EXTENSION OF PEACEMAN'S AND DING'S WELL INDEXES FOR APPLICATION IN 3D RESERVOIR SIMULATION WITH HORIZONTAL WELLS

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Abstract. *In reservoir simulation, one of the major difficulties is the scale difference between the reservoir and the wellbores. The reservoir scale is of order of kilometers whereas the wells diameter is of order of centimeters. Thus, an accurate approximation of the flow near the wells would require an extremely refined mesh in order to capture the pressure gradient. However, the use of a very refined mesh leads to a very large computational effort. Because of this, use of meshes having a size compatible with the scale of the reservoir and representing well flow as a source term in the mass balance for the control volumes where they are located, is a practical solution in reservoir simulation. In that case, it is necessary a model that mimics the local physics around the wells. The well models used in the petroleum industry normally employs a local analytic solution, which provides the pressure gradient near the well by means of a radial flow representation. These models, however, were developed for two-dimensional reservoirs. With the evolution of new technologies in well drilling, horizontal wells are more and more employed, requiring new developments for that situation. In the present work it is developed an extension of two-dimensional Peaceman's and Ding's well models to a three-dimensional situation, for application in partially penetrating horizontal wells. For that goal, a local analytical solution for a cylindrical radial flow for 3D situations were considered. All models led to good results, demonstrating their ability to represent the physics around the wells, avoiding grid refinement.*

Keywords: *Well Model, Well Index, Reservoir Simulation, Numerical Solution, Finite Volume Method.*

1. INTRODUCTION

Well models used in reservoir simulation employs a local analytical solution based on a radial flow. This local solution is used for representing the flow near the wells and applied in two or three dimension situations. In 2D cases the wellbore represented at the reservoir is fully penetrating, because just vertical wells are considered. In 3D cases when are used horizontal wells, the extremities of this kind of wells are inside of the reservoir, generating a different in the flow near of the extremities.

For application of that well models in 3D cases wellbore must be fully penetrating because the models can not represent the horizontal well extremities. The behavior at the ends of the well is not cylindrical radial flow, but spherical radial flow and models must be improved in order to represent that kind of flow.

Peaceman's and Ding's well models (Peaceman, 1978, 1983; Ding, 1998) do not include the spherical radial flow in well indexes because they were derived for 2D cases originally. The objective of this work is to extend the Peaceman's and Ding's models in order to apply them on 3D reservoir considering the spherical radial flow at the extremities of the horizontal well.

Those models were chosen because Peaceman's model is still one of the most used in reservoir simulation, and Ding's model was a extension of the Peaceman's for off-center wells, that is, wells whose location does not coincide with the grid-block center.

2. RESERVOIR SIMULATOR

2.1 Governing Equation

The reservoir model employed is a one-phase incompressible flow in consolidated, isotropic, homogeneous and isothermal porous medium. The main equation in the model is the Darcy's law proposed by Darcy (1856), given by

$$\vec{v} = -\frac{K}{\mu}\left(\nabla P - \rho\vec{g}\right) \tag{1}$$

where $\vec{v}$ is the Darcy's velocity vector, μ is the dynamic viscosity, ρ is the density and P is the pressure. K is the absolute permeability of the porous medium and $\vec{g}$ is the gravitational vector.

Replacing the Darcy's velocity on the mass conservation equation, one obtains,

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot\left[\rho\left(-\frac{K}{\mu}\left(\nabla P - \rho\vec{g}\right)\right)\right] = 0 \tag{2}$$

where ϕ is the porosity of porous medium.

As the porous medium is consolidated and the fluid is incompressible, the first term on the equation Eq. (2) is null, so the flow equation reduces to

$$\nabla\cdot\left[\rho\left(-\frac{K}{\mu}\left(\nabla P - \rho\vec{g}\right)\right)\right] = 0 \tag{3}$$

2.2 Discretization

The finite volume method is employed for the discretization of the flow equation. That method is based on a property balance in discrete control volumes (Aziz and Settari, 1979; Patankar, 1980; Maliska, 2004), in this case mass balance. The finite volume used is a box in a cartesian grid, with dimensions Δx, Δy and Δz, in directions *x*, *y* and *z*, respectively.

