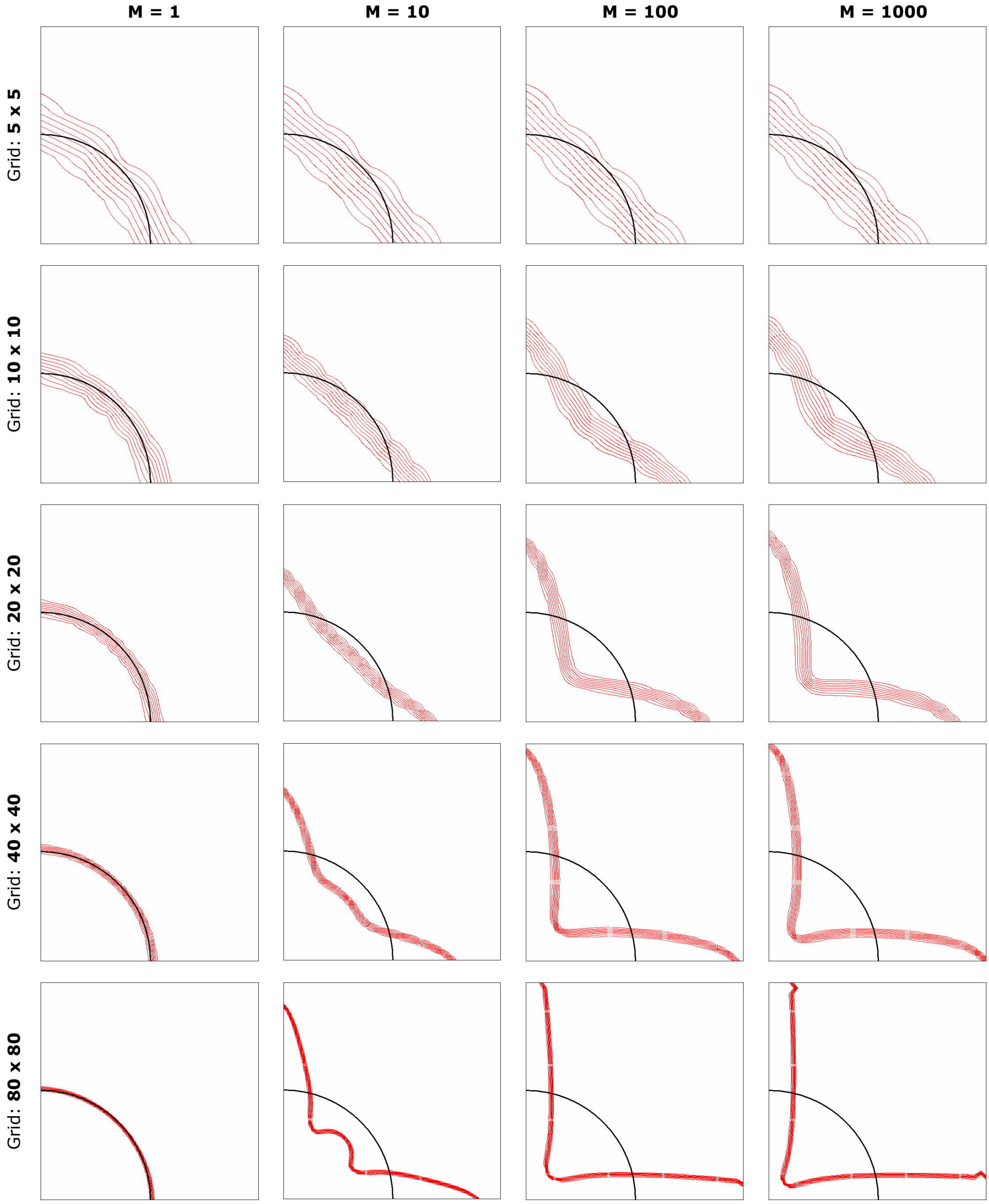

Figure 5. Comparison of numerical saturation fields with the analytical solution after injection of 0.2 pore volumes, for different mobility ratios and different grids. The bilinear scheme was used for total mobility in the pressure equation whereas the single-point upwind scheme was considered for the fractional flux function in the saturation equation.

