Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 8717-8724 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

CORRECT ANISOTROPIC AND HETEROGENEOUS TRANSMISSIBILITIES CALCULATION IN CORNER-POINT GRIDS

Maurício Pereira Tada , António F. Carvalho da Silva , Clovis R. Maliska

Computational Fluid Dynamics Laboratory SINMEC, Federal University of Santa Catarina UFSC, Florianópolis, Santa Catarina, Brazil, http://www.sinmec.ufsc.br

Keywords: nine point stencil, transmissibility, non-orthogonality, anisotropic, heterogeneous medium.

Abstract. The transmissibility coefficient, present in commercial codes in petroleum reservoir simulation, was firstly defined considering an orthogonal grid in an isotropic and homogeneous medium, applying the traditional cell center control volume method. This type of problem results in a five point stencil for two dimensional domains, and the mass flux over a control volume face is, in this particular case, evaluated correctly with two pressure points. Afterwards, Hegre and Dalen presented a useful calculation method, widely used in commercial reservoir simulators, to any distorted corner point grid, still resulting in a five point stencil for a heterogeneous and anisotropic medium. Recently, some works showed in distorted and two dimensional grids and/or anisotropic medium that the mass flux over a control volume face cannot be accurately evaluated with only two pressure points, i.e., a five point stencil. This can introduce significant errors in the problem solution and strong grid orientation effects. This work presents an extension of the traditional control volume method, based on Hegre's transmissibilities, considering the correct mass flux. The resultant stencil involves nine pressure points in a two dimensional case and twenty-three in a three dimensional case. The transmissibilities calculation proposed is designed to be applied in anisotropic (full tensor) and/or heterogeneous medium. The results of the tests performed are in good agreement with the correct nine point reference solution that considers the correct mass flux. In addition, the transmissibility method is straightforward to be extended in a computational code based on Hegre's transmissibility.

1 INTRODUCTION

Finite difference methods or control volume methods are widely used in petroleum reservoir simulation. For a general way, these methods do not compute correctly the mass flux over a control volume face, once this flux is always evaluated with two pressure points. It is fairly known that, for a general corner-point grid, and isotropic or anisotropic medium, the flux over a control volume face is not correctly evaluated using only two pressure points (Tada, 2009; Aavatsmark, 2007). This can introduce errors in the numerical solution and strong grid orientation effects. There are many works that treat this numerical problem by calculating the correct flux over a control volume face using more than two pressure points. Cordazzo (2006) shows an EbFVM method that computes the flux correctly. His scheme, in a three dimensional domain, results in twenty seven pressure points. Aavastmark (2002) shows a multipoint scheme for corner point grids and anisotropic medium. All of these methods are based in control volume methods, but they are different than the usual control volume or finite difference method, e.g., the scheme of the assembly global linear system matrix is different and the points where the mass flux is evaluated, in a control volume face, is different as well. In other words, their methods are new numerical methods comparing with traditional difference finite method, therefore, the programming code architecture is also new. The present work presents a nine point scheme, for two dimensional cases, easily extensible to a twenty and three point numerical scheme for three dimensional domains, using the finite difference methods. This work is an extension of the conventional finite difference method based on Hegre's (1986) transmissibility where it is considerate now the correct mass fluxes over a control volume face for anisotropic (full tensor) and heterogeneous medium. To do that, a new method to calculate the transmissibility is proposed. The tests were performed in a two dimensional domain and the results are compared with Cordazzo (2006) code that was previously validated. The results are in good agreement, showing that the new transmissibility method calculation is consistent and considers the correct mass flux over a control volume face in an anisotropic medium using non orthogonal corner point grid.

In short, this work proposes a new calculation transmissibility method based in a nine point scheme for two dimensional cases, or twenty three point scheme for three dimensional cases. As the classical Hegre's (1986) form, widely used in numerical codes, the transmissibility of this work is written in a vector form.

2 MATHEMATICAL AND NUMERICAL MODEL

The transmissibilities form proposed are obtained from a volumetric flux calculation over a control volume surface, considering a single phase flow in a heterogeneous and anisotropic porous medium. The Darcy law is considered to express the fluid velocity.

