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SOLUTION OF A MULTIPHASE FLOW IN HORIZONTAL WELLS USING A DRIFT-FLUX MODEL

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ABSTRACT

This works presents a procedure for solving two-phase (gas and liquid) flows along an oil well with lateral mass inflow coming from the reservoir. The flow is considered isothermal and one-dimensional. Equations are discretized using a *Finite Volume Method* with a C++ (OOP) code implementation. This algorithm is intended to be used with a reservoir simulator for solving the coupled flow between reservoir and well. A drift-flux model is used to model the two-phase flow and the solution is then validated with results available in the literature.

From the drift flux model, the gas (disperse phase) velocity is calculated as ([3]):

$$v_g = C_0 j + V_{gj} \tag{1.1}$$

where C_0 is the profile parameter, V_{gj} the drift velocity and j is the total volumetric flux, defined as:

$$j = \alpha_g v_g + \alpha_l v_l \tag{1.2}$$

The void fraction α_p of a phase p is given by: $\alpha_p = \frac{V_p}{V} = \frac{A_p}{A}$ (1.3)

The last equality results if we notice that, inside a control volume, all the properties are equal, so one can calculate the void fraction using the duct cross sectional area A and the cross sectional area occupied by the phase (A_p).

The two-phase drift-flux model is based on three balance equations [2]

Mixture Continuity E.:

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial(\rho_m v_m)}{\partial s} = \left(\frac{\dot{m}}{V}\right) \tag{1.4}$$

Gas-phase Continuity Eq.:

$$\frac{\partial(\alpha_g \rho_g)}{\partial t} + \frac{\partial(\alpha_g \rho_g v_m)}{\partial s} = \left(\frac{\dot{m}}{V}\right)_g - \frac{\partial}{\partial s}\left(\frac{\alpha_g \rho_g \rho_l}{\rho_m}\bar{V}_{gj}\right) \tag{1.5}$$

Mixture Momentum Eq.:

$$\frac{\partial(\rho_m v_m)}{\partial t} + \frac{\partial(\rho_m v_m v_m)}{\partial s} = -\frac{\partial P}{\partial s} - \rho_m g \sin(\theta) - \frac{f_m}{2D}\rho_m v_m |v_m| - \frac{\partial}{\partial s}\left(\frac{\alpha_g \rho_g \rho_l}{\alpha_l \rho_m}\bar{V}_{gj}^{\,2}\right) \tag{1.6}$$

where $\left(\frac{\dot{m}}{V}\right)$ and $\left(\frac{\dot{m}}{V}\right)_g$ are the total and gas mass influx (per unit volume) along the well lateral holes, respectively.

All the mixture properties (e.g. density, viscosity, etc.) are calculated by averaging values between the two phases. The term $\bar{V}_{gj}$ is called modified drift velocity and can be calculated in terms of the mixture velocity v_m ([4]):

$$\bar{V}_{gj} = V_{gj} + (C_0 - 1) j = \frac{V_{gj} + (C_0 - 1) v_m}{\left[1 - (C_0 - 1)\alpha_g \frac{(\rho_l - \rho_g)}{\rho_m}\right]} = \bar{V}_{gj}\left(\alpha_g, v_m, \rho_m(P, \alpha_g)\right) \tag{1.7}$$

The problem was solved using a Newton's Method ([1]), generating a tridiagonal block matrix structure, since it's a one-dimensional domain. Comparisons made with results available in [4], show that the results agreed well (Figure 1). The main advantage is that the approach presented in this work allows timesteps 100 times greater (or even more) than the maximum timesteps that could be used in the reference solution ([4] and [5]). Of course the greater timestep, the less accurate were the results of the transient solutions. Being able to use any timesteps, is an advantage not only when the steady state solution is the final goal, but also when the transient behavior is of interest. The great advantage of the method proposed herein is that one is not restricted to small time steps because of convergence. Recall that reservoir's timestep has an order of days, while the well's steady state is reached much earlier than that.

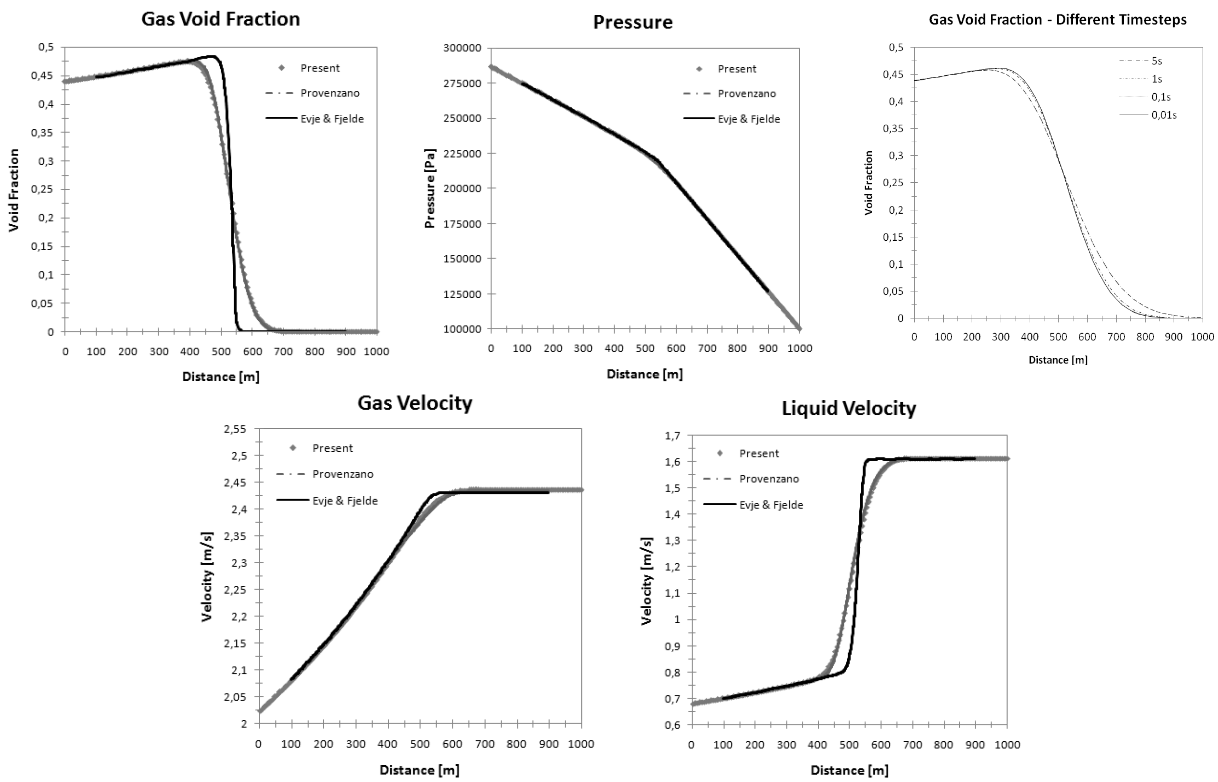

Figure 1 - Results obtained with 200 control volumes. Timestep equals 0.01s when not indicated. Pipe diameter: 0.1m, pipe lenght: 1000m.

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