NUMERICAL SIMULATION OF OIL SPILL TRAJECTORIES IN THE SEA

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Abstract

This paper presents a mathematical model and its numerical treatment for the forecasting oil spills trajectories in the sea. In order to estimate these slicks trajectories a new model, based on the mass and momentum conservation equations is presented. The model considers the spreading in the regimes when the inertial and viscous forces counterbalance gravity and takes into account the effects of winds and water currents. The numerical model is developed in generalized coordinates, making the model easily applicable to complex coastal geographies. Due to the similarity of the resulting equations with those used in Shallow Waters models, an adaptation to Finite Volume Method and generalized coordinates of the Semi-Implicit Finite Difference method presented by [Casulli and Cheng 1992] is used in this model.

INTRODUCTION

The sea transportation of crude oil by tankers or offshore pipelines has a significant associated risk of an accidental spill. When such spills occur near tourist or fishing regions, the damage is still worst. The recent oil spill in the Guanabara Bay, Rio de Janeiro, Brazil, caused by a pipeline rupture is a strong example of this broad impact.

The detailed knowledge of the spilled oil position and the area covered by the slick is of fundamental importance to take appropriate actions against pollution, like use of floating barriers, detergents, dispersants, etc. It is also important the estimation of potential risks in selecting pipeline routes, locating shoreline tanks and petrochemical industries. Therefore, a model to forecast the time-space evolution of the oil slick should make part of any environmental program that has the purpose of oil pollution combat.

As any fluid mechanics problem, two approaches for computing oil slicks trajectories are commonly encountered in the literature; Lagrangian and Eulerian models. Lagrangian models [Shen e Yapa 1988] consist basically in representing the oil slick by an ensemble of a large number of small parcels, which are advected by a velocity resulting from the combination of the action of winds and currents. Then, the slick is divided into pie shaped segments or strips, depending if the form of the slick is nearly circular or elongated and [Fay 1971] spreading formulas are applied to each segment to compute the spreading of the slick. Fay´s formulas consider the spreading of an oil slick in calm waters, where a slick, initially circular, will remains circular, just increasing its diameter. For the Eulerian approach, two model are usually encountered, those based in the mass and momentum equations applied to the oil slick [Hess and Kerr 1979], [Benqué *et. al.*1982], and those based on a convection-diffusion equation [Venkatesh 1988] (among others), in which the diffusive part of the equation represents the spreading of oil by itself and the convective terms represents the advection of oil by currents and winds. The model presented in this paper belongs to the first category of Eulerian models and it is based on the integration of mass and momentum equation over the thickness of the oil slick. It considers the spreading in inertial-gravity and viscous-gravity regimes, the slick transport by currents and wind and the oil evaporation.

MATHEMATICAL MODEL

The governing equations for the slick trajectory, are obtained by integrating the Navier-Stokes equations along the thickness of the slick.

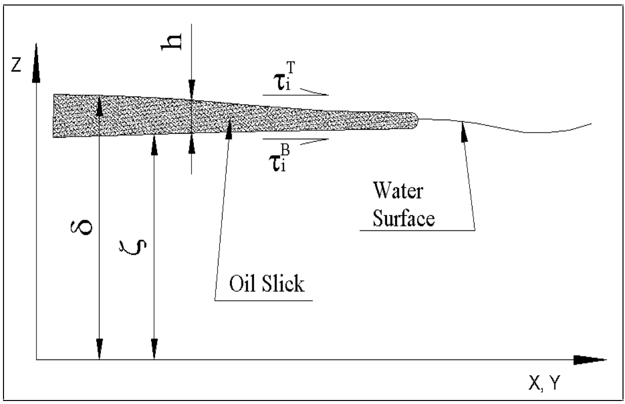


Figure 1: Variables considered in the vertical integration of governing equations.