Applying the mass balance on a hexahedral control volume it obtains

$$\sum_{nb} T_{nb}\left(P_{NB} - P_P\right) = \lambda\rho g\frac{\Delta x\Delta y}{\Delta z}\left(H_T + H_B - 2H_P\right) \tag{4}$$

where T_{nb} is the transmissibility between the control volume P and its neighbor NB, that share the face nb and H is control volume depth. The transmissibilities are defined by

$$T_{w,e} = \lambda K\frac{\Delta y\Delta z}{\Delta x}, \qquad T_{n,s} = \lambda K\frac{\Delta x\Delta z}{\Delta y} \qquad \text{and} \qquad T_{t,b} = \lambda K\frac{\Delta x\Delta y}{\Delta z} \tag{5}$$

where λ is mobility given by $1/\mu$.

For grid-blocks that have a wellbore inside, a source term is necessary in the balance equation, in order to represent the mass flow entering or leaving the control volume throughout the wellbore. The flow rate that must be used as source term in those control volume equations is

$$Q = \lambda WI\left(P_{well} - P_p\right) \tag{6}$$

where WI is the so-called well index, which depends on geometric parameters and porous medium properties and P_{well} is the wellbore pressure.

3. WELL MODELS

Well models are used for relating the well pressure with the grid-block pressure. So, in order to find that relation a local analytical solution for radial flow around the well is used. The well models define a well index able to insert the radial flow caracteristics through analytical solution. The well index is used in the Eq. (6), as previously shown.

3.1 Local Analytical Solution

In both original well models considered in this work, a radial flow with a local analytical solution is used to connect the wellbore pressure and the grid-block pressure. So the analytical solution for radial flow from the wellbore is

$$P(r) - P_{well} = \frac{Q\mu}{2\pi hK} \ln\left(\frac{r}{r_{well}}\right) \tag{7}$$

where r_{well} is well radius, i.e., the radius where the pressure is well pressure and $P(r)$ is the pressure on radius $r > r_{well}$

3.2 Well Index

The well index is defined as the ratio between the mass flow rate on wellbore and the pressure difference between wellbore and grid-block pressure. With well index definition and radial flow analytical solution, is easy to find

$$WI = \frac{2\pi hK}{\ln\left(\frac{r_{well}}{r_{eq}}\right)} \tag{8}$$

where r_{eq} is the equivalent radius, which contains geometric information, well radius and grid-block dimensions.

3.3 Peaceman's Well Model

Originally, the well model proposed by Peaceman (1978) was derived for the square grid-blocks and homogeneous and isotropic porous medium, in order to obtain the equivalent radius with the radial analytical solution.

In order to derive the equivalent radius equation, Peaceman considered a mass balance on well grid-block, and assumed a radial flow centered on well grid-block. As the well grid-block center and the well location are the same place, the well model assumes that well grid-block pressure is locate in a equivalent radius and the well pressure is in the well radius.

Applying Eq. (6) on well grid-block and using Eq. (8) is possible to obtain the equivalent radius equation. Considering rectangular grid-blocks, Peaceman's equivalent radius is

$$r_{eq} = 0.28\sqrt{\Delta x^2 + \Delta y^2}. \tag{9}$$

3.4 Ding's Well Model

Ding's well model (Ding, 1998) extended Peaceman's well model for off-center well locations. Then, that author made corrections in face transmissibilities dividing wellbore flow rate in proportion to its proximity to the face. That corrections are made in the transmissibility of each face based on the angle formed between well location and face vertices. The proportional grid-block face flux is defined by

$$f_e^{eq} = \frac{\Theta}{2\pi} Q \tag{10}$$

where Q is well mass flow rate, that will be divided by four grid-block faces using the angle Θ as proportionality factor.

