MATHEMATICAL MODELING AND NUMERICAL SIMULATION OF OIL SPILL TRAJECTORIES ON THE SEA

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Abstract

The aim of this paper is to present a mathematical model and its numerical treatment to forecast oil spills trajectories in the sea. The knowledge of the trajectory followed by an oil slick spilled on the sea is of fundamental importance in the estimation of potential risks and in combating the pollution using floating barriers, detergents, etc. In order to estimate these slicks trajectories a new model, based on mass and momentum conservation equations, is presented. This model considers the spreading in the regime when the inertial and viscous forces counterbalance gravity and takes into account the effects of winds and water currents. The mass loss caused by oil evaporation is also considered. The numerical model is developed in generalized coordinates, making the model easily applicable to complex coastal geographies.

Key Words: Environmental Flows - Oil Spill - Numerical Simulation - Generalized Coordinates

1 INTRODUCTION

The environment is today one of the main preoccupations of the potential pollutant industries and government authorities. This is particularly true in the petroleum branch, due to its high environmental risk. In the case of Brazil, the major petroleum exploitation is offshore, increasing considerably the risks of occurring oil spills in operation and transport tasks. These spills are much more damaging when they occur near shorelines because, besides the environmental impacts, the economical damages ranges reach from fishing to tourism. The recent oil spill in the Guanabara Bay is a strong example of this broad impact. The detailed knowledge of the spilled oil position and the area covered by the slick is of fundamental importance to take appropriate actions against pollution, like use of floating barriers, detergents, dispersants, etc. It is also important the estimation of potential risks in selecting pipeline routes, locating shoreline tanks and petrochemical industries. Therefore, a model to forecast the time-space evolution of the oil slick should make part of any environmental program that has the purpose of oil pollution combat.

The first studies attempting to model the movement of oil slicks (Fay (1969,1971), Fannelop and Waldmann (1971), Hoult (1972), Buckmaster (1973), etc.) consider the spreading as one-dimensional or axi-symmetric. These models consider the spreading of the oil in calm waters, where a slick, initially circular, will remains circular, just increasing its diameter. Considering the forces that governs the spreading process, Fay (1969), characterized the spreading by dividing it in three phases: Initially, when the thickness of the slick is big and so are the inertial forces, the gravity acts as the active force counterbalanced by inertial forces; this is called the gravity-inertial spreading regime. When the mean thickness of the slick begins to decrease, and the viscous forces exerted by the water boundary layer will eventually outweigh the inertia as the retarding force, it constitutes the gravity-viscous spreading. In the final instances, the slick will be so thin that the imbalances of surface tensions between air-water, air-oil and water-oil will substitute the gravity as active force, maintaining the tension exerted by the water as retarding force. This last regime is called viscous-surface tension spreading. For big spills (>10^4 m^3), these regimes last for 1 to 4 hours, four to ten days and several months, respectively.

Further models has tried to simulate more realistically the trajectories by including other process like dispersion caused by winds and water currents, and those process which represents mass exchanges between different environmental compartments (called fate processes) like evaporation, dissolution, emulsification, etc.

Two approaches for computing oil spills trajectories are commonly encountered in the literature; Lagrangian models and Eulerian models. The Lagrangian models (Shen e Yapa (1988)) consist basically in representing the oil slick by an ensemble of a large number of small parcels which are advected by a velocity which results from a combination of the action of winds and currents. Then, the slick is divided into pie shaped segments or strips, depending if the form of the slick is nearly circular or elongated. Fay (1969) spreading formulas are then applied to each segment. For the Eulerian approach, two model are usually encountered, those based in the mass and momentum equations applied to the oil slick (Hess and Kerr (1979), Benqué et. alii. (1982)), and those based on a convection-diffusion equation (Venkatesh (1988) among others), in which the diffusive part of the equation represents de spreading of oil by itself and the convective terms represents the advection of oil by currents and winds. The model presented in this paper belongs to the second category of Eulerian models and it is based on the integration of mass and momentum equation over the thickness of the oil slick.