To get these transmissibilities, the next steps are followed:

1. Auxiliary coordinate system ξ, η is used inside the control volume as shown in the Figure 1.
2. Permeability full tensor, pressure gradient and the single phase flux is written using the ξ, η coordinate system.
3. Variables available in the control volume face are calculated from properties of main control volumes neighbors.

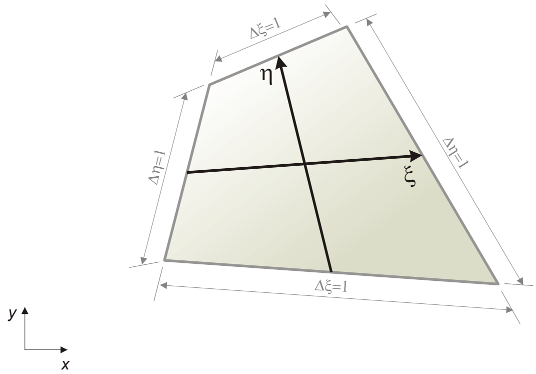

Figure 1: Coordinate system used inside a control volume.

2.1 Flux Evaluation

Single phase fluid flux over control volume face is generally calculated from

$$f_i = -\int_{S_i} \left(\bar{\bar{\mathbf{K}}} \cdot \nabla p \right) \cdot \mathbf{n}\, dS \tag{1}$$

where $\bar{\bar{\mathbf{K}}}$ is the full permeability tensor, ∇p is the pressure gradient $\mathbf{n}$ is the normal unitary vector pointing outside the control volume and S is the control volume surface area.

Tada (2009) shows that, using the ξ,η coordinate system (Figure 1), the flux over an east control volume face, is written as

$$f_e = -\frac{\Delta\eta}{J_e}\left(k_{\xi\xi}^{\mathrm{cov}} \frac{\partial p}{\partial \xi} + k_{\xi\eta}^{\mathrm{cov}} \frac{\partial p}{\partial \eta} \right)_e \tag{2}$$

where J_e is the jacobian of the coordinate transformation, $k_{\xi\xi}^{\mathrm{cov}}$ is the component of permeability tensor and the subscribe *e* denoting the east face point where all these variables are evaluated. Forward, the expressions of these parameters will be presented.

2.2 Permeability Tensor

Cartesian components of permeability tensor are, in a two dimensional domain, k_{xx}, k_{xy}, k_{yy}, having as basis the unitary vectors $\mathbf{i}$ e $\mathbf{j}$, denoted generally by $\mathbf{e}_i^c$. To get this tensor in a curvilinear basis, the following mathematical operation is necessary

$$\bar{\bar{\mathbf{K}}} = k_{ij}\mathbf{e}_i^c\mathbf{e}_j^c = k_{ij}\frac{\partial x^m}{\partial x_i}\frac{\partial x^n}{\partial x_j}\mathbf{e}_m\mathbf{e}_n \tag{3}$$

where $\bar{\bar{\mathbf{K}}}$ is the permeability tensor, k_{ij} are its components in the cartesian system, $\mathbf{e}_i^c$ is the cartesian basis been, $\mathbf{e}_1^c = \mathbf{i}$ and $\mathbf{e}_2^c = \mathbf{j}$, and $\mathbf{e}_m$ is the curvilinear covariant basis vectors (Tada, 2009).

The permeability tensor, written in a curvilinear system is

$$\bar{\bar{\mathbf{K}}} = k_{mn}^{\mathrm{cov}}\mathbf{e}_m\mathbf{e}_n \tag{4}$$

Comparing the equations (3) and (4), the permeability tensor components, in the curvilinear system are then

$$k_{mn}^{\mathrm{cov}} = k_{ij}\frac{\partial x^m}{\partial x_i}\frac{\partial x^n}{\partial x_j} \tag{5}$$

