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THE ELEMENT-BASED FINITE VOLUME METHOD APPLIED TO PETROLEUM RESERVOIR SIMULATION

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Resumo

Esse trabalho apresenta um modelo numérico para a simulação de reservatórios de petróleo utilizando o Método dos Volumes Finitos baseado em Elementos – EbFVM. Esse método emprega malhas não-estruturadas com elementos triangulares e/ou quadrangulares, de forma que reservatórios com geometrias complexas podem ser facilmente representados. Por ser um método conservativo, a conservação da massa em cada volume de controle é assegurada, o que permite uma interpretação direta das equações discretizadas resultantes. Além disso, é mostrado que pode-se lidar com mapas de permeabilidade absoluta sem a necessidade de realização de qualquer tipo de média nas interfaces dos volumes de controle em função do esquema de armazenamento das propriedades adotado. O tensor permeabilidade é incluído neste modelo sem dificuldades, o que é uma característica importante na simulação de reservatórios heterogêneos e anisotrópicos. Finalmente, uma comparação entre os resultados obtidos utilizando o esquema proposto neste trabalho com aqueles obtidos no esquema equivalente e comumente utilizando na simulação de reservatórios de petróleo é apresentada. É mostrado que o esquema proposto é menos susceptível ao efeito de orientação de malha.

Abstract

In this work a numerical model for simulating petroleum reservoirs using the Element-based Finite Volume Method (EbFVM) is presented. The method employs unstructured grids using triangular and/or quadrilateral elements, such that complex reservoir geometries can be easily represented. Due to the control-volume approach, local mass conservation is enforced, permitting a direct physical interpretation of the resulting discrete equations. It is demonstrated that this method can deal with the permeability maps without averaging procedures, since this scheme assumes uniform properties inside elements, instead inside of control volumes, avoiding the need of weighting the permeability values at the control volumes interfaces. Moreover, it is easy to include the full permeability tensor in this method, which is an important issue in simulating heterogeneous and anisotropic reservoirs. Finally, a comparison among the results obtained using the scheme proposed in this work in the EbFVM framework with those obtained employing the scheme commonly used in petroleum reservoir simulation is presented. It is also shown that the scheme proposed is less susceptible to the grid orientation effect with the increasing of the mobility ratio.

1. Introduction

The use of structured grids poses several difficulties in defining complex geometries and in refining the grid near faults and wells.

In this paper it is proposed a method that combines the flexibility obtained through the Finite Element Method with the local and global conservation enforcement obtained through the Finite Volume Method. It employs the ideas of Raw (1985) when developing the FIELDS method for solving the Navier-Stokes equations. This method is usually

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known by Control Volume Finite Element Method – CVFEM. However, a better denomination would be Element-based Finite Volume Method - EbFVM (Maliska, 2004), since, in fact, it is a finite volume methodology which only borrows from the finite element technique the concept of elements and their shape functions. CVFEM, by its turn, would erroneously suggest a finite element formulation that obeys the conservation principles at discrete level. Therefore, the denomination EbFVM will be used throughout this paper. The motivation for the use of the EbFVM is its flexibility, generality and suitability for a clean computational implementation.

The basic ideas of the EbFVM, often referred as CVFEM, have already been used in reservoir simulation. However, the approach employed to obtain the discrete equations imposes serious limitations for its practical use. These equations have been deduced starting from the single-phase flow differential equations and adapting them to multiphase flow (Forsyth, 1990; Fung *et al.*, 1991; Gottardi and Dall´Olio, 1992). It is not clear the advantages of using the single-phase flow equations when the interest is to solve the multiphase flow problems. There are no difficulties in starting with the correct physical equation, as is demonstrated in this paper.

This paper is organized as follows: initially the mathematical and numerical models are presented. Next, some computational details of the discretization of the equations for triangular and quadrilateral elements using EbFVM are addressed. Finally, some examples are included to demonstrate the geometric flexibility and the reduction of grid orientation effects. When appropriate, results from a commercial reservoir simulator are also included for comparison.