2 MATEMATICAL MODEL

This model is based on the integration of the mass and momentum equations along the thickness of the slick. Therefore, it takes into account the spreading of oil by itself and the transport caused by winds and water currents. As the surface tension is neglected and, therefore, only the first and second spreading regimes, i.e. gravity-inertial and gravity-viscous spreading are considered, the model is applicable up to about ten days after the spill, depending on its magnitude. The evaporation is considered through a logarithmic decay model presented by Stiver and Mackay (1984).

Following, it will be shown how the mathematical model for the motion of oil is obtained.

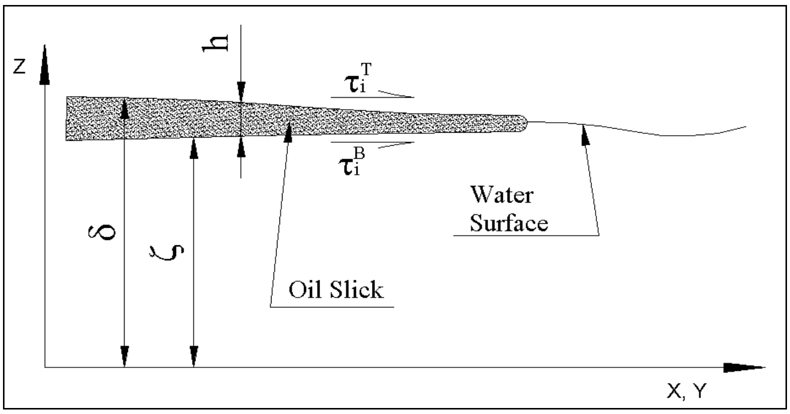


Figure 1. Variables considered in the vertical integration of governing equations

Fig. 1 shows schematically an oil slick being transported by the shear stresses exerted by water currents and winds. The oil flow is governed by mass and momentum equations for incompressible flows. These equations are:

$$\frac{\partial \mathbf{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \frac{\partial (\rho \mathbf{u}_i \mathbf{u}_j)}{\partial x_i} = \frac{\partial (\tau_{ij})}{\partial x_i} - \frac{\partial p}{\partial x_i} \tag{2}$$

Following Hoult (1972), we can consider that the oil viscosity is much large than the water viscosity. Thus, the vertical velocity gradients within the oil are much less than these gradients in the water or in the wind. It is, therefore, a good approximation to consider that the flow parameters (velocity and pressure) do not vary across the thickness of the slick. Integrating the governing equations, Eqs. (1) and (2), across the slick thickness as shown in Fig. 1, considering hydrostatic pressure distribution within the oil, we obtain

$$\frac{\partial h}{\partial t} + \frac{\partial (\bar{\mathbf{u}}_i h)}{\partial x_i} = 0 \tag{3}$$

$$\frac{\partial (\rho \bar{\mathbf{u}}_i h)}{\partial t} + \frac{\partial (\rho \bar{\mathbf{u}}_i \bar{\mathbf{u}}_j h)}{\partial x_i} = \frac{\partial}{\partial x_j}\left(h\mu \frac{\partial \bar{\mathbf{u}}_i}{\partial x_j} \right) + \tau_i^T - \tau_i^B - \rho g h \Delta \frac{\partial h}{\partial x_i} \tag{4}$$

Where the bar variables represent vertical integral averages, h is the oil slick thickness and Δ is a parameter which relates the oil and water densities $\Delta = (\rho_o - \rho_w)/\rho_w$. The terms τ represent the shear stresses on top and bottom of the slick exerted by winds and water currents, respectively. These stresses were calculated as (Benque *et. alii*. (1982), Cuesta *et. alii*. (1990)),

$$\tau_i^T = C_f^{wind} \mathbf{u}_i^{wind} \tag{5}$$

$$\tau_i^B = C_f^{water} \left(\mathbf{u}_i^{oil} - \mathbf{V}_i^{water} \right) \tag{6}$$

where, C_f^{wind} e C_f^{water} were made 3×10^{-5} and 1×10^{-6} respectively. Those constant values are commonly used in these models (Idelfonso Cuesta, personal communication). The C_f^{water} value is an empirically adjusted value, while C_f^{wind} value is calculated in such way that the final velocity of the slick mass center be about 3 % of wind velocity (3% rule).