Developing the last equation, the components, in a two dimensional domain are given by

$$\begin{aligned} k_{\xi\xi}^{\mathrm{cov}} &= J^2\left(k_{xx}y_\eta^2 - 2k_{xy}x_\eta y_\eta + k_{yy}x_\eta^2\right) \\ k_{\xi\eta}^{\mathrm{cov}} = k_{\eta\xi}^{\mathrm{cov}} &= -J^2\left(k_{xx}y_\xi y_\eta - k_{xy}\left(x_\xi y_\eta + x_\eta y_\xi\right) + k_{yy}x_\xi x_\eta\right) \\ k_{\eta\eta}^{\mathrm{cov}} &= J^2\left(k_{xx}y_\xi^2 - 2k_{xy}y_\xi x_\xi + k_{yy}x_\xi^2\right) \end{aligned} \tag{6}$$

where x_ξ, x_η, y_ξ, y_η and J are the metrics of the basis system transformation calculated from gridblock coordinates. The components k_{xx}, k_{xy} and k_{yy} are the cartesian components of the permeability tensor.

2.3 Transmissibility Model

The transmissibility model is based in the continuity of the flux over a surface of control. Consider for instance that the volumetric flux can be written from the follow equation

$$f_{AB} = -\left(T_{AB}^{\xi\xi}\Delta p^\xi + T_{AB}^{\xi\eta}\Delta p^\eta\right) \tag{7}$$

being f_{AB} the volumetric flux over the surface of the control volume between the gridblocks A and B, as showed in the Figure 2, $T_{AB}^{\xi\xi}$ and $T_{AB}^{\xi\eta}$ are the transmissibilities evaluated at the middle of the face. The purpose here is to get a formulation for transmissibilities in equation (7), this is done by considering the flux over the surface *AB* (Figure 2) is continuous and with same magnitude being calculated from *A* or *B* gridblock. In other words

$$f_A = f_B = f_{AB} \tag{8}$$

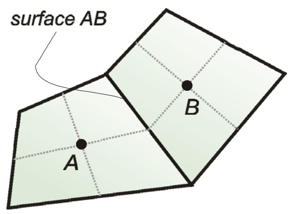

Figure 2: Transmissibility calculation between two gridblocks.

The flux f_A crossing surface AB, using only the properties from gridblock A is

$$f_A = -\frac{1}{J_A}\left(2k_{\xi\xi}^{\mathrm{cov}}\Delta p^\xi + k_{\xi\eta}^{\mathrm{cov}}\Delta p^\eta\right)_A \tag{9}$$

being J_A the transformation gridblock center jacobian, $\Delta p^\xi = P_i - P_A$, P_i the pressure in the middle of the surface AB and Δp^η the pressure variation along the face AB, calculated by interpolating the corner pressures of the surface AB from gridblocks neighbors. Note that, according to Figure 1, $\Delta\xi = 1/2$ and $\Delta\eta = 1$ reason why the number 2, multiplying the first term in equation (9) show up.

In the same way, the flux using the properties from gridblock B is given by

$$f_B = -\frac{1}{J_B}\left(2k_{\xi\xi}^{\mathrm{cov}}\Delta p^{\xi} + k_{\xi\eta}^{\mathrm{cov}}\Delta p^{\eta}\right)_B \tag{10}$$

Collecting the terms that multiply the Δp^{ξ} and the Δp^{η} terms, the expressions (9) and (10) can be rewritten as

$$f_A = -T_A^{\xi\xi}\Delta p_A^{\xi} - T_A^{\xi\eta}\Delta p_A^{\eta} \tag{11}$$

$$f_B = -T_B^{\xi\xi}\Delta p_B^{\xi} - T_B^{\xi\eta}\Delta p_B^{\eta} \tag{12}$$

where

$$T_A^{\xi\xi} = \frac{\left(2k_{\xi\xi}^{\mathrm{cov}}\right)_A}{J_A},\quad T_B^{\xi\xi} = \frac{\left(2k_{\xi\xi}^{\mathrm{cov}}\right)_B}{J_B},\quad T_A^{\xi\eta} = \frac{\left(k_{\xi\eta}^{\mathrm{cov}}\right)_A}{J_A},\quad T_B^{\xi\eta} = \frac{\left(k_{\xi\eta}^{\mathrm{cov}}\right)_B}{J_B} \tag{13}$$