2. Mathematical Model

The problem under analysis is the multiphase flow in a porous media. For the sake of simplicity, some effects are neglected in this work: gravity, chemical reactions and thermal and capillary effects. Using the conventional approach in which the velocity in the mass conservation equation is replaced by the one expressed by Darcy's law (Peaceman, 1977), the mass conservation for a *m* phase is given by

$$\frac{\partial}{\partial t}\left(\phi \frac{s_m}{B_m}\right) = \vec{\nabla} \cdot \left(\lambda_m \bar{\bar{k}} \vec{\nabla} p\right) + q_m \tag{1}$$

where p is the pressure, λ is the mobility ($\lambda = k_r/(B\mu)$), ϕ the porosity, s the saturation, B the formation volume factor, $\bar{\bar{k}}$ the permeability tensor and q is the flow-rate per unit of volume at reservoir conditions. Two immiscible phases will be considered: water and oil ($m = w$ and o, respectively).

For a two-phase flow model one gets a system of two equations and three unknowns: p, s_w and s_o. The closure equation comes from the condition that the two fluids jointly fill the void space:

$$s_w + s_o = 1 \tag{2}$$

3. Proposed Numerical Method

Differently from the traditional finite volume methods, often called finite difference methods in the petroleum literature, where elements and control volumes are coincident (cell centered construction), in the EbFVM control volumes are built around grid nodes, joining the center of the elements to its medians (cell vertex construction). The resulting control volume is formed by portions (sub-control volumes) of neighboring elements, as shown in Figure 1. As can be seen, the total control volume is formed by the sum of the sub-control volumes belonging to the elements surrounding node *P*. In this case, all fluxes at one specified integration point can be calculated using data from the element where the integration point lies. This feature makes this approach very adequate for reservoir simulation, because physical properties can be stored associated to the elements instead of to the control volumes. Thus, since the integration points lie inside the elements, there is no need to do any type of averaging procedure for calculating the permeability at these points, since the elements are homogeneous, i.e. the permeability is piecewise constant over each element. This approach has several advantages over the conventional ones, whereby permeability and porosity are piecewise constant over each control volume, as shown by Cordazzo (2004).

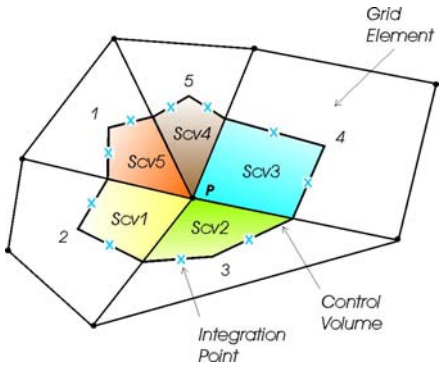


Figure 1. Control Volume in a unstructured grid of triangular and quadrilaterals elements

The mass conservation equation, Equation 1, integrated over the control volume and in time is given by

$$\left(\frac{s_m}{B_m}\right)_P = \left(\frac{s_m^o}{B_m^o}\right)_P + \left[(\bar{q}_m)_P + \sum_i \left(\lambda_m \bar{\bar{k}} \vec{\nabla} p \cdot \Delta \vec{S}\right)_i\right] \left(\frac{\Delta t}{\phi \Delta V}\right)_P \tag{3}$$

in which the variables evaluated at the grid nodes (or at the control volumes center) are labeled with the subscript *P*, and those ones evaluated at the integration points are denoted by the subscript *i.*

4. Computational Details of the Discretization Procedure for Triangular and Quadrilateral Elements

In the EbFVM, the equations can be solved in the computational domain using a standard element – triangular or quadrilateral in local coordinates (ξ,η). This procedure allows each element to become independent of other elements. In addition, each element is treated identically, no matter how distorted the element may actually be in terms of the global coordinates. Figure 2 shows examples of triangular and quadrilateral elements in *xy* and in $\xi\eta$ spaces.

Using a general procedure, the pressure, as well as other variables of interest, can be approximated over the whole domain in terms of shape functions N_i and the values of the variable at nodes, that is,

$$p = \sum_{i=1}^{NNE} N_i(\xi,\eta)\, p_i \tag{4}$$

where *NNE* is the number of nodes per element (3 for triangles and 4 for quadrilaterals), and N_i are the shape functions which are used here as interpolation functions (Maliska, 2004).

