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PEACEMAN'S WELL MODEL EVALUATION FOR NON-ISOLATED AND PARTIALLY PENETRATING WELLS

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Resumo

Com o avanço da tecnologia na aplicação de poços horizontais com trajetória arbitrária, alguns autores desenvolveram metodologias recentes que calculam o índice de poço de forma acurada, de modo que a maioria das hipóteses simplificativas do modelo tradicional de Peaceman não são mais necessárias. Nesses trabalhos, a complexidade da modelagem matemática é maior que o usual e o modelo têm aplicação mais ampla. Entretanto, nenhum deles mostra com clareza a diferença nas soluções que pode existir entre os modelos acurados e o modelo tradicional de Peaceman quando poços não isolados e/ou parcialmente penetrantes são empregados. O presente trabalho tem por objetivo avaliar a aplicabilidade do clássico modelo de Peaceman em situações onde três de suas hipóteses simplificativas são violadas, sendo elas: poço isolado, poço totalmente penetrante e regime permanente de escoamento. Dessa forma, nas avaliações são empregados poços parcialmente penetrantes próximos às fronteiras do reservatório operando em regime transiente. Para realizar essas avaliações, é obtida uma solução semi-analítica, baseada nas funções de Green, utilizada como referência. Os resultados mostram que o modelo de Peaceman não é exato quando poços não isolados são utilizados, por exemplo, para poços horizontais em reservatórios cuja extensão é muito maior que a espessura. Além disso, esse modelo também apresenta erros para poços parcialmente penetrantes, sendo acentuado em poços verticais, já que a sua formulação não considera o fluxo esférico nas extremidades do poço.

Abstract

With the advance of the technology in the application of horizontal wells, skewed or with arbitrary trajectory, some authors have been developing methodologies in order to calculate an accurate well index, so that most of Peaceman's model assumptions can be avoided. In these works, the mathematical formulation is more complex than the usual and the model is less restrictive. However, none of them show clearly the differences in the solutions that may exist between these recent models and the classical model when non-isolated and partially penetrating wells are employed. The objective of the present work is to evaluate Peaceman's formulation in situations where three of its assumptions are violated, namely: isolated well, fully penetrating well and steady-state flow. Thus, the evaluation is done for partially penetrating wells near the boundaries and operating during the transient regime. For this purpose, a semi-analytical solution is obtained, based on Green's function, and used as reference. The results demonstrate that Peaceman's model is not exact when non-isolated wells are used, e.g., horizontal wells in thin reservoirs. Furthermore, this model also presents errors for partially penetrating wells, especially in vertical wells, as his formulation does not consider the spherical flux in the extremities of the well.

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1. Introduction

One of the most employed models to calculate the well index in reservoir simulation is the classical model proposed by Peaceman [11] that is based on the following assumptions [10]:

1. Single phase flow;
2. Homogeneous medium;
3. Isolated well;
4. Fully penetrating well;
5. Cartesian grid;
6. Rectilinear well, parallel to one of the grid axes;
7. Pseudosteady or steady-state flow.

Even subjected to these restrictions, this model is often used in situations where they are not respected, as more accurate models are not simple to be obtained. Recently, some authors developed methodologies that calculate the well index for arbitrary well configurations so that the classical model assumptions can be avoided. These models, as well as Peaceman's [11], are based on analytical solutions in the near well region, and have been studied by some authors [1][2][3][4][5][7][10][13]. Ding [2] employs the potential layer theory to determinate an analytical pressure field near the well, and, with this solution, to change the transmissibilities in the wellblock faces. This model provides an accurate behavior for the wells, however the mathematical formulation is complex and there are few references applying this theory in reservoir simulation. Durlofsky [3], Economides [4], Oyang [7], Maizeret [10] and Wolfsteiner [13] obtained a semi-analytical solution, based on Green's function, in the well neighborhood. This methodology was first applied in reservoir problems by Gringarten and Ramey [5].

