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A METHODOLOGY USING FINITE VOLUMES AND FINITE ELEMENTS FOR AEROELASTIC ANALYSIS

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ABSTRACT

The solution of aeroelastic problems normally employs finite differences or finite element to solve the aerodynamic problem and finite element to solve the structural one. In general the mathematical model employed in the aerodynamic formulation does not follow the same degree of generality as the mathematical model used in the elastic problem. Moreover, numerical models for aerodynamic calculations differs significantly according to the flow regime. General aeroelastic models are, therefore, an important target to be pursued by numerical analysts. This paper presents a numerical model which goes in this direction. It employs a complete aerodynamic model with special feature of solving all speed flows. Preliminary results are present for the hemisphere cylinder subjected to a flow using the Euler equations.

INTRODUCTION

Across centuries necessity and curiosity have been important factors to help the man to make new discovers. The increase in aircraft necessities in man's life stimulate the growing of some branches in the aeronautical area, like aerodynamics and aeroelasticity. Aerodynamics is that part of Physics that studies the flow behaviour. Aeroelasticity is that part of Mechanics that studies the behaviour of structures when submitted to aerodynamic forces. The way these aerodynamic forces develop are strongly dependent on the flow regime.

Gilly (1970) classifies the flow regimes as incompressible, compressible subsonic, transonic, supersonic and hypersonic, refering to the Mach number. It is important, therefore, to obtain methodologies to analyse all these flow regimes, especially the region between Mach 0.8 and 1.2, denominated transonic regime, which presents a physical behaviour which is difficult to be modelled.

The calculation of transonic flows is a very important branch of computational aerodynamics due to its application to commercial aircrafts. However, the development of computational methods in this area is still insufficient and a lot of experiments have to be done for real designs. The development experienced by computers and numerical methods, however, indicates that the substitution of repetitive experimental work by numerical experiments is currently under way.

For transonic flows sharp changes in the aerodynamic coefficients are observed due to instabilities present in the flow. These instabilities are caused not only by strong viscous interactions in the boundary layer, but also by interactions between shock wave and boundary layer. This indicates that a model using the potential flow equations can not be used to analyse these effects, and one has to employ the Euler or the complete Navier-Stokes equations.

The design of an aircraft or a launch vehicle involves the calculation of the flow behaviour during a full flight passing through various flow regimes. Aeroelasticity plays an important role in the development of modern aircraft, since they tend to be more light and, consequently, more flexible. In order to analyse aeroelastic phenomena it is necessary to solve the Euler/Navier-Stokes and the dynamic equilibrium equations simultaneously (Guruswamy, 1990).

When solving aeroelastic problems it is common to use finite differences or finite elements to solve the aerodynamic problem, and finite elements to solve the structural one.

This work presents a computational methodology to solve aeroelastic problems using finite volumes and finite elements. Finite volumes are used to model the aerodynamic part of the problem. This includes the use of the all speed flow methodology and co-located variables in a boundary fitted frame. Finite elements are used to model the structural part as beam elements. To verify the behaviour of this methodology the aeroelastic solution is obtained for an hemisphere-cylinder subjected to a flow.

GOVERNING AND APPROXIMATE EQUATIONS

AERODYNAMIC PROBLEM

The governing equations for compressible flows are the well known Navier-Stokes equations that can be writen for a general escalar ϕ as

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x_i}\left(\rho u_i \phi - \Gamma^{\phi}\frac{\partial \phi}{\partial x_i}\right) + p^{\phi} = S^{\phi} , \tag{1}$$

where expressions for the terms appearing in the equation can be found in Marchi et al (1990).