Figure 1 shows schematically an oil slick being transported by the tensions exerted by water currents and winds. The oil flow is governed by mass and momentum equations for incompressible flows. These equations are:

$$\frac{\partial \mathbf{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \frac{\partial(\rho \mathbf{u}_i \mathbf{u}_j)}{\partial x_i} = \frac{\partial(\boldsymbol{\tau}_{ij})}{\partial x_i} - \frac{\partial p}{\partial x_i} \tag{2}$$

Following [Hoult 1972], we can consider that the oil viscosity is much large than the water viscosity. Thus, the vertical velocity gradients within the oil are much less than these gradients in the water or in the wind. It is, therefore, a good approximation to consider that the flow parameters (velocity and pressure) do not vary across the thickness of the slick.

After the integration, the governing equations obtained are,

$$\frac{\partial h}{\partial t}+\frac{\partial(\bar{\mathbf{u}}_i h)}{\partial x_i}=0 \quad (3)$$

$$\frac{\partial(\rho\bar{\mathbf{u}}_i h)}{\partial t}+\frac{\partial(\rho\bar{\mathbf{u}}_i\bar{\mathbf{u}}_j h)}{\partial x_i}=\frac{\partial}{\partial x_j}\left(h\mu\frac{\partial\bar{\mathbf{u}}_i}{\partial x_j}\right)+ + \boldsymbol{\tau}_i^T-\boldsymbol{\tau}_i^B-\rho g h\Delta\frac{\partial h}{\partial x_i} \quad (4)$$

where the bar variables represent vertical integral averages, h is the oil slick thickness and Δ is a parameter which relates the oil and water densities $\Delta=(\rho_o-\rho_w)/\rho_w$. The terms τ represent the shear tensions on top and bottom of the slick exerted by winds and water currents, respectively. These tensions were calculated as,

$$\boldsymbol{\tau}_i^T = C_f^{wind}\mathbf{V}_i^{wind} \quad (5)$$

$$\boldsymbol{\tau}_i^B = C_f^{water}\left(\mathbf{u}_i^{oil}-\mathbf{V}_i^{water}\right) \quad (6)$$

NUMERICAL SOLUTION

As can be seen the governing equations are similar to Shallow Waters equations. Then the semi-implicit method presented by [Casulli and Cheng 1992] is used for the treatment of the coupling between the thickness and velocity. In this case, the Finite Volume method with co-located variables and generalized coordinates were used. Then the model is applicable to arbitrary geometries such as complex coastal geographies. The transformed equations in terms of computational domain coordinates, ξ and η are,

$$\frac{\partial}{\partial t}\left(\frac{\rho h}{J}\right)+\frac{\partial(\rho h\tilde{U})}{\partial\xi}+\frac{\partial(\rho h\tilde{V})}{\partial\eta}=0 \quad (7)$$

$$\frac{\partial}{\partial t}\left(\frac{\rho h u}{J}\right)+\frac{\partial(\rho h\tilde{U}u)}{\partial\xi}+\frac{\partial(\rho h\tilde{V}u)}{\partial\eta}= \frac{\partial}{\partial\xi}\left(h\mu J\alpha\frac{\partial u}{\partial\xi}-h\mu J\beta\frac{\partial u}{\partial\eta}\right)+\frac{\partial}{\partial\eta}\left(h\mu J\gamma\frac{\partial u}{\partial\eta}-h\mu J\beta\frac{\partial u}{\partial\xi}\right)+ +\frac{\boldsymbol{\tau}_x^T}{J}-\frac{\boldsymbol{\tau}_x^B}{J}+\frac{\rho g h\Delta}{J}\left(\frac{\partial h}{\partial\xi}\frac{\partial\xi}{\partial x}+\frac{\partial h}{\partial\eta}\frac{\partial\eta}{\partial x}\right) \quad (8)$$

$$\frac{\partial}{\partial t}\left(\frac{\rho h v}{J}\right)+\frac{\partial(\rho h\tilde{U}v)}{\partial\xi}+\frac{\partial(\rho h\tilde{V}v)}{\partial\eta}= \frac{\partial}{\partial\xi}\left(h\mu J\alpha\frac{\partial v}{\partial\xi}-h\mu J\beta\frac{\partial v}{\partial\eta}\right)+\frac{\partial}{\partial\eta}\left(h\mu J\gamma\frac{\partial v}{\partial\eta}-h\mu J\beta\frac{\partial v}{\partial\xi}\right)+ +\frac{\boldsymbol{\tau}_y^T}{J}-\frac{\boldsymbol{\tau}_y^B}{J}+\frac{\rho g h\Delta}{J}\left(\frac{\partial h}{\partial\xi}\frac{\partial\xi}{\partial y}+\frac{\partial h}{\partial\eta}\frac{\partial\eta}{\partial y}\right) \quad (9)$$