In Wolfsteiner [13], for example, the idea is to generate a user-defined local well domain and to solve a particular single phase problem with inflow prescribed, both semi-analytically and numerically. With the flow rate $q_{w,i}$ and the pressure $p_{w,i}$ for each completion from the semi-analytical solution, and with the with pressure P_i from the numerical solution for each block i intercepted by the well, the well index is calculated as

$$WI_i = \pm \frac{\mu q_{w,i}}{P_i - p_{w,i}} \tag{1}$$

In these works, the mathematical formulation is more complex than the usual and the model is less restrictive. However, none of them show clearly the difference in the solutions that may exist between these recent models and the classical model when non-isolated and partially penetrating wells are employed. The objective of the present work is to evaluate Peaceman's formulation [11] in situations where assumptions 3, 4 and 7 are violated, that is, when non-isolated and partially penetrating wells during the transient regime are operating. For this purpose, the semi-analytical procedure described in Economides [4], Durlofsky [3] and Maizeret [10], based on Green's function, is used as reference.

Several tests were carried out, considering a closed box domain with different well configurations and different reservoir dimensions. The results demonstrate that the Peaceman's model [11] is not exact when non-isolated wells are used, e.g., horizontal wells in thin reservoirs. Furthermore, this model also presents errors for partially penetrating wells, especially in vertical wells, as his formulation does not consider the spherical flux in the extremities of the well.

2. Problem Description

According to the objectives previously described, Peaceman's model [11] is evaluated when assumptions 3, 4 and 7 are violated. In this way, the problem to be solved is a single-phase, incompressible and transient flow, with a homogeneous, isotropic and compressible medium in a box-shaped domain. The governing equation, in dimensionless variables, is

$$\nabla^2 p_D = \frac{\partial p_D}{\partial t_D} \tag{2}$$

The dimensionless variables are defined as

$$t_D = \frac{t}{x_e^2}\frac{K}{\mu c_r \phi_0}; \qquad p_D = \frac{K x_e}{q\mu}\left(p_{ini} - p\right) = \frac{p_{ini} - p}{q_D p_{ini}}; \qquad q_D = \frac{q\mu}{K x_e p_{ini}}; \tag{3}$$

where K is the absolute permeability, c_r the rock compressibility, ϕ_0 the reference porosity, μ the fluid viscosity, q the well total flow rate and x_e is the reservoir length in the x direction. The dimensionless coordinates are defined as

$$x_D = \frac{x}{x_e}; \quad y_D = \frac{y}{x_e}; \quad z_D = \frac{z}{x_e} \tag{4}$$

The initial condition is uniform pressure p_{ini} and the boundary condition is no-flux at the walls and uniform flux along the well. Mathematically, these conditions are written as

$$\begin{cases} p_D\left(x_D, y_D, z_D\right) = 0 \\ \left.\partial p_D \middle/ \partial n \right|_{S_i} = 0 \\ \left.\partial p_D \middle/ \partial r_D \right|_0 = \dfrac{1}{2\pi\, r_{Dw} L_D} \end{cases} \tag{5}$$

with $r_{Dw} = r_w / x_e$ and $L_D = L / x_e$ being the dimensionless well radius and length respectively.

3. Semi-analytical Solution

The procedure adopted to obtain the semi-analytical solution for the described problem is presented in the following steps [9][10][8]:

1. The dimensionless pressure *ipsrc* at any point $\left(x_D, y_D, z_D\right)$ subjected to an instantaneous point source/sink located at $\left(x_{D0}, y_{D0}, z_{D0}\right)$ is calculated by

$$ipsrc = \frac{1}{x_{D_e}} iplsrc\left(\frac{x_D}{x_{D_e}}, \frac{x_{D_0}}{x_{D_e}}, \frac{t_D}{x_{D_e}^2}\right) \times \frac{1}{y_{D_e}} iplsrc\left(\frac{y_D}{y_{D_e}}, \frac{y_{D_0}}{y_{D_e}}, \frac{t_D}{y_{D_e}^2}\right) \times \frac{1}{z_{D_e}} iplsrc\left(\frac{z_D}{z_{D_e}}, \frac{z_{D_0}}{z_{D_e}}, \frac{t_D}{z_{D_e}^2}\right) \tag{6}$$

where *iplsrc* represents the Green's function for an one-dimensional instantaneous source/sink solution considering an infinite plane in each direction [10]

$$iplsrc\left(x_D, x_{D_0}, t_D\right) = \frac{1}{\sqrt{4\pi t_D}} \sum_{n=-\infty}^{\infty} \exp\left[\frac{-\left(2n + x_{D_0} - x_D\right)^2}{4t_D}\right] + \exp\left[\frac{-\left(2n - x_{D_0} - x_D\right)^2}{4t_D}\right] \tag{7}$$