Recognizing that the 5-point numerical approximation leads to a highly anisotropic distribution of truncation error, taken as a major source of grid orientation effects, several authors suggested the use of the so-called 9-point schemes (Yanosik and McCracken, 1979; Ko and Au, 1979; Potempa 1985a). Those schemes are generally obtained combining the 5-point discretizations of two Cartesian grids rotated 45^o. Since it is not possible to extend this technique for non-Cartesian grids, we considered a more general approach for constructing a 9-point scheme.

One of the problems with the single-point scheme is that the actual flow direction is not considered. A flow with the local direction represented by the streamline in Fig. 3 must carry the influence of the value at node $p\text{-}u$ but also the influence of the value at the top left node. Since the single-point scheme includes only the $p\text{-}u$ nodal influence no matter which the local direction of flow is, it approximates the advection transport very poorly. Raw (1985) and Schneider and Raw (1986) proposed an alternative scheme in the context of the application of EbFVM to the solution of the Navier-Stokes equations. This scheme, which we call flow-weighted upwind scheme (FWUS), takes into account the flow direction and maintains at the same time the stability features of the single-point scheme. This is possible because the positivity of the assembled advection operator is guaranteed by construction.

The basic relation for interpolating an integration point value of F with FWUS can be written as

$$F_i = (1-\Lambda_i)F_{p-u} + \Lambda_i F_{i-u} \tag{10}$$

where F_{p-u} and F_{i-u} are the nearest upwind nodal value and integration point value, respectively, as Fig. 6(a) illustrates. Moreover, Λ_i is an interpolation factor which is related in the scheme with the ratio between the flow-rate across the upwind face and the flow-rate across the given face, that is,

$$\omega_i = \frac{(q_T)_{i-u}}{(q_T)_i} = \frac{(\vec{\mathbf{v}}_T)_{i-u} \cdot \Delta\vec{\mathbf{S}}_{i-u}}{(\vec{\mathbf{v}}_T)_i \cdot \Delta\vec{\mathbf{S}}_i} \tag{11}$$

The value of this flow ratio is directly connected to the local direction of the flow because it gives the proportion of fluid that crosses the i face and comes from the $i\text{-}u$ face. The main idea of the scheme is that this amount of fluid is somehow influenced by the F_{i-u} value, whereas the remaining fluid is influenced by the F_{p-u} value. The term associated to F_{i-u} in Eq. (11) relates the interpolated value with nodal values other than F_{p-u}. This arises because Eq. (11) should be applied to the four integration points inside an element, and therefore a system of four linear equations arises, relating the integration point values with the four nodal values. After assembling the control volume equations, a 9-point stencil for the saturation equation is obtained for Cartesian grids, as Fig. 6(b) illustrates. But FWUS approach can be applied also to unstructured grids discretizing heterogeneous and anisotropic media. More details about the implementation of FWUS can be found in Schneider and Raw (1985) and Hurtado (2005).

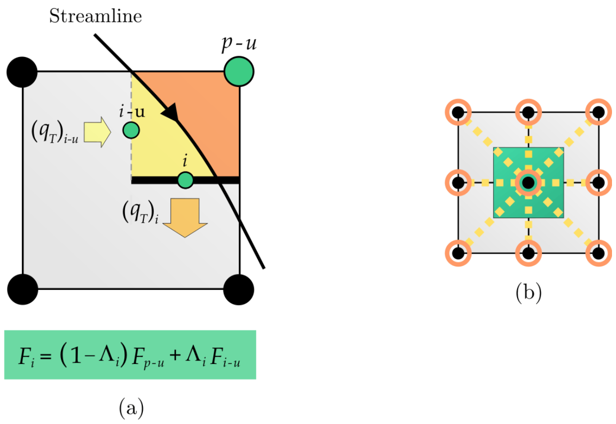

Figure 6. **(a)** Flow-weighted upwind interpolation of an integration point fractional flux function. **(b)** 9-point stencil resultant of that interpolation scheme