The variables ξ and η are the coordinates in the generalized coordinate system, and α, β and γ are the components of the covariant metric tensor, J is the Jacobian of the transformation and $\tilde{U}$ and $\tilde{V}$ are the contravariant velocities defined as

$$\tilde{U}=(y_\eta u-x_\eta v) \qquad \tilde{V}=(x_\xi v-y_\xi u) \quad (10)$$

Integrating these equations in the volume P showed in Figure 2 and using WUDS [Raithby & Torrance 1979] as interpolation function

$$\phi_e=\left(\frac{1}{2}+\bar{\alpha}_e\right)\phi_E+\left(\frac{1}{2}-\bar{\alpha}_e\right)\phi_P \quad (11)$$

$$\left.\frac{\partial\phi}{\partial\xi}\right|_e=\bar{\beta}_e\left(\frac{\phi_E-\phi_P}{\Delta\xi}\right) \quad (12)$$

and central differences for the cross derivatives,

$$\left.\frac{\partial\phi}{\partial\eta}\right|_e=\left(\frac{\phi_{NE}.+\phi_N-\phi_{SE}-\phi_S}{4\Delta\eta}\right) \quad (13)$$

one obtains for the volume P,

$$u_P=F[u]_P{}^0-\frac{\rho\Delta t g\Delta h}{M_P}\left[\left.\frac{\xi_x}{J}\right|_P\frac{(h_E-h_W)}{2\Delta\xi}+ +\left.\frac{\eta_x}{J}\right|_P\frac{(h_N-h_S)}{2\Delta\eta}\right] \quad (14)$$

$$v_P=F[v]_P{}^0-\frac{\rho\Delta t g\Delta h}{M_P}\left[\left.\frac{\xi_y}{J}\right|_P\frac{(h_N-h_S)}{2\Delta\eta}+ +\left.\frac{\eta_y}{J}\right|_P\frac{(h_E-h_W)}{2\Delta\xi}\right] \quad (15)$$

where $F[\ ]$ is an is an explicit convective-diffusive finite volume operator[1], given, for a generic scalar by

$$F[\phi_P]=\frac{\Delta t}{M_P}\left[\phi_P{}^0\left(\frac{M_P{}^0}{\Delta t}-A_P\right)+\sum A_{nb}\phi_{NB}{}^0+\hat{S}\Delta V\right] \quad (16)$$

and it represents the convective-diffusive balance of the scalar variable at the volume P, in generalized coordinates. In this case, ϕ represents the velocity components u and v.

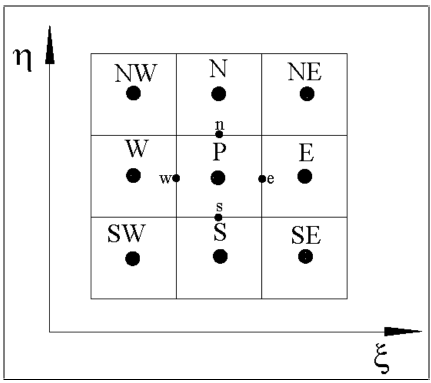


Figure 2: Control volume on the computational domain.