When $t_D > \pi/4$, *iplsrc* converges faster using Fourier series:

$$iplsrc\left(x_D, x_{D_0}, t_D\right) = 1 + 2\sum_{n=1}^{\infty} \exp\left(-n^{2\pi^2 t_D}\right) \cos\left(n\pi x_D\right) \cos\left(n\pi x_{D_0}\right) \tag{8}$$

2. Integrate *ipsrc* over time in order to obtain the continuous point source/sink solution

$$cpsrc = \int_0^{t_D} ipsrc\, dt_D \tag{9}$$

3. Integrate *cpsrc* along the well trajectory, obtaining the continuous line source/sink solution

$$clsrc = \int_0^{L_D} cpsrc\, dl \tag{10}$$

In this work, the procedure described is performed to determinate the dimensionless well pressure considering only one well in the domain with uniform flow rate condition. When more than one is present, the superposition principle must be used [8]. When uniform pressure is required, instead of uniform flow rate, the well must be split into *n* segments, treating each segment as a line source with uniform flux, using the procedure above and the superposition principle. Note that even when uniform pressure is used, the total flow rate for the well is prescribed and known, whereas the well pressure and the flow rate for each segment are unknown. Thus, a linear system with *n+1* equations must be solved in order to determinate the flow rate for each segment and the uniform well pressure [10].

The time and space integrations in Equations (9) and (10) are performed numerically using an adaptive stepsize control in Runge-Kutta method [6] and a Romberg integration method [12], respectively.

3.1. Semi-analytical well index

First, the semi-analytical solution must be founded using the method described above, providing the flow rate $q_{w,i}$ and the pressure $p_{w,i}$ for each completion. The same problem is applied to obtain the numerical solution for wellblock pressure P_i, prescribing the sources/sinks $q_{w,i}$ from the semi-analytical solution for each block i intersected by the well. The correct well index for block i is now computed using these three quantities:

$$WI_i = \pm \frac{\mu q_{w,i}}{P_i - p_{w,i}} \tag{11}$$

4. Results

Several tests were performed in order to verify the numerical implementation and the semi-analytical procedure, and all of them provided results that were in agreement with the ones available in the literature. One of these verification tests is shown in the following section, where the numerical solution using Peaceman's well index [11] and the semi-analytical solution are in good agreement.

4.1. Example 1: isolated and fully penetrating well

In this test one isolated and fully penetrating vertical well, depleting the reservoir, is simulated. The reservoir domain is a closed box (2,247.50 x 2,247.50 x 18.75 meters) and the well is located at the middle of the reservoir, working with uniform flux and constant flow rate ($80\, m^3 / day$) along the simulation. The input data is summarized in Table 1. The well index for the semi-analytical solution and for Peaceman's model [11] are illustrated in Figure 1a, and the correspondent well pressure for each method in Figure 1b. The problem has a transient period and the pseudosteady-state is reached in approximately 200 days. After this period, all assumptions from Peaceman's model [11] are obeyed, thus the well pressure from this model and from the semi-analytical solution must be the same. The difference between the well pressure solutions in the beginning of the simulation and during the pseudosteady-state regime was 0.70% and 0.08%, respectively. For the well index, it is observed 7.7% and 0.05% of difference, respectively.

Table 1. Input data for Example 1.