Add the equation (11) and (12), replacing $\Delta p_A^{\xi} = P_i - P_A$, $\Delta p_B^{\xi} = P_B - P_i$ and noting that $\Delta p_A^{\eta} = \Delta p_B^{\eta} = \Delta p^{\eta}$,

$$\frac{f_A}{T_A^{\xi\xi}} + \frac{f_B}{T_B^{\xi\xi}} = -\left(P_B - P_A\right) - \left(\frac{T_A^{\xi\eta}}{T_A^{\xi\xi}} + \frac{T_B^{\xi\eta}}{T_B^{\xi\xi}}\right)\Delta p^{\eta} \tag{14}$$

Using $f_A = f_B = f_{AB}$,

$$f_{AB} = -\left(\frac{1}{T_A^{\xi\xi}} + \frac{1}{T_B^{\xi\xi}}\right)^{-1}\left(P_B - P_A\right) - \left(\frac{T_B^{\xi\eta}T_A^{\xi\xi} + T_A^{\xi\eta}T_B^{\xi\xi}}{T_A^{\xi\xi} + T_B^{\xi\xi}}\right)\Delta p^{\eta} \tag{15}$$

Therefore, the flux can be written as

$$f_{AB} = -\left(T_{AB}^{\xi\xi}\Delta p^{\xi} + T_{AB}^{\xi\eta}\Delta p^{\eta}\right) \tag{16}$$

and the transmissibilities are given by

$$T_{AB}^{\xi\xi} = \left(\frac{1}{T_A^{\xi\xi}} + \frac{1}{T_B^{\xi\xi}}\right)^{-1},\quad T_{AB}^{\xi\eta} = \frac{T_B^{\xi\eta}T_A^{\xi\xi} + T_A^{\xi\eta}T_B^{\xi\xi}}{T_A^{\xi\xi} + T_B^{\xi\xi}} \tag{17}$$

These transmissibilities are applied to a full tensor anisotropic and heterogeneous medium using a nine point scheme to a two dimensional case. For a three dimensional case, the procedure to get the transmissibilities is the same. Tada (2009) writes the transmissibilities (13) for the gridblock A, in a vector form to be used and calculated according the architectures of the numerical code. They are given by

$$T_A^{\xi\xi} = \frac{\left(\mathbf{n}_A\cdot\overline{\overline{\mathbf{K}}}\cdot\mathbf{n}_A\right)\left(\mathbf{A}\cdot\mathbf{A}\right)}{\mathbf{D}_A\cdot\mathbf{A}},\qquad T_A^{\xi\eta} = \frac{\left(\mathbf{n}_A\cdot\overline{\overline{\mathbf{K}}}\cdot\mathbf{n}_D\right)\left|\mathbf{D}_A\right|\left|\mathbf{A}\right|}{\mathbf{D}_A\cdot\mathbf{A}} \tag{18}$$

where $\mathbf{D}_A$ is the vector from gridblock center to the center of the the gridblock face, $\mathbf{A}$ is the normal vector to the face with magnitude of the area face, $\mathbf{n}_A$ and $\mathbf{n}_D$ are respectively the unitary vector of $\mathbf{A}$ and the unitary normal vector of $\mathbf{D}_A$.

3 EXAMPLE RUN

The following synthetic problem is run to validate the methodology exposed. It is considered a two-phase, incompressible, anisotropic and heterogeneous problem in a two-dimensional domain, as showed in the Figure 3. The main rock and fluid data are summarized in Table 1. The capillary pressure is not considered, the relative permeability is made equal the fluid saturation and the initial conditions are uniform pressure (1.0 bar) and uniform oil saturation $S^{oil} = 1.0$.

