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CONJUGATE CONDUCTION, CONVECTION AND RADIATION PROBLEM IN WALLS CONTAINING CIRCULAR CELLS

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SUMMARY

The transient heat and mass transfer phenomenon through walls containing circular cells is of great importance in simulation of thermal behavior in buildings. This paper presents the numerical solution of the heat transfer problem, that is, conduction in the brick and convection and radiation in the cell. Due to geometry complexity the problem is solved using a boundary-fitted coordinate system. The results demonstrate that, for the outer cell's dimension studied, the heat flux is not significantly affected due the presence of the cavity, and that the radiation plays an important role.

INTRODUCTION

The analysis of the heat transfer through solid walls containing cavities is a challenging problem for the fluid dynamicist. First, the solid part of the wall is itself a porous media where a conjugate heat and mass transfer problem need to be solved. Coupled to this problem one has the convection-radiation problem inside the cavity. Associated to that, the boundary conditions in a real problem are always changing in time due to the transient nature of the solar radiation, temperature and relative humidity of the environment. To add a definite complicating factor, it suffices to say that the real problem is fully three-dimensional.

In spite of the complexity of the problem it is possible to formulate some simplifying hypothesis such that important engineering informations can still be obtained.

Numerical and experimental studies dealing with natural convection inside cavities without considering conduction in the walls are abundant in the literature and excellent reviews can be found [1,2], such that no attempt will be made here of reviewing them.

Analysis of the conjugate problem, considering conduction at the cavity walls and convection inside the cavity, has also received attention but mainly for square and rectangular geometries with geometrically similar cavities [3,4,5]. The problem considering also radiation at the cavity walls did not motivate too much research work [6], and again they are restricted to geometries whose boundaries follow the cartesian coordinate system.

This paper presents a two dimensional methodology with the analysis of a coupled problem, conduction in the solid, convection-radiation in the cavity, for arbitrary cavities. The methodology herein presented is a powerfull tool to simulate this coupled problem for different geometries as well as to serve as a basis for the solution of more complicated mathematical models which may include 3D effects.

PROBLEM FORMULATION

The cellular wall in consideration is depicted in figure 1a. For a large wall in the y and z directions, it is reasonable to assume no temperature changes in the z-direction as well as to assume that in the y-direction one has a repetitive boundary condition. As a result the analysis is performed in the cell shown in figure 1b. The additional simplifying

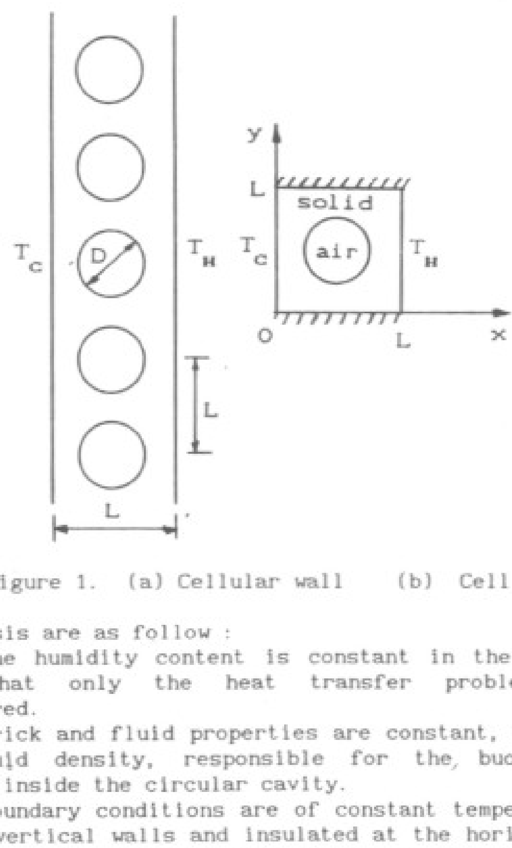

Figure 1. (a) Cellular wall (b) Cell

hypothesis are as follow :

1. The humidity content is constant in the brick such that only the heat transfer problem is considered.
2. Brick and fluid properties are constant, except the fluid density, responsible for the buoyances effects inside the circular cavity.
3. Boundary conditions are of constant temperature at the vertical walls and insulated at the horizontal ones. The latter condition will be again addressed.
4. At the solid fluid interface a continuous temperature and heat flux profiles are assumed. In addition it is assumed that the emissivity of the wall is 1.0 . This is not a severe assumption since normal building bricks have emissivity in the order of 0.93. This assuption allows to use the black body model for the radiation exchange problem.