Fluid and reservoir parameters		**Well parameters**	
Rock compressibility [1/kPa]	1.00E-07	Operation	Uniform and constant flux
Absolute permeability xx [mD]	10	x = y [m]	1123.75
Absolute permeability yy [mD]	10	z [m]	0 - 18.75
Absolute permeability zz [mD]	10	Flow rate [m³/day]	80
Fluid Density [kg/m³]	1000	Radius [m]	0.07
Fluid Vistosity [cP]	10	**Grid parameters**	
Initial Pressure [kPa]	5.00E+05	Domain [m]	2247.5 x 2247.5 x 18.75
Reference porosity	0.2	Nx x Ny x Nz	31 x 31 x 1

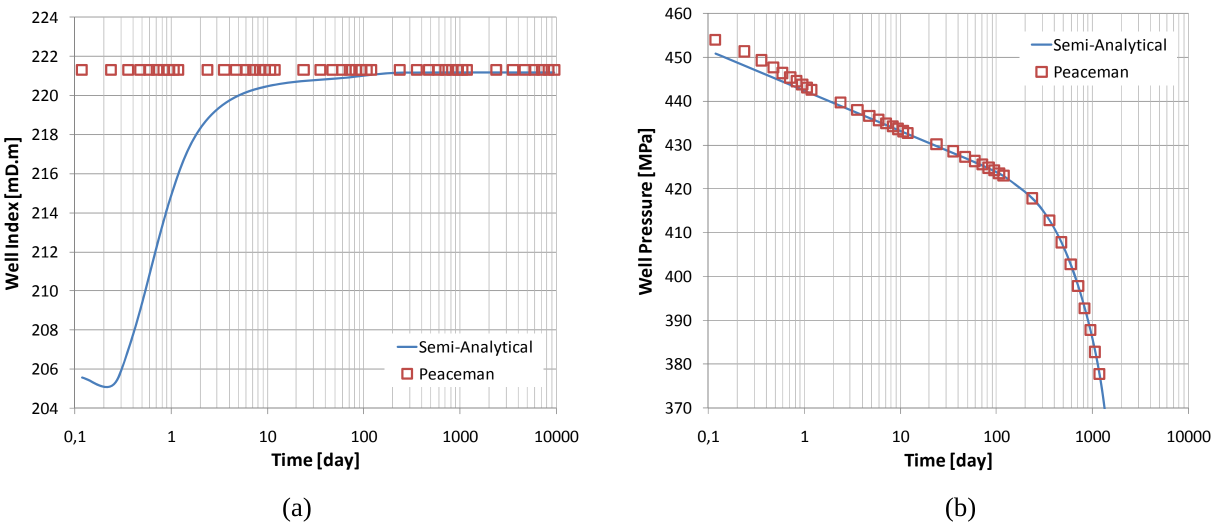


Figure 1. Well index (a) and well pressure (b) along the time.

4.2. Example 2: non-isolated and partially penetrating wells

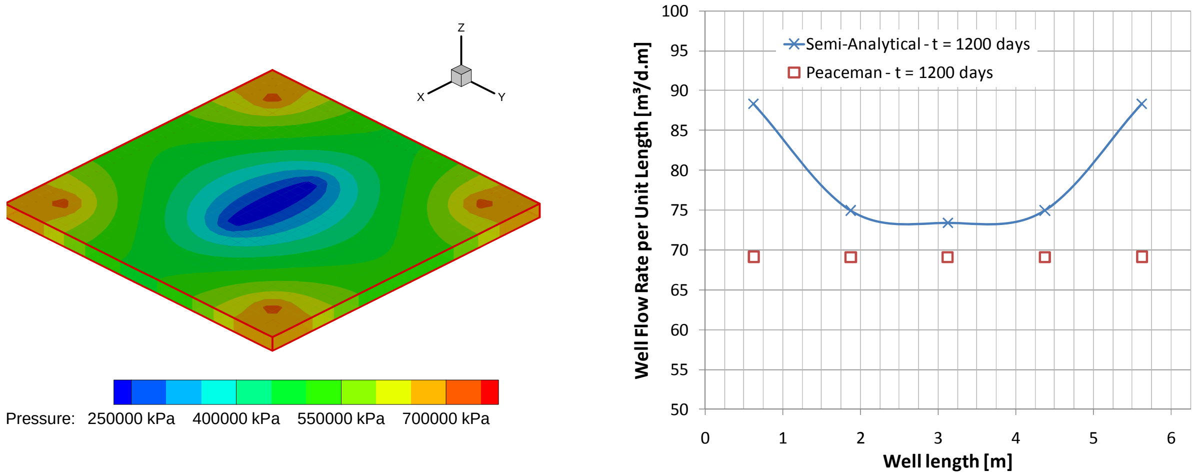
The following test was created in order to evaluate the numerical results when Peaceman's model [11] is used violating three assumptions: isolated well, fully penetrating well and steady-state regime. The domain is the same from the previous test, with one horizontal and partially penetrating producer well and four vertical and partially penetrating injector wells disposed symmetrically near the corners. The fluid and reservoir parameters are the same from Example 1, summarized previously in Table 1. The grid refinement in this case is 31 x 31 x 15 gridblocks and the well data is displayed in Table 2.

