XII CONGRESSO BRASILEIRO DE ENGENHARIA MECÂNICA - BRASÍLIA - BRASIL (DEZEMBRO, 1993)
12th BRAZILIAN CONGRESS OF MECHANICAL ENGINEERING - BRASÍLIA - BRAZIL (DECEMBER, 1993)

A FINITE-VOLUME TECHNIQUE FOR THE SOLUTION OF MISCIBLE DISPLACEMENT IN POROUS MEDIA

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ABSTRACT

This paper describes a numerical methodology for the solution of miscible displacement in porous media. The model is conceived for the simulation of tracer concentration in petroleum engineering. It uses structured boundary-fitted coordinates and the finite volume concept. The key question of finding an interpolation function for the diffusion/advection equation is addressed. The model is applied for the simulation of a tracer injection in a single phase flow in a five-spot arrangement. The numerical results are compared with other numerical and experimenal results. The agreement in very good.

1. INTRODUCTION

The solution of miscible displacement in porous media is of great interest in petroleum reservoir engineering, specially for the analysis of the behaviour of the reservoir and for the estimation of production capabilities. Miscible displacement occurs when tracers are injected in the water phase, such that the water and tracer forms a single phase. The solution of this flow amounts to solve the mass conservation equation and Darcy's equation for a single phase in porous media and of the convection/diffusion equation for the tracer concentration in that phase.

The traditional methods in petroleum engineering, as reported in Aziz and Settari (1979), employ, in general, cartesian grids for the domain discretization. This coordinate system has the disadvantage of not conforming the reservoir boundaries which are always of irregular shape. Recently efforts have been made in order to develop more general schemes, and curvilinear coordinate systems appear as a good alternative. Another observed trend in the petroleum engineering simulation is the use of control volume methods in conjunction with flexible grids. Examples can be seen in Sharpe and Anderson (1991), Rozon (1989), Forsyth (1991), Santos et al.(1991), among others.

The advantage of using control volume methods is claimed to be supported by the fact that the equations are satisfied at control volume level, despite the size of the control volume. To have this condition satisfied for every control volume has a tremendous influence when solving the system of equations encountered in fluid flow simulation. If one recognizes that the coefficients of the discretized equations for momentum and energy are dependent of the mass flow, it is crucial to have the mass conservation equation being satisfied at control volume level. After all, the conservation at control volume level avoids sinks and sources of mass, momentum and energy to happen at the control volume interfaces.

The present paper presents a finite volume numerical methodology using structured boundary-fitted grids for the solution of miscible displacement. For the solution of the convection/diffusion equation it is employed an interpolation function widely used in fluid flow simulations with the aim of reducing the numerical diffusion. The five-spot case is solved considering both active and inactive tracer. The results are compared with other numerical results which employ cartesian grids with five and nine diagonal and aligned elements. Comparisons are also made with experimental results and the agreement is very good.

2. THE PHYSICAL PROBLEM

Consider a petroleum reservoir with a given porosity and volume, with a steady flow of a fluid which constitutes the phase in consideration, to which will be added the tracer. This phase may be flowing together with other phases but, for the purpose of th present work, this is immaterial. At time t=0 one starts to inject in the phase a tracer with concentration equals to 1.0. The tracer and the fluid of the phase are miscible and form a single phase homogenous mixture. The viscosity of the mixture is related to the concentration of the tracer through

$$\mu = \frac{\mu_r}{[(M^{0.25} - 1)\, C + 1]^4} \tag{1}$$

where the mobility ratio M is here defined as

$$M = \frac{\mu_r}{\mu_t} \tag{2}$$

The goal is to obtain the transient concentration of the tracer, specially in the production wells closed to the injection well.

The condition in which the mobility ratio is equal to unity (M = 1) introduces a great simplification in the solution. In this case the steady pressure and velocity fields calculated for the phase are not altered when the tracer is injected. This means that during the transient solution it is only required to advance the concentration equation. In this situation the tracer is called inactive. For the general case, when the tracer is then called active, both mass conservation equation and concentration equation need to be solved simultaneously in time after the begining of the tracer injection.