Schneider and Raw (1985) derived Eq. (10) considering the following functional form for the interpolation factor

$$\Lambda_i = \max[\min(\omega_i, 1), 0] \tag{12}$$

which is linear in the interval $0 \le \Lambda_i \le 1$ and establishes the limiting values $\Lambda_i = 0$ for $\omega_i \le 0$ and $\Lambda_i = 1$ for $\omega_i \ge 1$. Figure 7 shows the numerical results obtained employing FWUS with the interpolation factor defined by Eq.

(12) for the fractional flow function in saturation equation. As before, bilinear interpolation was considered for total mobility in the pressure equation.

The numerical solutions in Fig. 7 show a different manifestation of grid orientation effect. While the advance of the front in direction parallel to the grid lines is privileged by the single-point scheme, the advance in the diagonal direction is favored by FWUS with the Schneider and Raw's interpolation factor. Thus, the spurious numerical fingers that appeared before near the symmetry boundaries now develop along the diagonal of the solution domain. The extent of these fingers is quite similar, however. So, the amount of grid orientation effect is nearly the same than before.

The relation given by Eq. (12) is not the only alternative for using with FWUS. There are actually innumerable relations for the interpolation factor that satisfies the positivity condition that guarantees oscillation-free solutions

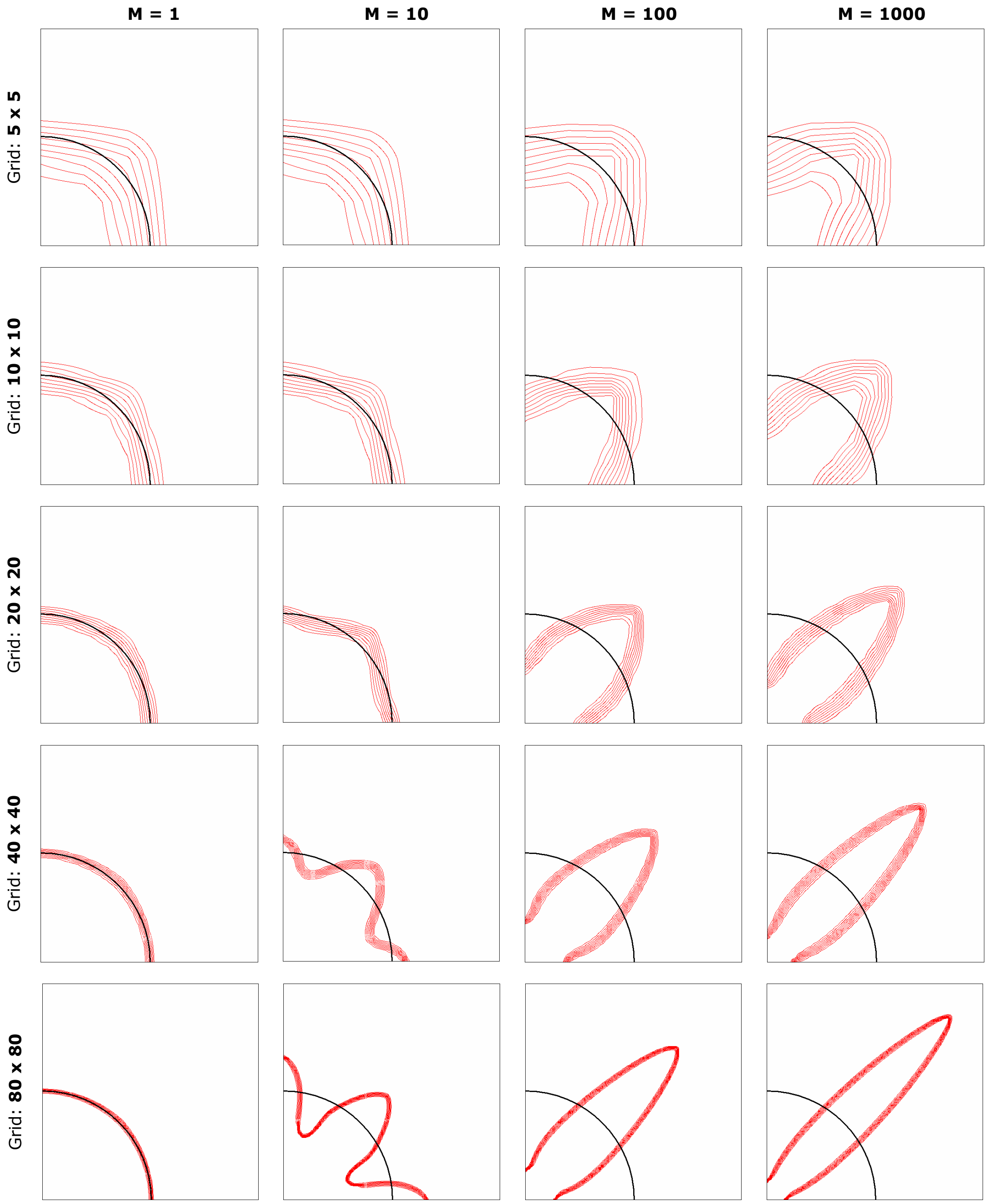

Figure 7. Comparison of numerical saturation fields with the analytical solution after injection of 0.2 pore volumes, for different mobility ratios and different grids. The bilinear scheme was used for total mobility in the pressure equation whereas the Schneider and Raw's form of FWUS was considered for the fractional flux function in the saturation equation.

(Hurtado, 2005). Several numerical experiments have been carried out employing different functional relationships. One relation that has produced better results is

$$\Lambda_i = \max[\omega_i / (1+\omega_i), 0] \tag{13}$$

which is tangent to $\Lambda_i = \omega_i$ at $\omega_i = 0$ and reaches asymptotically the limiting value $\Lambda_i = 1$ as $\omega_i \to \infty$. Results obtained using that relation are shown in Fig. 8. As can be observed, no traces of grid orientation effects appear in the solutions corresponding to the coarser grids. But they progressively degrade as the grids are refined, however, as occurred with the other schemes. This behavior seems to confirm the assertion of Brand et al. (1991) that the numerical