GOVERNING EQUATIONS

As mentioned, the governing equations need to be solved in the solid domain and in the circular cavity. With the listed assumptions these equations have the following forms. For the solid part,

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$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (1)$$

and for the fluid inside the cavity,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho vu)}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] \qquad (3)$$

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} = - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu \frac{\partial v}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu \frac{\partial v}{\partial y}\right] + \rho g \qquad (4)$$

$$\frac{\partial(\rho uT)}{\partial x} + \frac{\partial(\rho vT)}{\partial y} = \frac{\partial}{\partial x}\left[\frac{k}{c_p} \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k}{c_p} \frac{\partial T}{\partial y}\right] \qquad (5)$$

It is important to note that the usual Boussinesq approximation is not being used. Instead the density changes locally and is related to temperature and pressure through the state equation for ideal gases.

BOUNDARY CONDITIONS

For the solid wall, according to figure 1 , one has

$$T(0,y) = T_C \qquad (6)$$

$$T(L,y) = T_H \qquad (7)$$

$$\frac{\partial T}{\partial y}(x,0) = 0 \qquad (8)$$

$$\frac{\partial T}{\partial y}(x,L) = 0 \qquad (9)$$

The symmetry condition, originating eqs. (8) and (9) is an approximation since the natural convection creates different temperature profiles for the top and bottom of the circular cavity. This, through conduction in the solid, will also interfere in the temperature distribution at the top and bottom of the solid part. This difference will decrease as the ratio D/L decreases. In fact, for the situation tested (D/L = 0.5) the approximation is good. It is worth to note that the repetitive boundary condition could have been used without introducing any simplification. The insulated boundaries were used for the sake of simplicity only.

For the solid/fluid interface one has

$$u|_O = 0 \qquad (10)$$

$$v|_O = 0 \qquad (11)$$

$$T_a|_O = T_s|_O \qquad (12)$$

$$- k_s \frac{\partial T_s}{\partial n}\Big|_O = - k_a \frac{\partial T_a}{\partial n}\Big|_O + q_i \qquad (13)$$

where the symbols "O", "a" and "s" indicate the cavity wall, air and solid, respectively, and the second term in the right hand side of eq. (13) is given by

$$q_i = \sum_{j=1}^{n} \sigma F_{ij} \left(T_i^4 - T_j^4 \right) \qquad (14)$$

where q_i is the radiant heat flux exchanged by the element "i" with the "n" elements that can be "seen" by element "i". It is not a difficult matter to calculate the shape factor F_{ij} for this situation.

NUMERICAL METHODOLOGY

The problem is solved using a two-domain procedure, that is : the conduction equation, eq. (1), is solved using the heat flux from the convective problem added to the radiative heat flux given by eq. (14). The temperature from this solution serves as boundary condition for eqs. (2-5). The procedure is repeated until convergence.

Since the geometry in consideration is non-cartesian, it is employed two boundary-fitted coordinate systems, as can be seen in figure 2. The one employed for the fluid equations is simply connected, and its transformation onto a rectangular domain retains as boundary conditions, in the four sides of the transformed plane, the existing boundary conditions in the physical plane. For the solid equation the domain is doubly-connected and for mapping again onto a rectangular domain it is necessary to use the cutting procedure. This results in a computational plane with repetitive boundary conditions along the cuts, as shown in figure 3. Or, in other words, this means to say that there is no need of specifying boundary conditions in the cuts. This, of course, is in accordance with the physical problem which does not provide boundary conditions in these regions. As can be seen in figure 3, the boundary conditions are all associated with the south and north frontiers of the computational domain.