To analyse complex problems it is necessary to transform these equations to a general coordinate frame. This can be done using the chain rule. Eq. (1) transformed to the (ξ,η,γ,τ) coordinate system results

$$\begin{aligned} &\frac{\partial}{\partial \tau}\left(\frac{\rho\phi}{J}\right) + \frac{\partial}{\partial \xi}\left\{\rho \tilde{U}\phi - \Gamma^{\phi}\frac{\partial}{\partial \xi}\left[J\left(\alpha_{11}\frac{\partial \phi}{\partial \xi} + \alpha_{12}\frac{\partial \phi}{\partial \eta} + \alpha_{13}\frac{\partial \phi}{\partial \gamma}\right)\right]\right\} \\ &+ \frac{\partial}{\partial \eta}\left\{\rho \tilde{V}\phi - \Gamma^{\phi}\frac{\partial}{\partial \eta}\left[J\left(\alpha_{12}\frac{\partial \phi}{\partial \xi} + \alpha_{22}\frac{\partial \phi}{\partial \eta} + \alpha_{23}\frac{\partial \phi}{\partial \gamma}\right)\right]\right\} \\ &+ \frac{\partial}{\partial \gamma}\left\{\rho \tilde{W}\phi - \Gamma^{\phi}\frac{\partial}{\partial \gamma}\left[J\left(\alpha_{13}\frac{\partial \phi}{\partial \xi} + \alpha_{23}\frac{\partial \phi}{\partial \eta} + \alpha_{33}\frac{\partial \phi}{\partial \gamma}\right)\right]\right\} = \\ &- \frac{p^{\phi}}{J} + \frac{S^{\phi}}{J} + GCL , \end{aligned} \tag{2}$$

where GCL is the Geometric Conservation Law (Trulio and Trigger, 1961) (Dimirdzic and Peric, 1988) (Batina, 1991). $\tilde{U}$, $\tilde{V}$ and $\tilde{W}$ are the contravariant components of the relative vetor velocity given by, taking $\tilde{U}$ as example,

$$\tilde{U} = \xi_x(u - x_\tau) + \xi_y(v - y_\tau) + \xi_z(w - z_\tau) .$$

Finally, α_{ij} and J are the metrics and the Jacobian of the coordinates transformation, respectively.

Algebraic equations are obtained using the finite volume method. Integrating Eq. (2) over the elemental control volume shown in Fig. 1,

$$\int_{v,\tau} [Eq.(2)]\ dVd\tau \ ,$$

it results

$$a_P^\phi \phi_P = \sum a_{nb}^\phi \phi_{NB} + b^\phi \ . \tag{3}$$

where

$$\sum a_{nb}^\phi \phi_{NB} = a_e\phi_E + a_w\phi_W + a_n\phi_N + a_s\phi_S$$
$$+ a_d\phi_D + a_f\phi_F + a_{nw}\phi_{NW} + a_{se}\phi_{SE} + a_{sw}\phi_{SW} + a_{de}\phi_{DE}$$
$$+ a_{dw}\phi_{DW} + a_{fe}\phi_{FE} + a_{fs}\phi_{FS} + a_{dn}\phi_{DN} + a_{ds}\phi_{DS} + a_{fn}\phi_{FN} \ .$$

Here the subscripts **e**, **w**, **n**, **s**, **d**, and **f** indicate the interface of the control volume, and **E**, **W**, **NW**, **SE**, etc, the **P** neighbouring control volumes. The coefficients of Eq. (3) are the same for **u**, **v**, **w** and temperature **T** since the co-located arrangement of variables is employed.

In order to obtain an adequate coupling between pressure and velocity it is necessary to evaluate the velocity vector components at the interfaces of the control volumes as functions of the velocities located at the center of the control volume (De Bortoli, 1990) (Marchi et al, 1990). For the **u** velocity written for the **P** control volume (Fig. 1) one has

$$u_P = \frac{1}{a_P^u}\left\{\sum (a_{nb}u_{NB})_P + \frac{M_P^\circ u_P^\circ}{\Delta\tau} + L[S^u]_P\frac{\Delta V}{J}\right\} + \frac{\Delta V}{a_P^u J}L[p^u]_P \ . \tag{4}$$