In this section it is shown that other important difference between the EbFVM and other conventional finite volume methods is the way the fluxes, which involve the pressure gradient ∇p , are discretized in Equation 3. It is easy to show (Cordazzo, 2004) that in this method the expression that determines the pressure gradient at any point within an element yields

$$[\nabla p] = [J]^{-1}[D]\{p\}_e \tag{5}$$

where $\{p\}_e$ is the vector of pressure nodal values (dimension of *NNE*), and $[J]$ is the Jacobian transformation matrix, which can be calculated by

$$[J] = [D][Z]_e \tag{6}$$

where $[Z]_e$ is the node coordinates matrix of a grid element, and [*D*] is the shape function local derivatives matrix, which may have different forms according to the element type:

$$[D] = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad [D] = \frac{1}{4}\begin{bmatrix} 1+\eta & -1-\eta & -1+\eta & 1-\eta \\ 1+\xi & 1-\xi & -1+\xi & -1-\xi \end{bmatrix} \tag{7}$$

for triangular and quadrilateral elements, respectively.

Other important geometrical information arising in Equation 3 is the control volume area vector $\{\Delta S\}$. Even though in 2D this surface is simply seen as a line, the denomination "area" will remain throughout this paper. The area vector can be determined according to expression (Hurtado, 2004):

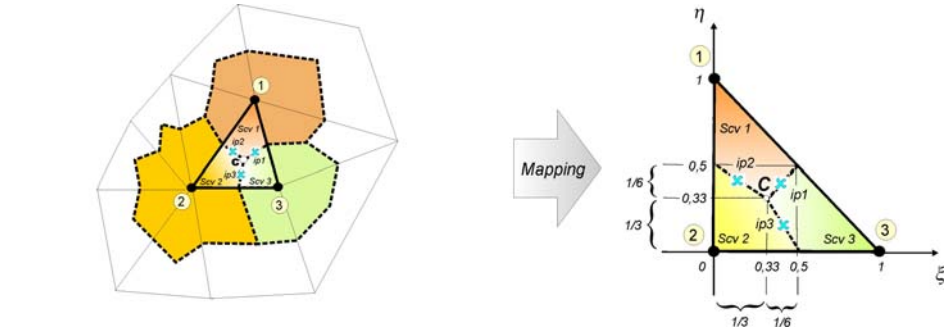
$$\{\Delta S\}_i = [R][J]_i^T \{\Delta\}_i \tag{8}$$

where *i* is the integration point located at the center of the surface whose area is being calculated, and *R* is the rotation matrix given by

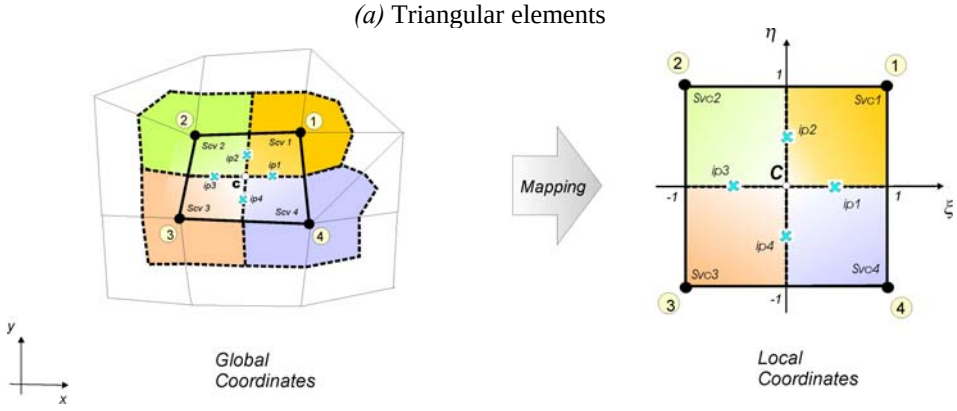
$$[R] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{9}$$

and $\{\Delta\}_i$ is the sub-control volume area vector in local coordinates, which is easily identified in Figure 2.