Table 2. Well data for Example 2.

Injector 1	**Injector 2**	**Injector 3**	**Injector 4**	**Producer**
x [m]: 253.75 y [m]: 253.75 z [m]: 6.25 - 12.5	x [m]: 253.75 y [m]: 1993.75 z [m]: 6.25 - 12.5	x [m]: 1993.75 y [m]: 253.75 z [m]: 6.25 - 12.5	x [m]: 1993.75 y [m]: 1993.75 z [m]: 6.25 - 12.5	x [m]: 725.0 - 1522.5 y [m]: 1123.75 z [m]: 9.375
Operation: Uniform pressure Analytical flow rate [m³/day]: 500 Radius [m]: 0.07				Operation: Uniform pressure Analytical flow rate [m³/day]: 2000 Radius [m]: 0.07

The problem is solved semi-analytically and then numerically with Peaceman's model [11]. The semi-analytical procedure uses constant total well flow rate and uniform, but unknown, well pressure as prescribed conditions. The well flow rate distribution and the well pressure are the solutions for the problem. To Peaceman's numerical solution, in this particular problem, the uniform well pressure was chosen to be prescribed, with the total flow rate and the flow rate distribution to be determined. This uniform pressure arrives from the semi-analytical solution, varying in each time step. Note that even though the analytical well pressure varies along the time, it is always uniform along the well. Hence, Peaceman's numerical solution, with these conditions prescribed, provides the well flow rate distribution, varying with time and differing from the semi-analytical solution.

For qualitative purpose, regarding well positioning, the solution for the pressure field in the steady-state regime, when the semi-analytical approach is used, is shown in Figure 2a. The flow rate per unit of length is the same for all injectors, due to symmetry, but varies during the transient period and along the well length. This distribution for the steady-state is shown in Figure 2b for one of the injectors.



(a) (b)

Figure 2. (a) Pressure field; (b) well flow rate per unit length for each injector in the steady-state regime.

The first interesting result of this work is the flow rate distribution obtained from Peaceman's well index [11]. It cannot reproduce the spherical flux in the extremities of the well, while the solution from the semi-analytical well index reproduces this behavior. As the reservoir thickness is small compared to its length, the pressure field is almost uniform in the *z* direction. Therefore, in this case the only way to reproduce the spherical flux at the extremities of the injector wells would be with higher values for the well index in that region. However, this is not possible with Peaceman's model [11], as the well index is the same for all wellblocks. The result is a uniform and wrong distribution for the flux along the injector wells. For the well index and flow rate in the injector wells, it is found 22% of error in the extremities and 6% in the center.

The flow rate for the producer well is shown in Figure 3 for three simulation times: in the early transient period ($t = 0.12$ days), in the time $t = 12$ days and in the steady-state period ($t > 400$ days). The absolute difference for the flow rate in the extremity of the well, for each time, is respectively 13%, 4% and 16%, changing its signal during the transient.