3 NUMERICAL SOLUTION

Due the similarity of the governing equations with those used in Shallow Waters Flows, an adaptation of the semi-implicit method presented by Casulli and Cheng (1992) is used here for generalized coordinates, a finite volume procedure and co-located variables. This fact makes the model easily applicable to complex coastal geographies. Transforming Eqs. (3) and (4) to generalized coordinates following the procedure described in details in Maliska (1995), we obtain

$$\frac{\partial}{\partial t}\left(\frac{\rho h}{J}\right)+\frac{\partial\left(\rho h\tilde{U}\right)}{\partial \xi}+\frac{\partial\left(\rho h\tilde{V}\right)}{\partial \eta}=0 \tag{7}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\rho hu}{J}\right)+\frac{\partial\left(\rho h\tilde{U}u\right)}{\partial \xi}+\frac{\partial\left(\rho h\tilde{V}u\right)}{\partial \eta}=\frac{\partial}{\partial \xi}\left(h\mu J\alpha\frac{\partial u}{\partial \xi}-h\mu J\beta\frac{\partial u}{\partial \eta}\right)+\\+\frac{\partial}{\partial \eta}\left(h\mu J\gamma\frac{\partial u}{\partial \eta}-h\mu J\beta\frac{\partial u}{\partial \xi}\right)+\frac{\tau_y^T}{J}-\frac{\tau_y^B}{J}+\frac{\rho g h\Delta}{J}\left(\frac{\partial h}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial h}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\rho hv}{J}\right)+\frac{\partial\left(\rho h\tilde{U}v\right)}{\partial \xi}+\frac{\partial\left(\rho h\tilde{V}v\right)}{\partial \eta}=\frac{\partial}{\partial \xi}\left(h\mu J\alpha\frac{\partial v}{\partial \xi}-h\mu J\beta\frac{\partial v}{\partial \eta}\right)+\\+\frac{\partial}{\partial \eta}\left(h\mu J\gamma\frac{\partial v}{\partial \eta}-h\mu J\beta\frac{\partial v}{\partial \xi}\right)+\frac{\tau_y^T}{J}-\frac{\tau_y^B}{J}+\frac{\rho g h\Delta}{J}\left(\frac{\partial h}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial h}{\partial \eta}\frac{\partial \eta}{\partial y}\right)\end{aligned} \tag{9}$$

The variables ξ and η are the coordinates in the generalized coordinate system, α, β and γ are the components of the covariant metric tensor, J is the Jacobian of the transformation and $\tilde{U}$ and $\tilde{V}$ are the contravariant velocities defined as

$$\begin{aligned}\tilde{U}&=\left(y_\eta u-x_\eta v\right)\\ \tilde{V}&=\left(x_\xi v-y_\xi u\right)\end{aligned} \tag{10}$$

These equations were discretized using a finite volume approach, the time variation was considered explicitly in momentum equations and implicitly for the mass conservation equation used to calculate the oil thickness distribution. Fig. 2 shows a control volume in the computational domain used for the equations discretization.

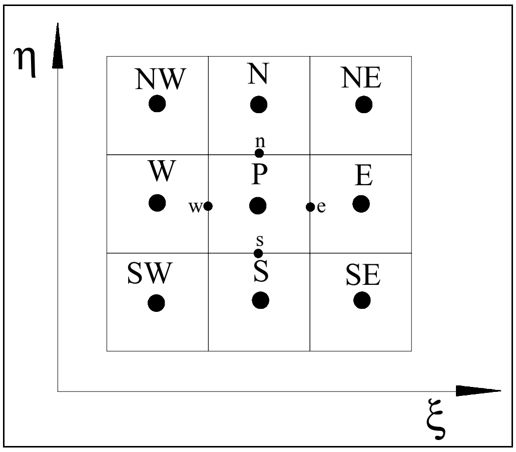


Figure 1: Control Volume on the Computational Domain

Using WUDS (Raithby & Torrance (1979)) as interpolation function and evaluating explicitly the time derivative, we have, taking the east face as example, the velocities at this face given by