The total volume of a control volume, ΔV, in Equation 3, on the other hand, can be determined at the beginning of the simulation, "visiting element by element", summing the contribution of each sub-control volume.



(a) Triangular elements



(b) Quadrilateral elements

Figure 2. Triangular and quadrilateral elements in *xy* and $\xi\eta$ spaces, showing the sub-control volumes areas and the integration points (*x*)

Regarding the evaluation of mobility term, λ, in Equation 3, the scheme employed in this work is the upstream mobility weighting, which is the technique most commonly used. In this approach, the relative permeability utilized for estimating the mobility at the interface between two control volumes is calculated with the saturation of the upstream control volume.

At this point one has all terms of the governing equations completely discretized. To end up with a robust algorithm, it is now required that an efficient computer implementation be employed. Although there are several options, the most effective way seems to create submatrices in each element, computing the contributions of each one, followed by the assemble procedure, in a element-by-element base, to generate the global matrix This technique is, of course, in fully agreement with the philosophy of the EbFVM which computes all required quantities at the integration points with information contained in a single element. Hence, this technique based on element is employed in this work.

5. Numerical Examples

In this section, two examples involving reservoirs under water-flood (secondary recovery) are analyzed. In the first one, the grid orientation sensitivity of the EbFVM is examined, and its results are compared with the ones obtained through a commercial simulator - CMG's STARS (STARS User's Guide, 2002) with the option CVFEM (Fung *et al.*, 1991). From now on, this method will be referred here as the conventional CVFEM. In the last problem, a more realistic water-flood simulation is carried out using a grid composed by triangles and quadrilaterals.

5.1. *Test case 1* - Grid formed only by obtuse triangles

The first case analyzed in this paper consists of a petroleum reservoir discretized using a grid composed only by obtuse triangular elements, i. e. triangles that have an angle greater than 90º, as depicted in Figure 3. According to what is reported in the literature, elements with these characteristics should be avoided because they would origin negative transmissibilities (Fung *et al.,* 1993; Sonier *et al.*, 1993). In fact, the literature recommends not using this type of elements by a misinterpretation of these negative coefficients, which are recognized as transmissibilities, but they are not. It is shown (Cordazzo *et al*., 2004) that these coefficients can assume negative values with no prejudice for the numerical scheme.

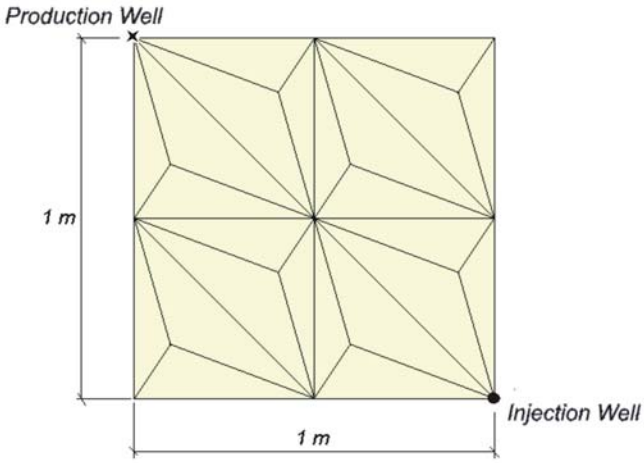


Figure 3. Grid for the *Test case 1* (9 nodes and 24 elements)

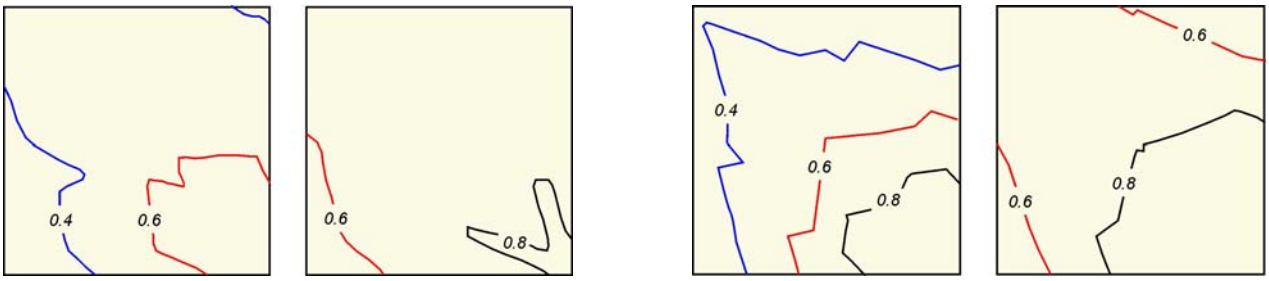
In this example the water injection well is at the lower-right corner, whereas the production well is at the opposite diagonal corner, according to Figure 3. The data utilized in this test problem are listed in Table 1.