3. GOVERNING EQUATIONS

Starting from the mass conservation equation given by

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{3}$$

where u is the veocity field, ρ the density and ϕ the porosity, combined with the Darcy's equation

$$V = -\frac{k}{\mu}\nabla P \tag{4}$$

one obtains the mass conservation equation written in terms of pressure as

$$\nabla.(\nabla P) = 0 \tag{5}$$

In the above equation it was considered density as constant. It is also usual to consider the fluid quasi-incompressible, retaining in the right hand side of Eq. (5) a temporal term for the pressure involving a compressibility coefficient. Since in this work all runs were made with zero compressibility coefficient this term is not retained in the equation.

To obtain the equation which governs the transient concentration of the tracer one has to combine the mass conservation with the diffusion equation of the tracer in the mixture. The resulting equation is

$$\frac{\partial}{\partial t}(\phi C) + \nabla.J + \nabla.(uC) + qC_i = 0 \tag{6}$$

where $J = -\phi D\nabla.C$, ϕ is the porosity, C the concentration of the tracer in the phase and D the diffusion coefficient given by

$$D_x = \frac{\alpha u^2}{|V|} \tag{7}$$

$$D_y = \frac{\alpha v^2}{|V|} \tag{8}$$

where α is the dispersion coefficient. Recalling the solution procedure, Eqs. (5) and (6) need to be solved simultaneously when M is different from unity.

4. TRANSFORMED AND DISCRETIZED EQUATIONS

The above governing equations are to be solved in a boundary-fitted curvilinear coordinate system, which requires the equations be transformed to this new system. Due to the different character of Eqs. (5) (diffusive) and (6) (convective/diffusive) they will be transformed and discretized independently, permitting that physical interpretation of its terms, useful in applying boundary conditions, could be given.

4.1 - PRESSURE EQUATION. Using a transformation of the type

$$\xi = \xi\,(x, y) \tag{9.1}$$

$$\eta = \eta\,(x, y) \tag{9.2}$$

Eq. (5) transformed to the (ξ, η) domain and integrated in the elemental volume has the following form

$$\frac{\partial}{\partial \xi}\left[D_1 \frac{\partial P}{\partial \xi} + D_2 \frac{\partial P}{\partial \eta}\right] + \frac{\partial}{\partial \eta}\left[D_3 \frac{\partial P}{\partial \eta} + D_4 \frac{\partial P}{\partial \xi}\right] = 0 \tag{10}$$

where the diffusion coefficient are given by

$$D_1 = J\,\alpha \qquad D_2 = D_4 = -J\,\beta \qquad D_3 = J\,\gamma \tag{11}$$

and α, β, γ and J are the components of the metric tensor and the jacobian of the transformation, respectively.

Since one is dealing with a finite volume method it is important to take advantage of the procedure recognizing the physics of the terms in brackets in Eq. (10). This recognition allows one to easily implement boundary conditions of prescribed flow across reservoir boundaries. The components of the velocity vector using Darcy's equation are given by

$$u = -\frac{k}{\mu}\frac{\partial P}{\partial x} \tag{12.1}$$

$$v = -\frac{k}{\mu}\frac{\partial P}{\partial y} \tag{12.2}$$

If a cartesian coordinate system is used the velocity components u and v are the responsible for carrying mass across the control volume interfaces. For a generalized coordinate system this are realized by the contravariant velocity components, which are related to the cartesian components by

$$U = u\,y_\eta - v\,x_\eta \tag{13.1}$$

$$V = v\,x_\xi - u\,y_\xi \tag{13.2}$$

Using Eqs. (12) and (13) one obtains

$$U = -\frac{k}{\mu}\left[J\alpha \frac{\partial P}{\partial \xi} - J\beta \frac{\partial P}{\partial \eta}\right] \tag{14.1}$$

$$V = -\frac{k}{\mu}\left[J\gamma \frac{\partial P}{\partial \eta} - J\beta \frac{\partial P}{\partial \xi}\right] \tag{14.2}$$

It can be seen that the terms in brackets in Eq. (10) represents, respectively $-U\mu/k$ and $-V\mu/k$. As the mass flow in the east and west faces are related to U, and in the north and south faces to V, acording to Fig.1, when the mass flow is precribed the full term in brackets is replaced by the prescribed mass flow using the contravariant velocity components. When the pressure, instead of the mass flow is prescribed, the pressure derivatives inside the brackets need to be evaluated using the prescribed pressure and the surrounding unknowns pressure points. These pressure derivatives are then substituted in Eq. (10) and the final discretized form of the pressure equation reads

$$A_P P_P = A_N P_N + A_S P_S + A_E P_E + A_W P_W + A_{NE} P_{NE} + A_{NW} P_{NW} + A_{SE} P_{SE} + A_{SW} P_{SW} + b \tag{15}$$