In order to introduce the corrections, Ding redefined the well grid-block transmissibility using a equivalent length to representing the discrete length of flux as

$$T_e^{eq} = \lambda K h \frac{\Delta y}{L_{eq,e}} \tag{11}$$

where T_e^{eq} is the equivalent transmissibility and $L_{eq,e}$ is the equivalent length. $L_{eq,e}$ is equal to Δx when the well location is the center of the grid-block, returning to the original transmissibility. In that way it is possible to find the correction factor as the ratio between the original transmissibility and the equivalent transmissibility, obtaining

$$\alpha_e = \frac{T_e^{eq}}{T_e} = \frac{\Delta x}{L_{eq,e}} \tag{12}$$

where α is reduced just a ratio between the discrete length, Δx, and equivalent length ,L_{eq}.

That kind of correction allows existing reservoir simulators use the Ding's well model without any change in the original computational implementation, just adding the corretion factor calculation. Note that in order to obtain the correction factor is just needed to find the expression for the equivalent discrete length $L_{eq,e}$.

Equivalent discrete length is obtained using Eq. (10) and aplying the analitycal solution on neighbor grid-blocks that shares face e. Therefore, the equivalent length is given by,

$$L_{eq,e} = \Delta y \frac{\ln\left(\frac{r_E}{r_P}\right)}{\Theta_e} \tag{13}$$

where r_E and r_P are the distances between the well location and the centers of grid-blocks E and P, respectively, determined by

$$r_i = \sqrt{(x_i - x_{well})^2 + (y_i - y_{well})^2} \tag{14}$$

Then the face angle is obtained with,

$$\Theta_e = \arctan\left(\frac{y_P + 0.5\Delta y - y_{well}}{x_P + 0.5\Delta x - x_{well}}\right) - \arctan\left(\frac{y_P - 0.5\Delta y - y_{well}}{x_P + 0.5\Delta x - x_{well}}\right) \tag{15}$$

where x_P and y_P are the well grid-block center coordinates.

Therefore, the correction factor can be calculated using Eqs. (12), (13) and (15). Now it is necessary to multiply original transmissibility. Finally, the well index must be introduced into the source term at well grid-block balance equation. It requires a small change to apply the well index, as can be seen in the next relationship

$$WI = \frac{2\pi h K}{\ln\left(\dfrac{r_{well}}{r_P}\right)} \tag{16}$$

where r_P is calculated by Eq. (14). When $r_P \leq r_{well}$ Peaceman's equivalent radius must be used in place of r_P. Note that when the well is located at the well grid-block center, Ding's well model reduces to Peaceman's well model by two reasons: the L_e^{eq} is equal Δx and r_p on WI is equal r_{eq}.

4. MODIFIED WELL MODELS

To modify the well models were inserted at well extremities the spherical radial flow, just adding a new flux on the well grid-block corresponding to well extremities.

4.1 Modified Peaceman's Well Model

In order to consider spherical flow at well extremities, it is necessary to obtain an analytical solution, corresponding to the spherical radial flow, that solution is given by

$$P(r) - P_{well} = \frac{Q\mu}{4\pi K}\left(\frac{1}{r_{well}} - \frac{1}{r}\right) \tag{17}$$

whereas, the corresponding spherical well index and equivalent radius are

$$WI = \frac{4\pi K}{\left(\frac{1}{r_{well}} - \frac{1}{r_{eq}}\right)} \quad \text{and} \quad r_{eq} = \frac{T_w + T_e + T_b + T_t + T_s}{\frac{T_w}{r_w} + \frac{T_e}{r_e} + \frac{T_b}{r_b} + \frac{T_t}{r_t} + \frac{T_s}{r_s} + 4\pi\lambda K} \tag{18}$$

where r_w, r_e, r_b, r_t and r_s are the distances between well grid-block center and the center of a neighbor grid-block.

4.2 Modified Ding's Well Model

In order to extend Ding's well model, adding the semi-spherical radial flow, it is necessary to work with solid angle to divide the well mass flow rate proportionally in the well grid-block faces, for representing the spherical radial flow. But this kind of flow must be considered only at well extremities, whereas the flow around the rest of the well remains radial cylindrical as considered before.