Although the water-cut curves at the production well (not shown here) obtained for both methods are similar, the water-saturation contour, in contrast, is very different in both methods, as one can see in Figure 4. Despite the fact that the grid used is not symmetric (c.f. Figure 3), the conventional CVFEM results show exaggerated asymmetry, Figure 4*a*, while the results of the method advanced in this paper presents good symmetry (Figure 4*b*). Therefore, once again the method advanced in this paper yields more physically consistent results, having no difficulties in handling obtuse triangles in the grid.

Table 1. Data used for $M = 1$ in *Test case 1*

Porosity	0.2
Permeability (isotropic)	150 mD
Reservoir thickness	1 m
Viscosity of water	1 cp
Viscosity of oil	1 cp
Compressibility of water	0
Compressibility of oil	0
Formation volume factor of water	1
Formation volume factor of oil	1
s_{wi}	0
k_{rw} @ s_{wi}	1
s_{or}	0
k_{ro} @ s_{or}	1
Curve of k_{rw}	linear
Curve of k_{ro}	linear
Water flow-rate at the injection well	0.1 m^3/day
Bottom-hole pressure at the production well	100 kPa
Well index	1 mD.m
Time step	0.001 day

As can be seen in Figure 4*b*, the grid orientation effects are not greatly influenced by type of triangular grid used in the proposed method. However, in practical cases it is always preferred to have a random orientation of triangles in order to avoid additional grid orientation effects caused by the mobility evaluation scheme.



t = 1.25 days t = 2.09 days t = 1.25 days t = 2.09 days

(a) Conventional CVFEM *(b)* Present Work

Figure 4. Water-saturation contour at different simulation times obtained from different methods

5.2. *Test case 2* – Faulted reservoir discretized with triangles and quadrilaterals

In this case a more realistic water-flood simulation is carried out using a grid composed by triangles and quadrilaterals. Problems with complex geometry, like the one considered here, can be treated with considerable flexibility by the proposed numerical technique. For instance, triangulation can be performed over any irregular domain with smaller triangles in more important regions. In other regions, quadrilaterals elements can be used, such as the grid depicted in Figure 5.

The porous media matrix was assumed to be homogeneous and isotropic, with absolute permeability equal to 500 mD and porosity equal to 0.2. The fluids are incompressible and their viscosities are equal to 1 cp. The relative permeability data curves used are given in Table 2. The water flow-rate at the injection well is 0.1 m^3/day, and the

bottom-hole pressure at the production well is 100 kPa. The location of geological fault, as well as the location of injection and production wells, is depicted in Figure 5. The time step used throughout the simulation was 0.03 day.

Table 2. Relative Permeabilities of water and oil versus water saturation

s_w	k_{rw}	k_{ro}
0.00	0.000	1.000
0.30	0.510	0.490
0.50	0.750	0.250
0.75	0.937	0.090
0.90	0.990	0.003
1.00	1.000	0.000