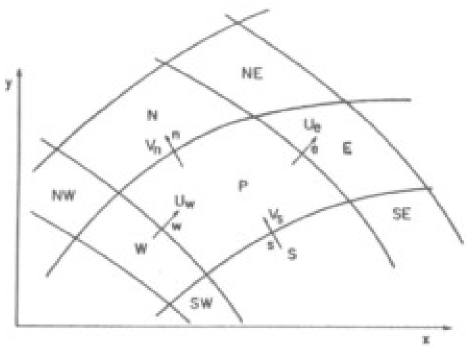

Fig. 1 - Elemental volume showing the velocity components which carries mass across the boundary.

The coefficients of Eq. (15) can be found in Maliska et al. (1991) where internal control volumes as well as boundary control volumes are considered.

4.2 - EQUATION FOR THE TRACER CONCENTRATION. Using the transformation given by Eqs. (9.1) and (9.2), the equation for the concentration of the tracer in the (ξ, η) domain is

$$\frac{1}{J}\frac{\partial}{\partial t}(\phi C) + \frac{\partial}{\partial \xi}(UC) + \frac{\partial}{\partial \eta}(VC) = \frac{\partial}{\partial \xi}\left[D_1 \frac{\partial C}{\partial \xi} + D_2 \frac{\partial C}{\partial \eta}\right] + \frac{\partial}{\partial \eta}\left[D_3 \frac{\partial C}{\partial \eta} + D_4 \frac{\partial C}{\partial \xi}\right] \tag{16}$$

where the diffusion coefficients are the ones given by Eq. (11) multiplied by k/μ. The integration of the above equation in time and space gives

$$\frac{[(\phi C)_P - (\phi C)_P^0]\,\Delta V}{J\,\Delta t} + [(UC)_e - (UC)_w]\,\Delta\eta + [(VC)_n - (VC)_s]\,\Delta\xi =$$

$$\left[D_1 \frac{\partial C}{\partial \xi} + D_2 \frac{\partial C}{\partial \eta}\right]_e \Delta\eta - \left[D_1 \frac{\partial C}{\partial \xi} + D_2 \frac{\partial C}{\partial \eta}\right]_w \Delta\eta + \left[D_3 \frac{\partial C}{\partial \eta} + D_4 \frac{\partial C}{\partial \xi}\right]_n \Delta\xi - \left[D_3 \frac{\partial C}{\partial \eta} + D_4 \frac{\partial C}{\partial \xi}\right]_s \Delta\xi \tag{17}$$

The values of the concentration and its derivatives at the control volume interfaces are obtained using the WUDS (Weighted Upstream Differencing Scheme) of Raithby and Torrance (1974). This scheme uses one-dimensional interpolation functions obtained from the exact solution of the convection diffusion equation. According to the local Peclet number this scheme has, as limiting cases, the Central Differencing Scheme (CDS), when only diffusive effects are present, and the Upstream Differencing Scheme (UDS) when convection dominates. In this work only results for the limiting cases are reported. To illustrate, the interpolation function for the east face of a control volume is

$$C_e = (1/2 + \bar{\alpha}_e)\, C_P + (1/2 - \bar{\alpha}_e)\, C_E \tag{18}$$

$$\left.\frac{\partial C}{\partial \xi}\right|_e = \frac{\bar{\beta}_e\,(C_E - C_P)}{\Delta\xi} \tag{19}$$

Introducing Eqs. (18) and (19), and the similar equations for the other faces, into Eq. (17), one obtains

$$a_P C_P = a_N C_N + a_S C_S + a_E C_E + a_W C_W + a_{NE} C_{NE} + a_{NW} C_{NW} + a_{SE} C_{SE} + a_{SW} C_{SW} + b \tag{20}$$

Expressions for $\bar{\alpha}$ and $\bar{\beta}$ in Eqs. (18) and (19) and for the coefficients of Eq. (20) can be found in Maliska et al. (1991). They are not given here due to space limitations.