A special average is done using the above equation for volumes **P** and **E** to obtain a pseudo equation for **u** at the interface **e**, as follows

$$u_e = \frac{1}{a_P^u + a_E^u}\left\{ \sum (a_{nb}u_{NB})_P + \sum (a_{nb}u_{NB})_E + \frac{M_P^\circ u_P^\circ + M_E^\circ u_E^\circ}{\Delta\tau} + (L[S^u]_P + L[S^u]_E)\frac{\Delta V}{J}\right\} - 2\frac{\Delta V}{a_P^u + a_E^u}\frac{L[p^u]_e}{J} \ , \tag{5}$$

where

$$\frac{L[p^u]_e}{J} = \frac{p_E - p_P}{\Delta\xi}\xi_{x_e} + \frac{p_N + p_{NE} - p_S - p_{SE}}{4\Delta\eta}\eta_{x_e} + \frac{p_D + p_{DE} - p_F - p_{FE}}{4\Delta\gamma}\gamma_{x_e} \ .$$

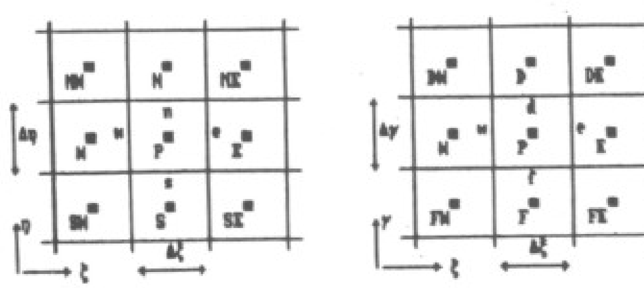

Figure 1 Elemental control volume for P

It can be observed that the pressure gradient acting on the u_e velocity component resembles the use of staggered grids. This promotes an adequate coupling between velocity and pressure since the pressure correction equation will have the same form as when a staggered grid is used (Peric et al, 1988) (Marchi et al 1989) (De Bortoli, 1990).

Another important step in the methodology used here is the mass flow linearization. To start with, the discretized mass conservation equation can be written as follows

$$\frac{M_P - M_P^\circ}{\Delta\tau} + M_e - M_w + M_n - M_s + M_d - M_f = 0 \ . \tag{6}$$

Taking the mass flow at the east face, as example, the following linearization allows the solution for all speed flows (Harlow and Amsdem, 1971) (Patankar, 1971) (Van Doormaal, 1985) (Silva and Maliska, 1988)

$$M_e = (\rho^*\bar U + \rho\bar U^* - \rho^*\bar U^*)_e\Delta\eta\Delta\gamma \ , \tag{7}$$

where the (*) represents the best estimate values of the variables. When introducing Eq. (7), and its analogues for the remaining faces, in Eq. (6), it is clear that both density and velocity will be unknowns. When replacing ρ as a function of pressure, from the state equation, and velocity as a function of pressure, from the momentum equations, the resulting equation will be an equation for pressure, as shown below. Following the SIMPLEC (Van Doormaal and Raithby, 1984) procedure to obtain expressions for the velocity and density corrections, one has

$$\bar U_e = \bar U_e^* - d_P^u(p'_E - p'_P)\alpha_{11_e} \ ,$$

$$\rho_P = \rho_P^* + c_P^P p'_P \ ,$$

where the densities at the interfaces are obtained by

$$\rho_e = \left(\frac{1}{2} + \varpi_e\right)\rho_P + \left(\frac{1}{2} - \varpi_e\right)\rho_E \ .$$

Here ϖ is $\mp$ 0.5, depending on the sign of the velocity component at the interface. Introducing the equations for the density and velocity corrections in the mass conservation equation, one obtains an equation for pressure as follows

$$a_P^{P'} p'_P = \sum a_{nb}^{P'} p'_{NB} + b^{P'} \ . \tag{8}$$

STRUCTURAL PROBLEM

When the FEM (Finite Element Method) is used for solving structural problems, one has to select an element formulation which includes the most important structural behaviour as well as to be simple and effective. Along the remaining of this section, one presents the Timoshenko beam formulation for plane deflections and, for considering space ones, it is sufficient to add similar relations relative the plane orthogonal to the first one. The Timoshenko beam theory (Dym and Shames, 1973) considers that shearing deformation is constant in every beam cross-section. The displacement can then be considered as the sum of those due to pure bending and shearing deformation, as follows

$$u_1(x,y,z,t) = -z\left(\frac{\partial u_3}{\partial x} - \beta(x)\right) \ ,$$
$$u_2(x,y,z,t) = 0 \ , \tag{9}$$
$$u_3(x,y,z,t) = u_3(x,t) \ ,$$

where $\beta(x)$ is the shear angle formed by a given cross section with the neutral axis.