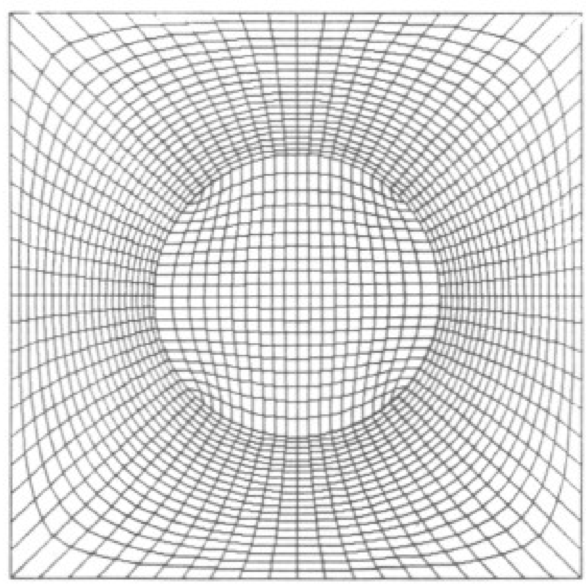

Figure 2. Computational grid for D/L = 0.5

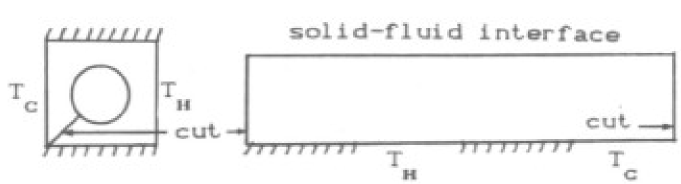

Figure 3. Mapping of the solid domain

The transformed equations are not presented here but the procedure to obtain them and details of the transformation can be found in [7]. The computational grid shown in figure 2 was generated using elliptic equations [8], with 20 x 20 volumes in the fluid domain and 80 x 20 in the solid part. It is to observe that the grids fits well at the boundaries making it easier to transfer informations (boundary conditions) from one domain to the other.

The solution of the natural convection problem was realized using the computer code MACH2D [9] with the appropriate simplifications. This code employs a co-located variable arrangement [10]. To handle the pressure-velocity coupling the SIMPLEC [11] method is employed, and the solution of the linear system of equations from the convective problem is obtained using MSI [12], and Gauss-Seidel for the conduction problem in the solid domain.

SOLUTION PROCEDURE

a) To save computer time during the iterative procedure the average temperature of T_H and T_C is initially assigned to all volumes in the domain.

b) Solve the equation in the solid wall assuming, in the first iteration, no heat flux at the solid fluid interface. Compute interface temperature using interpolation.

c) Solve for velocity and temperature for the fluid using as boundary condition the interface temperatures obtained in item b.

d) Compute the heat flux due to the convective flow.

e) With temperatures of item b it is calculated the flux by radiation using eq. (14).

f) The total heat flux, item d plus item e, is now used as boundary condition for the solid domain in item b, iterating until the prescribed convergence criterion is reached.

BASIC TEST CASES

When computing a numerical solution it is always desirable to have analytical solutions to test the numerical model. Of course, the problem one is dealing with precludes the determination of the analytical solution, but it is possible to analyse limiting cases. Using the full solution procedure, iterating from one domain to the other, density was kept constant and the air conductivity was made equal to the conductivity of the brick. The problem is then one-dimensional. It can be seen in figure 4 that the isotherms computed in each domain matches very well at the interface, reproducing the one-dimensional conduction problem. The maximum error of the numerical solution for this case, with T_H= 320 K , T_C= 300 K , k_{brick} = 0.69 W/mK, L = 0.1 m and D = L/2 is 4.3 %. For D = L/10 the error increases to 5.1 % .