5. THE PROBLEM UNDER CONSIDERATION

To apply the described methodology the usual five-spot problem is considered. This problem is chosen because it is widely used in the petroleum literature for testing new numerical methods. Fig. 2 shows the geometry and the 19x50 grid employed. It is to be noted that the grid is refined at well level, such that there is no need of employing a well model. The wells are at the corners with prescribed pressure and the remaining boundaries are of symmetry type. The grid employed, in this particular case, is very well aligned with the flow and it is expected that it will help in reducing the numerical diffusion.

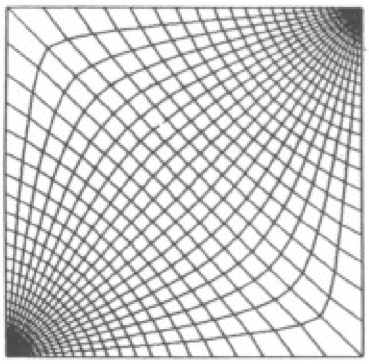

Fig. 2 - Geometry and the 19x50 grid. Wells are at the corners where the grid is concentrated.

At the symmetry boundaries the normal pressure gradient is zero, which is equivalent in having no flow. Due to the symmetry the normal concentration gradient is also equal to zero. At the wells the pressure is prescribed and the concentration is prescribed at the injection well, with a local parabolic boundary condition at the production well.

6. NUMERICAL RESULTS

The problem was solved initially for M=1.0, that is, when the tracer is inactive. In this case, as already mentioned, the solution of the pressure field is obtained only once. The grid employed was a 31x90 and both systems of linear equations for pressure and concentration are solved using the MSI procedure of Schneider and Zedan (1982).

Fig. 3 depicts the concentration profile, along the diagonal (line joining the two wells), for the time when the volume of tracer injected is equal to 50% of the porous volume of the reservoir. These results were obtained for a Peclet number of 1.0 with $\bar{\alpha}$ = 0. and $\bar{\beta}$ = 1, that is using the CDS scheme. Using the UDS scheme, that is, $\bar{\alpha}$ = 0.5 and $\bar{\beta}$ = 1, the results are almost the same due to the fact that the Peclet number is small. The Peclet number under consideration here is defined by

(21)

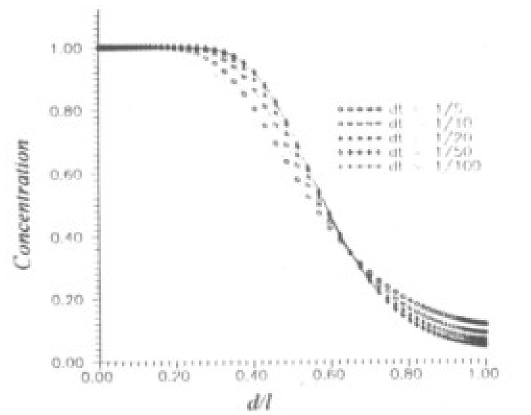

Fig. 3 - Concentration profile along the diagonal.

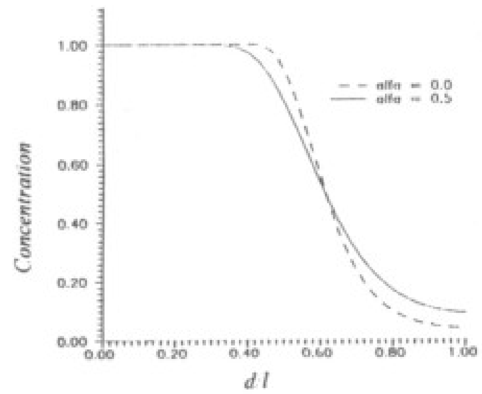

Fig. 4 - Concentration along the diagonal for Pe = 100 for the CDS and UDS schemes.

Still considering Fig. 3 , one can see that for the time step 1/50 and lower, the solution is already independent of the time step. Fig. 4 shows the concentration along the diagonal for Peclet number of 100 using the CDS and UDS scheme. It can be seen that for equals to zero (CDS) the sharp front of the tracer is better captured than with $\bar{\alpha}$ = 0.50 (UDS). The smeared profile for the UDS scheme resembles the existence of numerical diffusion. For this particular problem the grid lines are fully aligned with the streamlines and so, numerical diffusion due to grid inclination with respect to flow is absent. The smearing can be attributed to the fact that the UDS approximation is one order less accurate than the CDS approximation. It is not shown here but, for the same Peclet number of 100, with the CDS scheme and smaller time steps, the results are physically wrong.