[1] Further details could be seen in Paladino (2000)

By the integration of the mass equation, in space and time, we have

$$h_P = h_P{}^0 - \rho J_P \frac{\Delta t}{\Delta \xi}\left[\left(h^0 \tilde{U}\Big|_e - h^0 \tilde{U}\Big|_w \right)\right] + \\ - \frac{\Delta t}{\Delta \eta}\left[\left(h^0 \tilde{V}\Big|_n - h^0 \tilde{V}\Big|_s \right)\right] \tag{17}$$

Here, we need the contravariant velocities at the volume faces. Using Eqs. (10), and the velocities at the control volume interfaces given by,

$$u_e = F[u]_e{}^0 - \frac{\rho \Delta t g \Delta h_e}{M_e}\left[\frac{\xi_x}{J}\Big|_e \frac{(h_E - h_P)}{\Delta \xi} + \\ + \frac{\eta_x}{J}\Big|_e \frac{(h_{NE} + h_N - h_{SE} - h_S)}{4\Delta \eta} \right] \tag{18}$$

$$v_e = F[v]_e{}^0 - \frac{\rho \Delta t g \Delta h_e}{M_e}\left[\frac{\xi_y}{J}\Big|_e \frac{(h_E - h_P)}{\Delta \xi} + \\ + \frac{\eta_y}{J}\Big|_e \frac{(h_{NE} + h_N - h_{SE} - h_S)}{4\Delta \eta} \right] \tag{19}$$

we have for the east face ,

$$\tilde{U}_e = \tilde{U}_e{}^* - \frac{\rho \Delta t g J \Delta h_e}{M_e}\left[\begin{array}{l} \alpha_e \dfrac{(h_E - h_P)}{\Delta \xi} - \\ \beta_e \dfrac{(h_N + h_{NE} - h_S - h_{SE})}{4\Delta \eta} \end{array} \right] \tag{20}$$

were

$$\tilde{U}^*{}_e = y_\eta F[u]_e - x_\eta F[v]_e \tag{21}$$

and similarly for the other faces of the control volume. Note that for the evaluation of the velocity components at the interface we need the convective-diffusive operator evaluated at the volume interfaces. As the variables arrangement used is co-located, the velocities at the interfaces are not available. The proposition here is to evaluate this operator by an average of the operators calculated at the centers of the adjacent volumes, i.e.,

$$F[\phi]_e = \frac{F[\phi]_E + F[\phi]_P}{2} \tag{22}$$

Note that it not represents an arithmetical average of the velocity components, which is know to generate strong instabilities in the solution procedure, but it is an average of the equations of motion as suggested by [Marchi and Maliska 1994].
Substituting the contravariant velocities in Eq.(17) and operating, we find for calculating the slick thickness field like,

$$A_P h_P = A_e h_E + A_w h_W + A_n h_N + A_s h_S + A_{ne} h_{NE} \\ + A_{se} h_{SE} + A_{nw} h_{NW} + A_{sw} h_{SW} + B \tag{23}$$

Note that for compute the thickness field we need to solve a linear system of equations while the velocity field is calculated explicitly.

BOUNDARY CONDITIONS

Two type of boundary conditions were used in this model, where the domain limits coincides with shorelines no mass flux was prescribed and at the open sea locally parabolic conditions were assumed. This last condition type allows the slick to leave the computational domain without affecting the thickness distribution of the slick inside the domain. Then it is possible to define the domain just for the region of interest because the presence of the boundaries will not affect the results inside the domain.
For the no mass flux condition, we have fro the velocity components a condition of prescribed variable, i.e., a Dirichlet condition with prescribed value equal to zero. If ϕ represents any component of the velocity vector, we have,

$$\phi_e = \phi_f = 0 \tag{24}$$

$$\frac{\partial \phi}{\partial \xi}\Big|_e = \frac{2(\phi_f - \phi_P)}{\Delta \xi} = \frac{-2\phi_P}{\Delta \xi} \tag{25}$$

The cross derivatives are zero because the prescribed value is constant along he frontier.

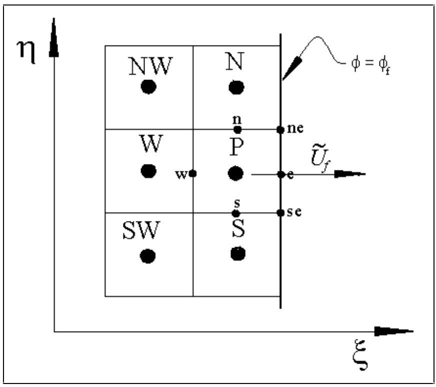


Figure 3: East boundary at the computational domain.