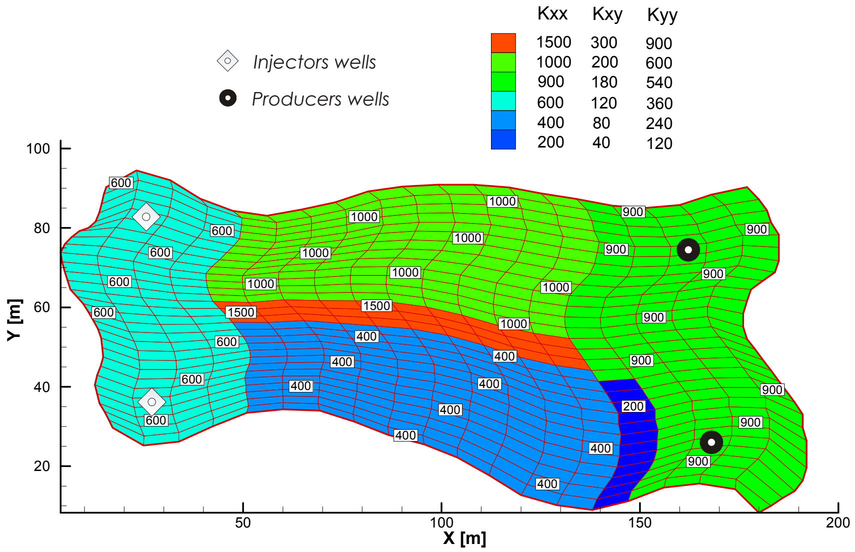

Figure 3: Synthetic problem proposed.

Porosity	0.20
Water viscosity	1.0 cp
Oil viscosity	10.0 cp
Injectors wells with flow type condition	50 m³/day
Producers wells BHP	1.0 bar (absolute)
Well index	10 mD*m

Table 1: Main input data.

3.1 Solution Comparison with EbFVM simulator

The solution of the problem above was compared with a validated simulator, the EbFVM – Element based Finite Volume Method developed in the SINMEC/UFSC laboratory (Cordazzo, 2006). As previously commented, this simulator is a nine point scheme in two dimensional domains which means that the volumetric flux over a control volume face is correctly evaluated considering truncations errors of second order.

The run is performed with a 19x29 and 38x118 corner point grid and the solution for the pressure and water saturation is shown in Figure 4 after 200 days of simulation. Some comments must be done about these results. Despite the EbFVM to be a control volume method, as well as the present work, their numerical schemes are different, e.g., the EbFVM is a cell vertex method while the present finite difference method is cell center. Furthermore, there are many other details that can cause difference between the present work and EbFVM solutions. However, as it can be seen in the Figure 4, when a finer 38x118 grid is used, the results are similar showing that the method proposed is consistent.