The comparison between Peaceman's well index [11] and the semi-analytical well index during all the simulation time is shown next. Note that the semi-analytical well index varies along the well, being shown for two completions (extremity and center) in Figure 4. The maximum difference between the well indexes occurs in the center of the well, presenting 28% of error in the steady-state regime. Note that the difference in the flow rate per unit of length is not the same as the difference in the well index. Even though the semi-analytical well index presents a higher value in the center of the well, the flow rate is higher in the extremity. Thus, the semi-analytical well index is addressed to result in the correct flow rate distribution for the correspondent pressure field.

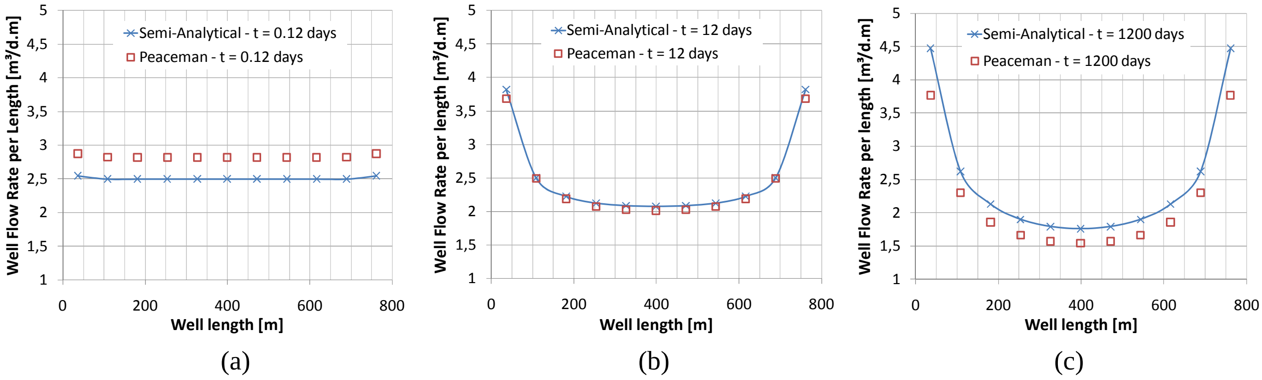


Figure 3. Well flow rate per length: (a) t = 0.12 days; (b) t = 12 days; (c) t = 1200 days.

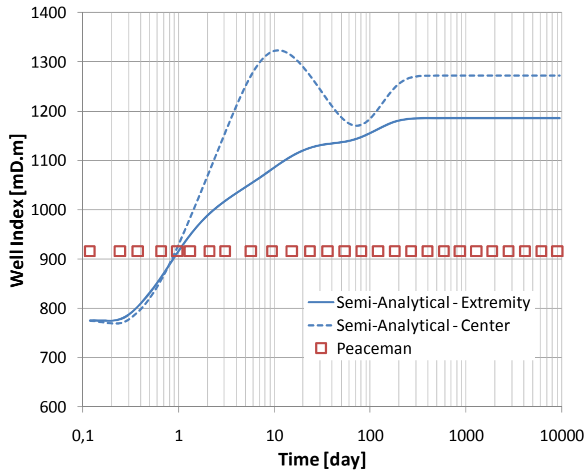


Figure 4. Peaceman's and semi-analytical well index along the simulation time.

It is important to mention that tests with other well configurations for this same problem were carried out and it was observed that when fully penetrating wells were used, Peaceman's well index [11] provided better results, without significant errors in the steady-state regime. In the same way, for isolated, but partially penetrating wells, Peaceman's solution [11] also provided better results.

5. Conclusion

In this paper, Peaceman's well model [11] was evaluated when some of its assumptions were violated. For this purpose, a semi-analytical procedure was applied to determinate the well pressure and flow rate, and, in conjunction with the correspondent numerical solution, to calculate the correct well indexes.

The results for the test cases presented and for other ones not shown here, among the three assumptions analyzed, two of them must be violated together in order to result in considerable errors, that are: isolated and fully penetrating well. Besides this, the use of Peaceman's model [11] during the transient is also not accurate.

As stated previously, Peaceman's model [11] is based on several assumptions. In this work, only three of them were violated and poor results were obtained. When non-conventional wells are used, i.e., wells with arbitrary trajectory or multiple branches, other assumptions will be violated and even more significant errors are expected.

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