For the mass conservation equation, it will be used a methodology proposed by [Maliska 1981] and [Van Doormaal and Raithby 1984]. To avoid the necessity of prescribe thickness values at the boundary, it is proposed to substitute the contravariant velocity at the frontier in the mass conservation equation, then the mass balance for the volume *P* of the Figure 3 results,

$$h_P = h_P{}^0 - \rho J_P \frac{\Delta t}{\Delta \xi}\left[\left(\overbrace{h_P{}^0 \tilde{U}\Big|_f}^{0} - h^0 \tilde{U}\Big|_w \right)\right] \\ - \frac{\Delta t}{\Delta \eta}\left[\left(h^0 \tilde{V}\Big|_n - h^0 \tilde{V}\Big|_s \right)\right] \tag{26}$$

The other velocities are evaluated in the same way of the internal volumes. Then the equation for the east boundary volume is,

$$A_P h_P = A_w h_W + A_n h_N + A_s h_S + \\ A_{nw} h_{NW} + A_{sw} h_{SW} + B \tag{27}$$

Note that in this case the contravariant velocity at the frontier is zero and then the source term for Eq. (27) is,

$$B = h_P{}^0 - \Delta t J_P \left(- h_w{}^0 \tilde{U}_w{}^* + h_n{}^0 \tilde{V}_n{}^* - h_s{}^0 \tilde{V}_s{}^* \right) \tag{28}$$

For the open sea boundaries (Locally parabolic condition), we state that there is no variation in any variable normally to the frontier. For the volume *P* of the Figure 3, it means that,

$$\tilde{U}_e = \tilde{U}_w \qquad (29)$$

and zero diffusive flux normal to the frontier,

$$h\Gamma^{\phi} J\alpha \frac{\partial\phi}{\partial\xi}\Delta\eta\bigg|_e - h\Gamma^{\phi} J\beta \frac{\partial\phi}{\partial\eta}\Delta\eta\bigg|_e = 0 \qquad (30)$$

Considering the equation (29) we have, for the mass balance at the frontier volume,

$$h_P = h_P{}^0 - \rho J_P \frac{\Delta t}{\Delta\xi}\left[\left(h_P{}^0\tilde{U}\Big|'_w - h^0\tilde{U}\Big|_w\right)\right] - \frac{\Delta t}{\Delta\eta}\left[\left(h^0\tilde{V}\Big|_n - h^0\tilde{V}\Big|_s\right)\right] \qquad (31)$$

The prime indicates the variable is calculated in function of the available values and this term will take part of the independent term *B*. In this case it is given by,

$$B = h_P{}^0 - \Delta t J_P\left(h_P{}^0\tilde{U}'_w - h_w{}^0\tilde{U}_w{}^* + h_n{}^0\tilde{V}_n{}^* - h_s{}^0\tilde{V}_s{}^*\right) \qquad (32)$$

The solution procedure for the coupled system is:

- Initialize all variables at *t=0*. The thickness of the oil for the whole domain is initialized with a small value (say 1×10^{-15}) to avoid division by zero. Define the region and the thickness of the initial oil slick, if an instantaneous spill is considered.
- Calculate the coefficients of the momentum equations. Determine the velocity field explicitly, i.e. no linear system has to be solved here.
- With the most recent velocities, calculate the coefficients of the momentum equation. Compute the convective-diffusive operator to enter the evaluation of the source term of the mass equation.
- Calculates the coefficients and source term of the mass equation and solve the oil thickness.
- Recalculate the oil thickness field taking into account the mass transfer processes like evaporation, sinking, etc.
- Advance a time step, update all fields and cycle back to step one.