$$u_e=F[u]_e^0-\frac{\rho\Delta t g\Delta h_e}{M_e}\left[\left.\frac{\xi_x}{J}\right|_e\frac{\left(h_E-h_P\right)}{\Delta\xi}+\left.\frac{\eta_x}{J}\right|_e\frac{\left(h_{NE}+h_N-h_{SE}-h_S\right)}{4\Delta\eta}\right] \tag{11}$$

$$v_e = F[v]_e^0 - \frac{\rho \Delta t g \Delta h_e}{M_e}\left[\left.\frac{\xi_y}{J}\right|_e \frac{(h_E - h_P)}{\Delta\xi} + \left.\frac{\eta_y}{J}\right|_e \frac{(h_{NE} + h_N - h_{SE} - h_S)}{4\Delta\eta}\right] \tag{12}$$

where $F[\;]$ is an explicit convective-diffusive finite volume operator[1] and represents the explicit convection-diffusion balance of the variable for a control volume. It is expressed for a generic variable ϕ as,

$$F[\phi_P] = \frac{\Delta t}{M_P}\left[\phi_P{}^0\left(\frac{M_P{}^0}{\Delta t} - A_P\right) + \sum A_{nb}\phi_{NB}{}^0 + \hat{S}\Delta V\right] \tag{13}$$

The mass balance in the volume P which is obtained by the discretization of Eq. (7), is given by,

$$h_P = h_P{}^0 - \rho J_P \frac{\Delta t}{\Delta\xi}\left[\left(h^0\tilde{U}\Big|_e - h^0\tilde{U}\Big|_w\right)\right] - \frac{\Delta t}{\Delta\eta}\left[\left(h^0\tilde{V}\Big|_n - h^0\tilde{V}\Big|_s\right)\right] \tag{14}$$

Substituting the Cartesian velocities into the expressions for the contravariant velocities, and then into the mass equation, we obtain an equation for the oil thickness as[2]:

$$\begin{aligned} A_P h_P &= A_e h_E + A_w h_W + A_n h_N + A_s h_S + \\ &+ A_{ne} h_{NE} + A_{se} h_{SE} + A_{nw} h_{NW} + A_{sw} h_{SW} + B \end{aligned} \tag{15}$$

This equation is solved using the Gauss-Seidel method. Note that for the momentum equations no linear system of equations has to be solved. The solution procedure for the coupled system is:

1. Initialize all variables at *t=0*. The thickness of the oil for the whole domain is initialized with a small value (say 1×10^{-15}) to avoid division by zero. Define the region and the thickness of the oil spill.
2. Calculate the coefficient of the momentum equations. Determine the velocity field explicitly, i.e. no linear system has to be solved here.
3. With the most recent velocities, calculate the coefficients of the momentum equation. Compute the convective-diffusive operator to enter the evaluation of the source term of the mass equation.
4. Calculates the coefficients and source term of the mass equation and solve the oil thickness.
5. Recalculate the oil thickness field taking into account the mass transfer processes like evaporation, sinking, etc.
6. Advance a time step, update all fields and cycle back to step one.

Two type boundary conditions were used[3]. Where the domain coincides with shorelines no mass flux was prescribed and at the open sea locally parabolic conditions were assumed. This allows the slick to leave the computational domain without affecting the thickness distribution of the slick inside the domain.

[1,2,3] Further details could be seen in Paladino (2000)

These conditions were applied as suggested in Maliska (1981) and Van Doormaal and Raithby (1984). For the impermeable frontier, the velocity components are zero (taking in account the no-slip condition) and, in those frontiers in which locally parabolic flux is considered, the velocities derivatives in a normal direction to the frontier are zero.