The kinetic energy for this element is given by

$$T = \frac{1}{2}\int_V \varrho(\dot{u}_i)^2 dV \ , \tag{10}$$

while the deformation energy associated with the potential energy is given by

$$\pi_p = \int_0^L \left\{ \frac{EI}{2}\left(\frac{\partial \psi}{\partial x}\right)^2 + \frac{\kappa GA}{2}\left(\frac{\partial u_3}{\partial x} - \psi\right)^2 - f\, u_3 \right\} dx \ . \tag{11}$$

Here ϱ, I, E, κ, ψ, G, A and f are the material density, moment of inertia, linear elasticity module, a constant to correct the shear, the rotation of the line elements along the centerline due to bending only, a Lamé constant, area and a loading component, respectively.

Employing Hamilton's principle,

$$\delta \int_{t_1}^{t_2} \{ T - \pi_p \} \ dt = 0 \ , \tag{12}$$

for E, I, G and A constants, and approximating the solution by finite elements, results

$$[M]\{\ddot{q}\} + [K]\{q\} = \{N\} \ , \tag{13}$$

where q is the generalized displacement vector.

This element can be employed to analyse small displacement problems, but, with some changes in the reference system, it can also be employed to solve for large displacements as the ones that appear in wings (Cook, 1981). For nonsymmetrical bodies, it is also necessary to calculate the shear center (Timoshenko and Goodier, 1984).

AEROELASTIC PROBLEM

Since the aeroelastic deformation resulting from the flexibility of an aerodynamic body can alter the flow conditions, it is necessary to solve the aerodynamic and structural problems simultaneously.

The solution procedure consists in solving the aerodynamic problem for a given instant of time, obtaining the initial excitation (forces and moments) which originate from friction and shape effects, that is

$$\vec{N} = \mu A \frac{\partial \vec{V}}{\partial \vec{n}} + p(\vec{n}.A) \ . \tag{14}$$

This excitation will deform the structure, modifying the flow. Then it is necessary to obtain the displacements and rotations at each node to calculate the new aerodynamic grid.

SOLUTION PROCEDURE

Basically, the computational procedure for this aeroelastic analysis is:

A. Aerodynamic problem

1. Domain discretization
2. Estimation of variables for the present time cycle
3. Calculation of coefficients of Eq. (3)
4. Calculation of source terms of Eq. (3)
5. Calculation of u^*, v^*, and w^* using the MSI (Modified Strongly Implicit Procedure) (Schneider and Zedan, 1981)
6. Calculation of $\tilde{U}$, $\tilde{V}$ and $\tilde{W}$
7. Calculation of p' using the MSI, Eq. (8)
8. Correction of $\tilde{U}^*$, $\tilde{V}^*$ and $\tilde{W}^*$
9. Correction of ρ
10. Calculation of T using the MSI, Eq. (3)
11. Calculation of ρ using the state equation
12. Return to item 3 until aerodynamic cycle converges

B. Structural Problem

13. Calculation of aerodynamic force, Eq. (14)
14. Obtain the structure system matrices [M] and [K], Eq. (13)
15. Calculation of displacements and rotations, Eq. (13)
16. Return to item 1 for a new time step cycle, changing the grid.

NUMERICAL RESULTS

Since this is a very complex three-dimensional coupled problem the first step in checking the methodology is to solve for aerodynamic and elastic problems independently. To verify the aerodynamic computational code, the flow over a hemisphere-cylinder at Mach 0.85 is calculated and the results are compared with the numerical ones obtained by Azevedo (1988) and the experimental ones from Hsieh (1977), presented by Azevedo.

The results depicted in Fig. 3 and Fig. 4, obtained using the grid of Fig. 2, show that the computational code has the ability to represent aerodynamic problems properly. Better results can be obtained if a mesh refinement is done (Maliska et al, 1991).