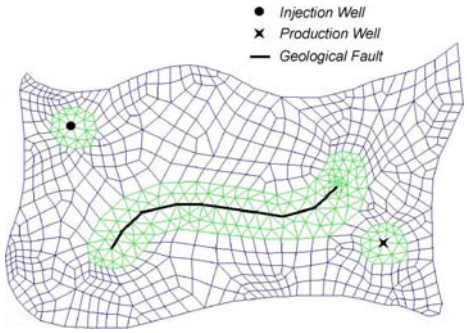
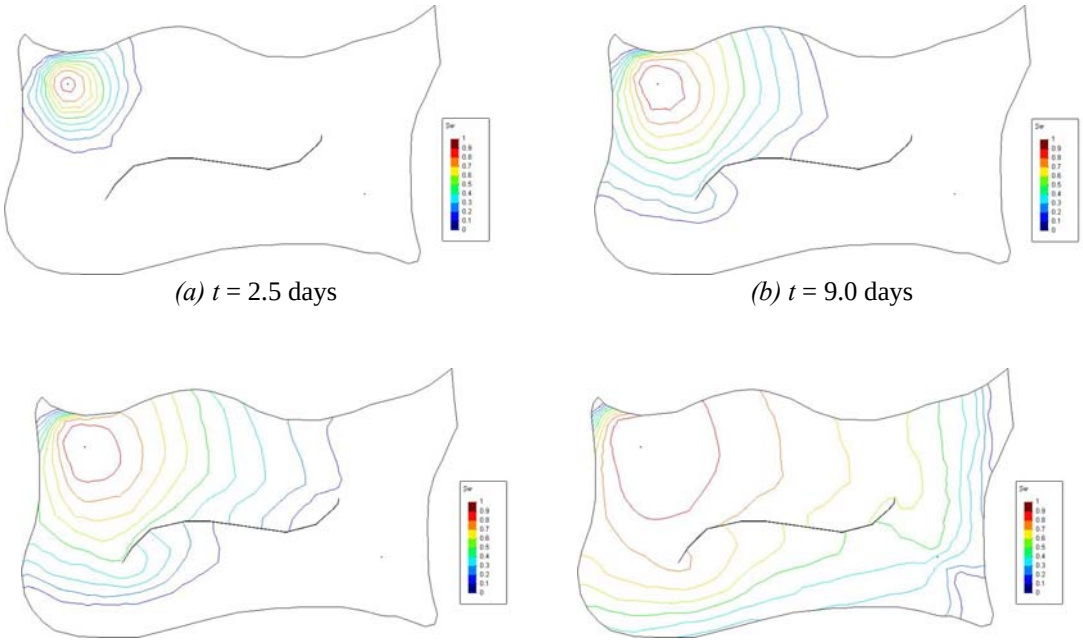


Figure 5. A two-dimensional faulted reservoir discretized using triangular and quadrilateral elements (867 nodes and 935 elements)

The water saturation profile at different simulation times is presented in Figure 6. One can note that these results are qualitatively reproducing the expected physical behavior. Besides, the facility as different elements can be handled in a same grid indicates that this method is promising for simulating petroleum reservoirs. Note, also, that the grid refinement around wells and faults can be easily done.



(a) t = 2.5 days *(b)* t = 9.0 days

(c) t = 14.5 days *(d)* t = 43.0 days

Figure 6. Water saturation profile at different simulation times

6. Conclusions

This work presented a model for simulating petroleum reservoirs using the Element-based Finite Volume Method (EbFVM). This method permits to discretize the domain in an unstructured fashion using triangular and/or quadrilateral elements, such that complex reservoir geometries can be easily represented. Furthermore, due to the control-volume approach, local mass conservation is enforced, permitting a direct physical interpretation of the resulting discrete equations.

The local refinement is naturally done in this method, without the difficulties found in structured methods related to the loss of accuracy in flow calculation between blocks at the intersection of coarse and fine grids.

It was demonstrated that this method can deal with the permeability map without averaging procedures, since this scheme assumes uniform properties inside elements, instead of inside of control volumes, as usual in the petroleum reservoir simulation literature. The approach adopted in this work avoids the need of weighting the permeability values at the control volumes boundaries.

Moreover, in the proposed method it is easy to include the full permeability tensor, which is an important issue in simulating heterogeneous and anisotropic reservoirs. The element-based calculations and the element-by-element assemble allows the construction of a general scheme with a clean computational implementation and easiness in maintenance and reusability.

The flexibility and accuracy of this method were demonstrated through numerical examples. They showed that it is possible to reduce the grid orientation effect by using a physically consistent mathematical model. This physically consistent approach reveals that right-angle or obtuse triangles can be used without restrictions in the grid, opposed to what is recommended in the literature for the conventional CVFEM.

7. Acknowledgements

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