4 MODEL VALIDATION AND RESULTS

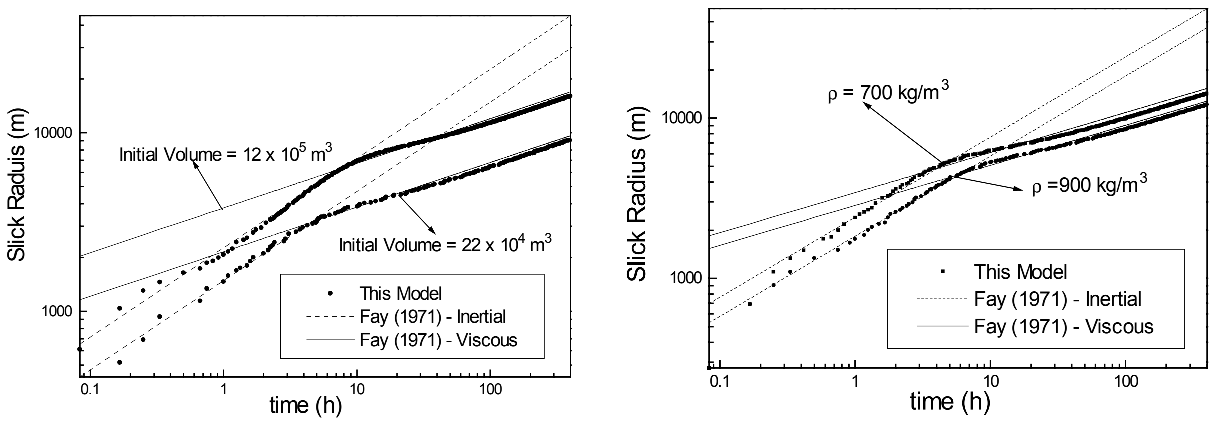
The first step in validating a numerical model is to compare with available analytical solutions. For this problem the semi-analytical solution of Fay (1971) are adequate. Physical validation requires field measurements. As was already mentioned, Fay's results describe the spreading of an instantaneous spill in calm waters. The results for the gravity-inertial and gravity-viscous spreading regimes are, respectively

$$R = K_{g-i}\left(\Delta g V t^2\right)^{1/4} \tag{16}$$

$$R = K_{g-v}\left(\frac{\Delta g V^2 t^{3/2}}{\nu^{1/2}}\right)^{1/6} \tag{17}$$

In the above equations R is the slick radio (in calm waters the spreading is axi-symmetric) as a function of elapsed time after the spill.

The following figures shows the results for the two spreading regimes considered by the model, for different oil densities and different initial spills.



(a) **(b)**

Figure 2: Comparison of theoretical (Fay (1971)) and numerical solutions for axi-symmetric spreading in calm water, for (a) different volumes spilled and (b) different oil densities.

In the first problem, the water body was considered initially quiescent, with the water movement induced by the oil movement. The next figure shows the one-dimensional evolution of an oil slick, considering an instantaneous spill, in the case that the water is moving. In this case, it was considered a spatially and temporally constant current of magnitude of 0.5 m/s in the *x*-direction.