To analyse dynamic problems it is necessary to verify if the mass and the stiffness matrices are correctly formed, and if the system of equations is correctly solved. The use of integration methods (Bathe, 1982) make it easy to analyse the beam elements behaviour. In this work Houbold's method is used.

The second natural frequency, found for a steel beam 30 m long, with 1 m external radius and moment of inertia of 0.09425 m^4, is 14.06 using the Euler-Bernoulli beam or 14.01 using the Timoshenko beam theory. Comparison with the value 14.0, found by Feng and Quevat (1990), allows one to conclude that the results are in good agreement.

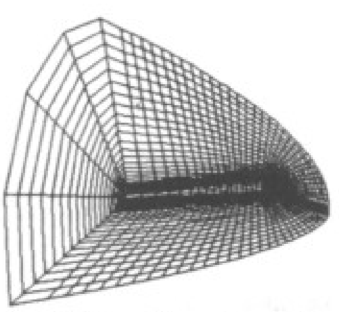

Fig. 2 Three-dimensional view of hemisphere-cylinder grid (34x26x19 volumes)

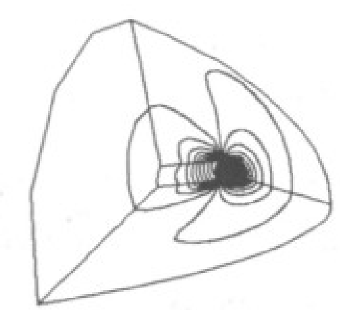

Fig. 3 Hemisphere-cylinder density contour lines for Mach 0.85 and $\alpha = 6^\circ$

SIMPLE AEROELASTIC SOLUTIONS

To obtain preliminary results with the methodology, the aeroelastic analysis of the hemisphere-cylinder case, clamped at one of its end with 6 degrees angle of attack, 10 cm wall thickness, 5.00 m length and 0.75 m external radius, subjected to Mach 0.85 is done. The angle of attack is with respect to the vertical direction. The computational grid used is 17x15x16 volumes. This is, of course, a coarse grid and can be used only for a qualitative analysis of the problem.

Fig. 5 presents the beam deflections versus time, showing that the structural behaviour of this problem is very similar to the mass-spring system behaviour submitted to a simple force. It can be seen that the vertical deflection follows the sinusoidal movement, as expected.

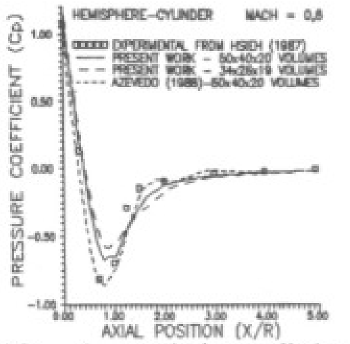

Fig. 4 Hemisphere-cylinder pressure coefficient distribution for Mach 0.6 and $\alpha = 0°$

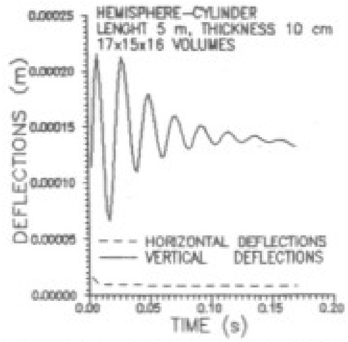

Fig.5 Deflections versus time for the hemisphere-cylinder nose

CONCLUSIONS

Preliminary results show that aeroelastic analysis can be done using finite volumes and finite elements. Much more studies have to be done to evaluate the efficiency of this methodology. One drawback is the limited time step necessary for the solution of the aerodynamic problem. This is not new and is well reported in Azevedo (1988) and Guruswamy (1990). Several alternatives to alleviate this problem are under consideration. Among them, the multigrid techniques (Radespiel, 1989) can be employed to improve the methodology efficiency, trying to remove the time step limitations. It is the authors opinion that the present methodology is promising and can be used to analyse aeroelastic problems in a consistent way, because the finite volume method is based on the physical aspects of flow.

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