RESULTS

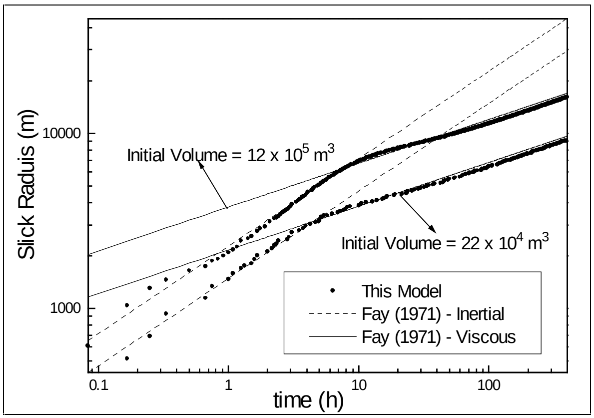
To validate the model, the first step will be the comparison with available analytical solutions. For this problem the semi-analytical solution of [Fay 1971] are adequate. Physical validation requires field measurements. As was already mentioned, Fay's results describe the spreading of an instantaneous spill in calm waters. The results for the gravity-inertial and gravity-viscous spreading regimes are, respectively

$$R = K_{g-i}\left(\Delta g V t^2\right)^{1/4} \qquad (33)$$

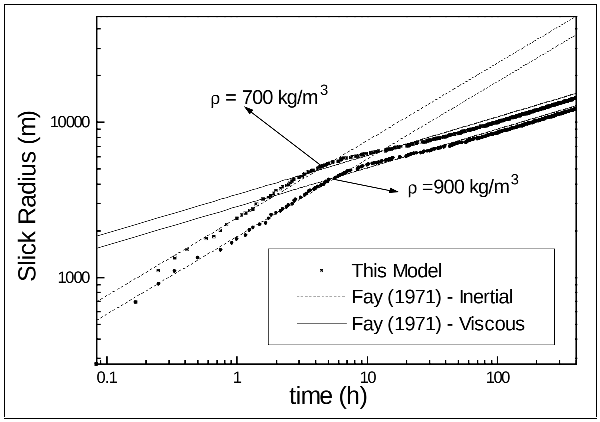
$$R = K_{g-v}\left(\frac{\Delta g V^2 t^{3/2}}{\nu^{1/2}}\right)^{1/6} \qquad (34)$$

In the above equations R is the slick radio (in calm waters the spreading is axi-symmetric) as a function of elapsed time after the spill and *K* is an empirical proportionality factor depending on the spreading regime.

The following figures shows the results for the two spreading regimes considered by the model, for different oil densities and different initial spills.



(a)

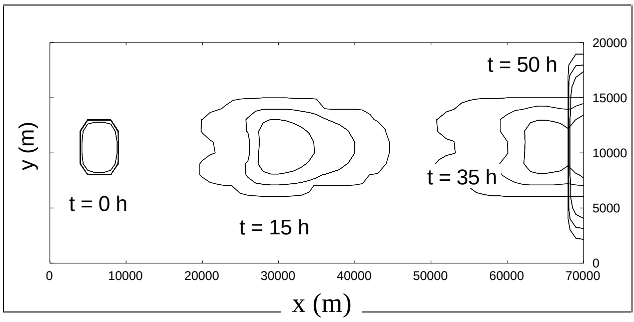


(b)

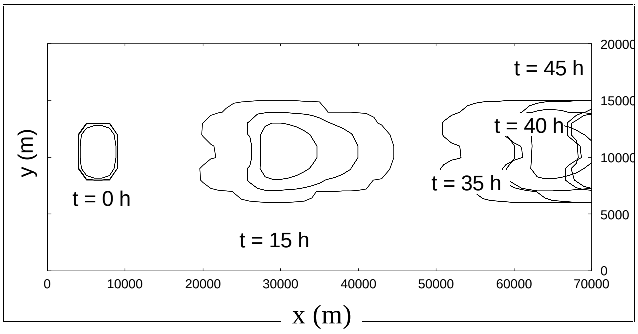
Figure 4: Comparison of theoretical [Fay 1971] and numerical solutions for axi-symmetric spreading in calm water, for (a) different volumes spilled and (b) different oil densities.

In the first problem, the water body was considered initially quiescent, with the water movement induced by the oil movement.