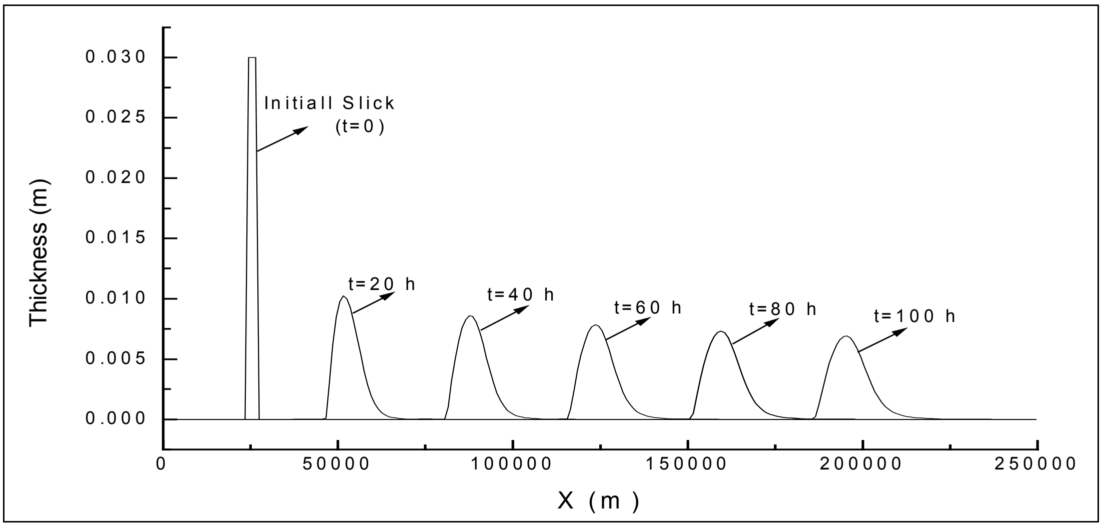


Figure 3: One-dimensional evolution of an slick subjected to a constant water current of 0.5 m/s. Note that the scales are distorted, the maximum thickness is 30 mm and the whole domain has 250 km.

As it was expected, after a period of time in which the slick accelerates, the mass center of the slick moves with the water velocity.

Finally, to show the model features, it was applied to simulate an eventual spill at the vicinity of the harbor at São Fransisco do Sul, Santa Catarina, where there is an oil charge/discharge point at 9 km. off shore. Therefore, this is a local with high spill risk which could be caused by pipeline rupture or failure in charge/discharge operations.

The following figure shows the generalized grid used in the simulations. The line $\xi = 1$ represents the coast in the region of São Francisco do Sul. The domain has been extended into the sea just to cover the region of interest, reminding that, due to the locally parabolic condition far from the shoreline, if the slick passes through these boundaries, this does not affect the slick position inside the domain.

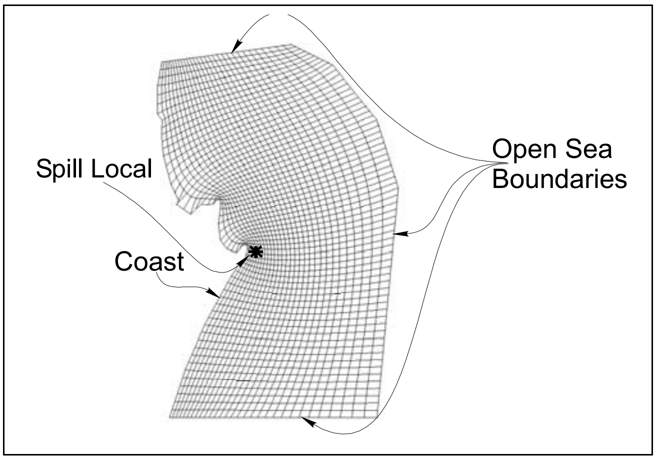


Figure 4: Generalized grid used in the simulations at the port of São Francisco do Sul.

As this simulation has the only purpose to show the generality of the model to a coastal spill, and not to simulate accurately a real problem, the current field was considered spatially constant and variable as a sine function of time, trying to represent approximately the tidal currents. Reports of experimental measurements at the region show predominantly south-southwest currents with residual currents of approximately 0.05 m/s and maximum tidal currents of 0.16 m/s. The wind was considered from south-southeast blowing at 30 km/h.