As a second basic test the coupled problem is solved using no radiation. The qualitative behaviour of the natural convection flow is in accordance with the solution of the natural convection inside a circu lar cavity with no conduction considered at the walls. The isotherms for this case is shown in figure 5.

NUMERICAL RESULTS AND DISCUSSIONS

With the main goal of better understanding the heat transfer phenomenon involved, six differents situations are simulated, as below.

1) Vacuum in the cavity with and without radiation.

2) Conduction in the air in the cavity with and without radiation.

3) Convection in the air in the cavity with and without radiation.

For each situation above it is varied the diameter of the cavity.

This problem is very rich in informations if all parameters are varied, forming the dimensionless groups which governs the phenomenon. In this paper this complete analysis is not realized and this is the reason why the results will be shown keeping the dimensions in the variables involved. The full analysis is under preparation and will appear in a coming publication.

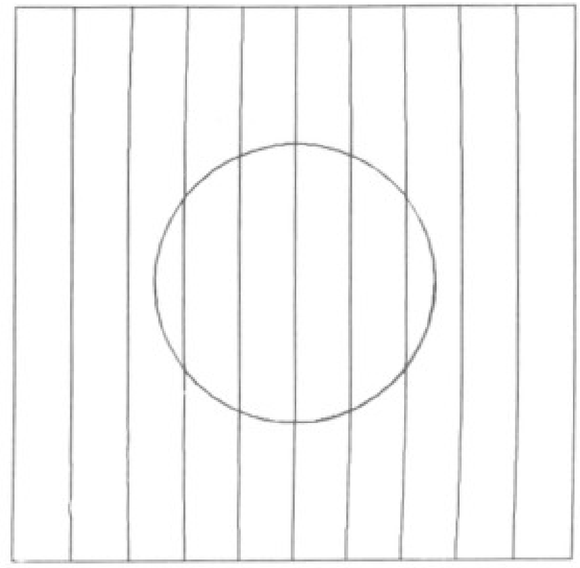

Figure 4. Isotherms of the one-dimensional conduction problem

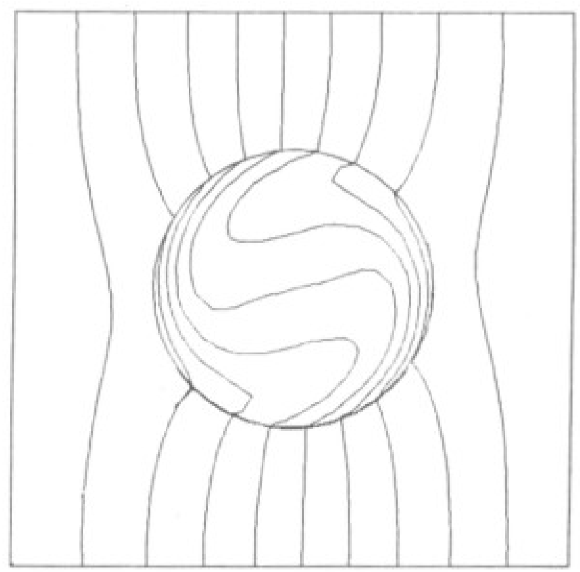

Figure 5. Isothems of the convective problem

The results are plotted using the concept of equivalent conductivity, defined as

$$k_{eq} = \frac{\text{heat transfer considering the cavity}}{\text{heat transfer considering the wall fully solid}}$$

where the denominator of the above equation is easily obtained exactly. Figure 6 shows the equivalent conductivity for the six different situations as a function of D , using T_H = 320 K , T_C = 300 K , k_{brick} = 0.69 W/mK and L = 0.1 m .

One important finding is the fact that the existence of the cavity, considering convection and radiation does not impair too much the heat transfer. In fact, the minimum value for k_{eq} encountered was 0.87 , which means that the brick with the cavity exchanges 13 % less heat than the wall fully solid. The results of figure 6 also demonstrate the importance played by radiation in the process. If radiation is not considered k_{eq} drops to 0.59 for a diameter equal to 0.08 m .