Figure 5 shows the effect of the boundary condition type as the slick approximates to the frontier. In the case of impermeable frontier oil accumulates against the wall increasing the slick thickness and begins to spread in transversal direction. For the locally parabolic condition, it could be seen that the slick leaves the domain without changes in the part remaining inside the domain.



(a)



(b)

Figure 5: Slick spreading and drifting with a constant water velocity of 0.5 m/s, in x direction, (a) no mass flux east boundary and (b) locally parabolic east boundary.

Finally, to show the model features, it was applied to simulate an eventual spill at the vicinity of the harbor of São Fransisco do Sul, Santa Catarina, where there is an oil charge/discharge point at 9 km off shore. Therefore, this is a local with high spill risk, which could be caused by pipeline rupture or failure in charge/discharge operations.

Figure 6, shows the domain definition at region of the port of São Francisco do Sul, the oil duct break locals and the definition of boundary conditions for the simulations. The domain has been extended into the sea just to cover the region of interest, reminding that, due to the locally parabolic condition far from the shoreline, if the slick passes through these boundaries, this does not affect the slick position inside the domain.

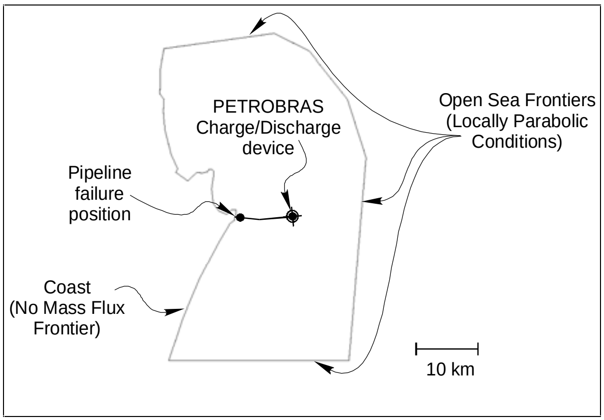
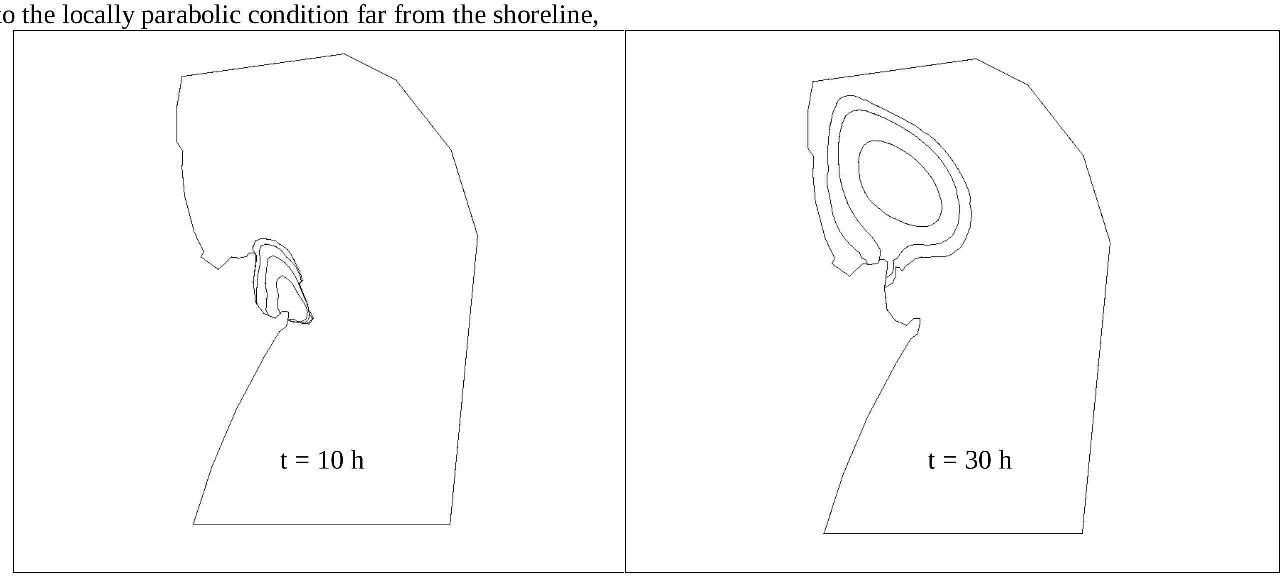


Figure 6: Domain definition for the simulations at the port of São Francisco do Sul.