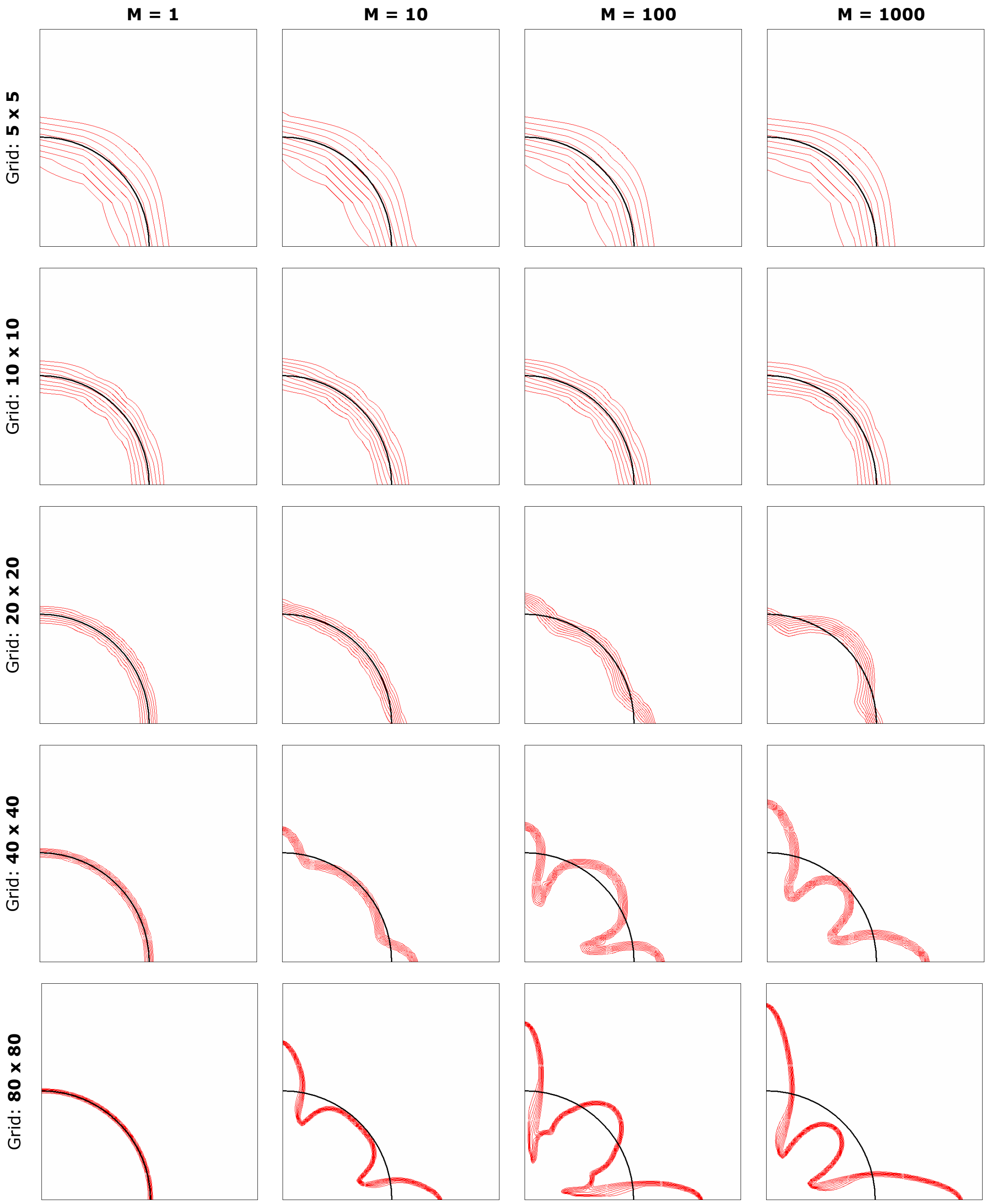

Figure 8. Comparison of numerical saturation fields with the analytical solution after injection of 0.2 pore volumes, for different mobility ratios and different grids. The bilinear scheme was used for total mobility in the pressure equation whereas the flow-weighted upwind scheme with $\Lambda_i = \omega_i / (\omega_i + 1)$ was considered for the fractional flux function in the saturation equation.

instabilities associated to high mobility ratios eventually give rise to grid orientation effect at certain grid refinement level where the numerical diffusion cannot damp out anymore that instabilities. For a suitable interpolation scheme such as the considered for obtaining the results shown in Fig. 8, it must exist a range of grids for which the discretization error actually decreases as grids are refined, but after certain refinement level this tendency is reversed and the error starts to grow.

5. CONCLUDING REMARKS

Several results showing the extent of grid orientation effect in the numerical simulation of radial flow problem have been presented in this paper. The influence of factors such mobility ratio, grid refinement level, and interpolation schemes has been considered in the numerical experiments performed. Some of the interpolation schemes considered are novel in the reservoir simulation context. They have a much better performance that the conventional single-point schemes and are much more general than the traditional 9-point schemes, since it can be straightforwardly applied to unstructured-grid formulations.

The results presented herein can be useful as comparison basis in further research about grid orientation effect. As shown in this work, this problem is still unresolved since for relatively refined grids its manifestations become evident, no matter which interpolation scheme is utilized. New approaches are needed in order to understand better the mechanisms by which this numerical phenomenon arises and to propose definitive solution strategies.

6. ACKNOWLEDGEMENTS

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