In order to check the numerical model the results of the present work were compared with the numerical results reported in Santos et al (1992). Fig. 5 presents the fractional recovery as a function of time for M = 10 for the present paper and for four cartesian schemes reported in Santos et al. (1992).

Tab.1 - Data used in the simulation and experiment

	Simulation	Experiment
Dimension (m)	300x300	0.15x0.15
Permeability (md)	200	519
Porosity (%)	20	17.75
Injected flow rate(cm^3/s)	500	0.0041166
Dispersivity (m)	7.5	0.00115

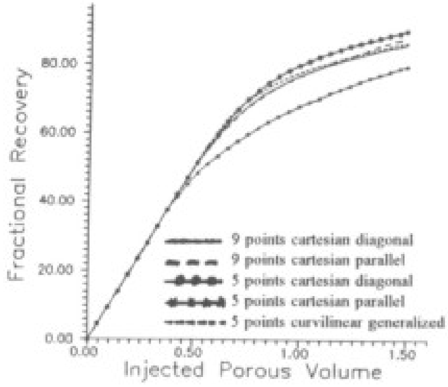

Fig. 5 - Fractional recovery for M = 10

The outcome of these tests demonstrates that the use of curvilinear grids reduces the numerical diffusion and produces results comparable with more sophisticated and elaborated 9-point schemes. It is important to point out that the method herein presented is not a 9-point scheme. It may become a 9-point stencil due to the nonorthogonality of the grid. However, it does not contain any special scheme which would be equivalent of using two-dimensional interpolation functions, as is done in the 9-point cartesian schemes.

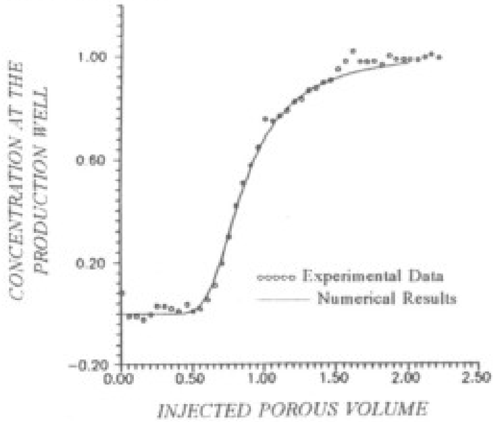

Fig. 6 - Concentration at the production well. Comparison with experimental results.

As a final test of the model the numerical results are compared with the experimental ones of Santos et al. (1992), only available for M = 1. Table 1 and 2 give the data of the reservoir analized. A 19x50 curvilinear grid was employed with the same scheme used to compute the results presented above. Fig. 6 shows the concentration at the production well, compared with the experimental results of Santos et al. (1992) and continuos injection, and Fig. 7 for a tracer pulse of 0.4 porous volume. Again the numerical model performed well.

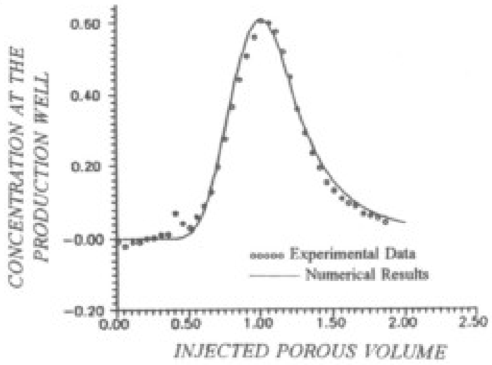

Fig. 7 - Concentration at the production well. Comparisons with experiment.

7. CONCLUSIONS

The present work described a numerical model using curvilinear grids and one dimensional interpolation functions for the solution of miscible displacement. The method uses the control volume concept which is helpful in understanding the transformed equations and prescribing the boundary conditions. The numerical results obtained for mobilities ratios of 1 and 10 were compared with other numerical and experimental results and good agreement was found. The overall outcome of the present work encourages further development using boundary-fitted meshes. Imiscible displacement are already under investigation and the results are going to appear in a companion paper.

8. ACKNOWLEDGEMENTS

The authors are grateful to PETROBRAS - PETROLEO BRASILEIRO S/A for the financial support of this work.

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