As this simulation has the only purpose to show the generality of the model applied to a coastal spill, and not to simulate accurately a real problem, the current field was considered spatially constant and variable as a sine function of time, trying to represent approximately the tidal currents. Reports of experimental measurements at the region show predominantly south-southwest currents with residual currents of approximately 0.05 m/s and maximum tidal currents of 0.16 m/s. The wind was considered from south-southeast blowing at 30 km/h.

To simulate the pipeline break, it was considered a pollutant source with constant mass flux injecting 1000 kg/s during 10 h.

Figure 7 shows th time space evolution of an oil slick caused by the pipeline brek. It could be seen the effects of the boundary conditions, at the shoreline, were no mass flux condition was imposed, the oil accumulates, increasing the slick thickness. In the case of an open sea boundaries, the slick leaves the domain without affecting its shape upstream.



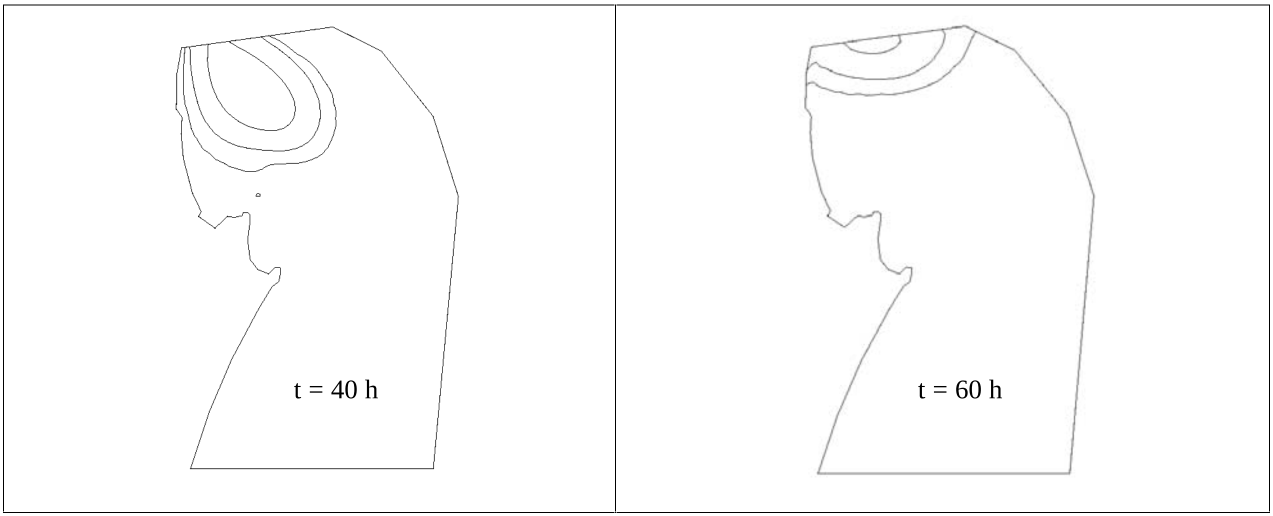


Figure 7: Temporal-spatial evolution of an oil slick spilled at the harbor of São Francisco do Sul

CONCLUSIONS

This paper presented a mathematical and numerical model to predict oil spill movements in the sea. Results for the spreading in the calm water were compared with semi-analytical solutions and the agreement was good. Although there are no benchmark solutions available for the case where the water moves, the results for a general problem, where the water moves periodically in time, follow the expected physical trends and the mass center of the slick moves with the water current velocity.

The model can be used to simulate *in situ* oil spills in order to assist pollution combat tasks, so it is an important tool in any oil spill contingency plan. It can be also used to estimate potential risks in decision support for tankers and oil ducts route selection, distilleries and ground tanks location, among other oil storing tasks.

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