The assumption which considered thermal insulation in the horizontal faces is now addressed. Figure 7 shows the heat flux along the 360 degree of the cavity and the temperature along the insulated boundaries. The existence of the convection causes a slight unsymmetry in the heat flux at the solid-fluid interface, deviating slightly the used boundary condition from the repetitive one.

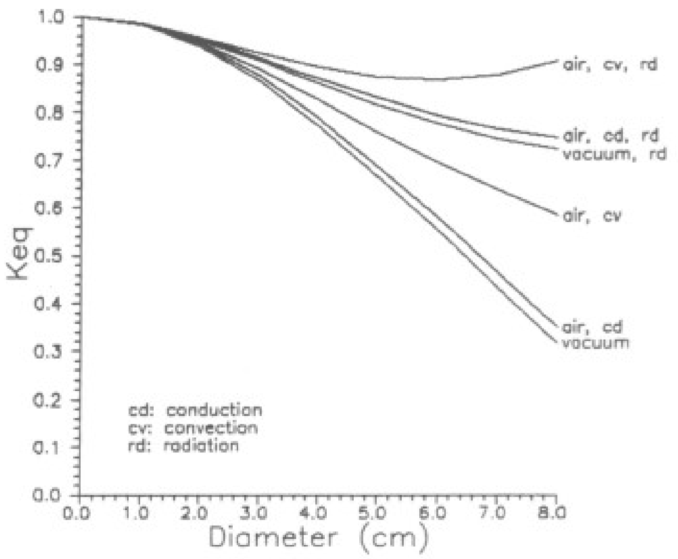

Figure 6 Equivalent conductivity versus cavity diameter

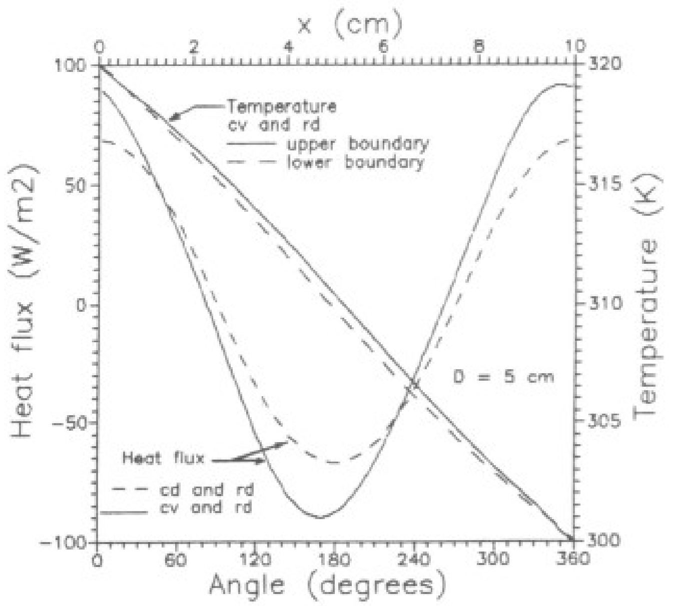

Figure 7 Heat flux along the solid-air interface and temperature along the insulated boundaries

CONCLUSIONS

The main conclusions of this paper are twofold.

First, it became clear that the analysis of complex problem involving complex domains can be done efficiently using a boundary-fitted grid. Several analysis can now be done for different shapes of the solid part and cavity.

Secondly, considering the physics of the phenomena, it was seen that, for the case tested, the existence of the cavity does not reduce significantly the heat transfer rates. The results also confirm that the radiation plays an important role in this type of problem, besides the low temperatures involved. However, as L and D decrease the relative importance of the radiation and convection in the cavity also decreases. As in insulating materials, conduction tends to be the principal heat transfer mechanism in the air cavities. As already mentioned, in a coming paper the conclusions will be generalized for the problem, considering the dimensionless groups involved.

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