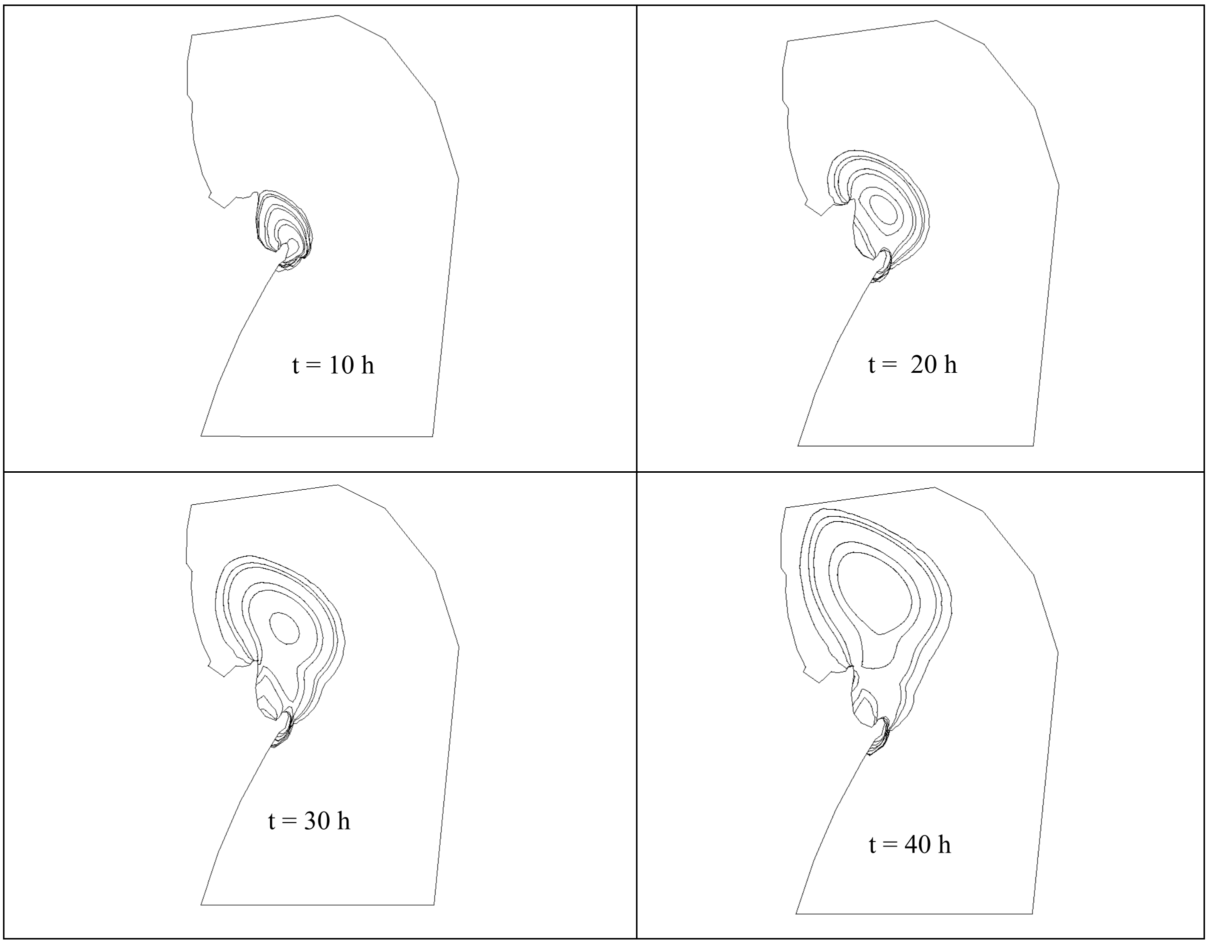


Figure 5: Temporal-spatial evolution of an oil slick spilled at the harbor of São Franico do Sul

Due to the periodic behavior of the tidal currents, the movement of the slick is caused primarily by the action of the residual currents. Since they are small, the slick movement is also small.

5 Conclusions

This paper presented a mathematical and numerical model to predict oil spill movements in the sea. Results for the spreading in the calm water were compared with semi-analytical solutions and the agreement was good. Although there are no benchmark solutions available for the case where the water moves, the results for a general problem, where the water moves periodically in time, follow the expected physical trends and the mass center of the slick moves with the water current velocity.

The model can be used to simulate *in situ* oil spills in order to assist pollution combat tasks, so it is an important tool in any oil spill contingency plan. It can be also used to estimate potential risks in decision support for tankers and oil ducts route selection, distilleries and ground tanks location, among other oil storing tasks.

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