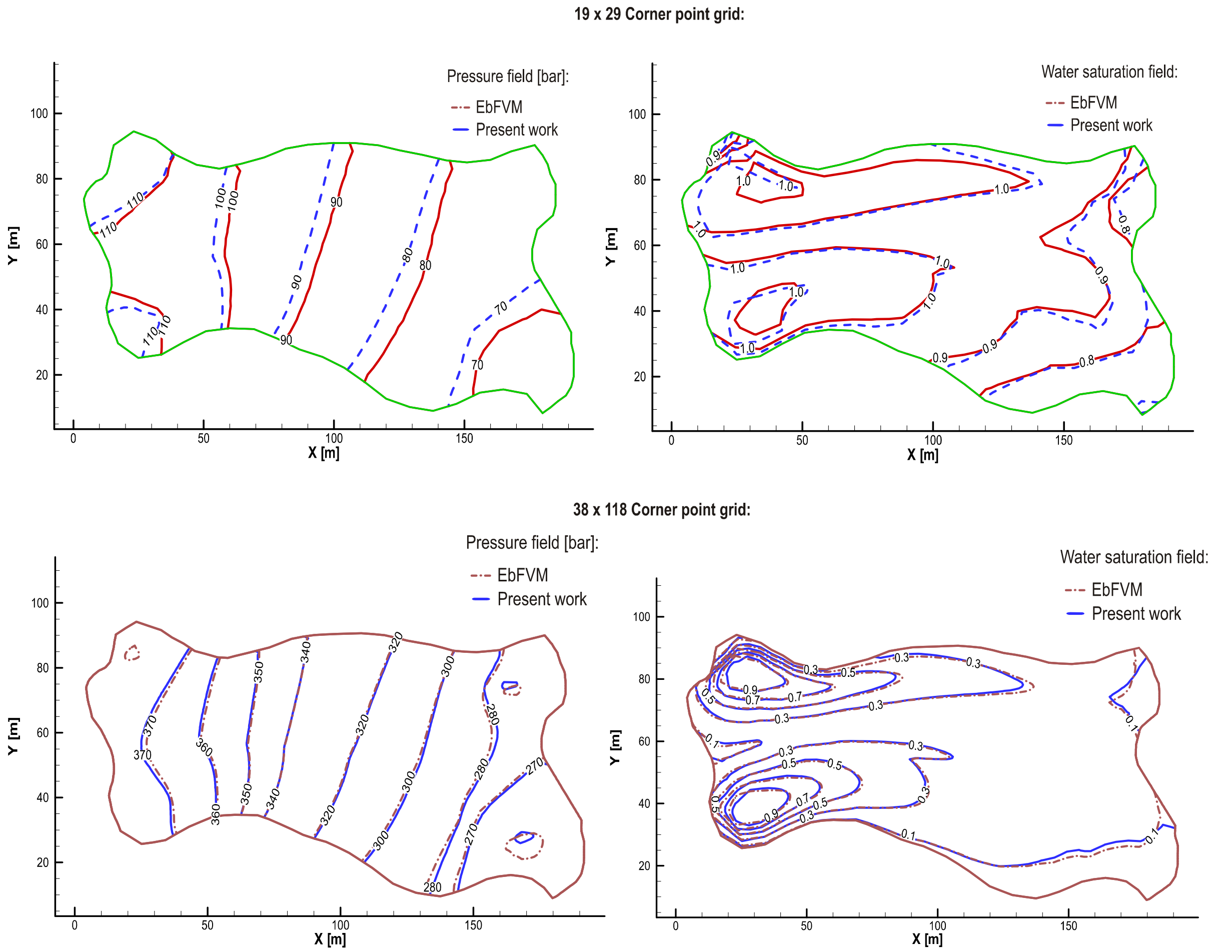

Figure 4: Pressure and water saturation solution with a 19x29 and 38x118 corner point grid.

Tada (2009) presents additional runs showing that the same problem when simulated using a five point scheme, with Hegre's (1986) transmissibilities, generates strong grid effects orientation and the solution does not match with the corresponding nine point solution, even for fine grid simulations. Details about errors of Hegre's (1986) transmissibility applied in non orthogonal grids and anisotropic medium can be found in Tada (2009).

4 CONCLUSIONS

The present work showed a correct and simple way to calculate the transmissibilities for full anisotropic tensor and heterogeneous medium using non orthogonal corner point grids and the traditional difference finite method. The transmissibilities presented were written in a vector form being easy to implement in numerical codes that uses the traditional finite difference method.

REFERENCES

Aavastsmark, I., An Introduction to Multipoint Flux Approximations for Quadrilateral Grids. *Comput Geosci*, v.6, p.408-432, 2002.

Aavastsmark, I., Interpretation of a Two-point Flux Stencil for Skew Parallelogram Grids.

Comput Geosci, v.11, p.199-206, 2007.

Cordazzo, J., Simulação de Reservatórios de Petróleo Usando o Método EbFVM e Multigrid Algébrico. *Doctoral degree Dissertation*, Mechanical Department Engineering, Federal University of Santa Catarina, Florianópolis, Brazil, 2006.

Hegre, T. M., Generalized Transmissibilities for Distorted Grids in Reservoir Simulation. *Annual Tech Conf And Exhi of SPE*, New Orleans, LA, October 5-8, 1986.

Tada, M. P., Análise de Esquemas Numéricos de Cinco e Nove Pontos para Malhas Tipo Corner Point. *Master degree Dissertation*, Mechanical Department Engineering, Federal University of Santa Catarina, Florianópolis, Brazil, 2009.