Following the same procedure of Ding's model and using the solid angle Ω in place of plane angle Θ, a equation for calculating the equivalent discrete length, can be found as

$$L_{eq,e} = \Delta y \Delta z \frac{\beta\left(\frac{1}{r_P} - \frac{1}{r_E}\right)}{\Omega_e} \tag{19}$$

where parameter β is defined by

$$\beta = \frac{\sum_{nb=1}^{5} \Omega_{nb}}{4\pi} \tag{20}$$

that parameter is necessary because one of the well grid-block faces is traversed by the well orthogonally, so the well flow rate is not divided with that face. That situation occurs because the flow rate considered in the extremities is hemispheric, thus the well flow is divided by others 5 faces. The solid angle Ω_e is obtained by vectors formed from well extremity and vertices of face. The solid angle can be calculated using the procedure shown by Oosterom and Strackee (1983).

5. RESULTS

To evaluate the accuracy of modified models three tests are considered, two refinement tests and one test to evaluate the spherical portion inserted on original well models. The refinement tests are separate for modified models one for each. The spherical portion test is only to compare the modified models. A numerical solution on a very refined unstrutured mesh is used as reference solution. That solution was obtained with the Element based Finite Volume Method (EbFVM), with a mesh of 6,603,809 elements ((Maliska *et al.*, 2010)). The EbFVM and applications in reservoir simulation can be found in Hurtado (2005) and Cordazzo (2006).

5.1 Base Problem

The base problem consists of a box reservoir domain with one production horizontal well and one injection vertical well, both wells with constant pressure considering infinite condutivity, i.e., without solving internal well flow. The simulation is a one-phase flow with null flux in the reservoir boundaries. The reservoir domain can be seen in Fig. (1). The dimensions of reservoir domain and the tests features will be indicated afterward in each test subsection.

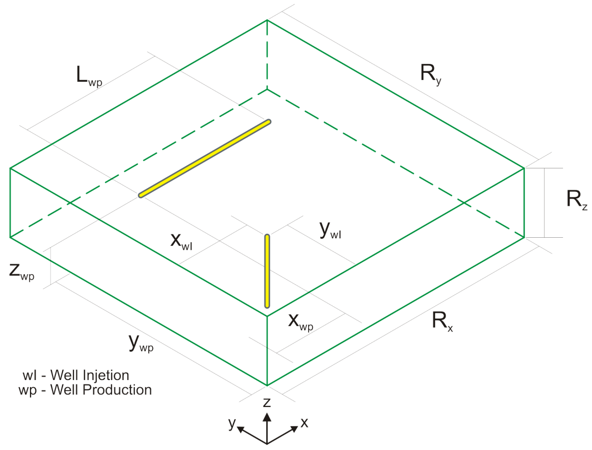

Figure 1. Reservoir Domain

The fluid properties, porous medium properties and operating conditions used in the tests can be found in Tab. 1. Those values were used in all tests.

Table 1. Tests Input Data.

Data		Value	Unit
Porous Medium	Porosity	0.227	-
	Permeability	8.0×10^{-13}	m^2
Fluid	Density	1000	kg/m^3
	Viscosity	1.43×10^{-3}	$Pa.s$
Operating Conditions	Well Radius	0.1	m
	Injection Pressure	1.0×10^7	Pa
	Production Pressure	1.0×10^3	Pa

5.2 Refinement Tests

The refinement tests were made one for each modified well model, the reservoir domain will be shown following.

5.2.1 Modified Peaceman's Model

The reservoir domain data can be seen in Tab. 2 and flow rate erros in Tab. 3. In all cases the reservoir dimensions are $R_x = 1000m$, $R_y = 1000m$ and $R_z = 60m$

Table 2. Refinement Test Well Position for Modified Peaceman's Model.

Well Injection	
X_{wi}	$83.334m$
Y_{wi}	$83.334m$
Well Production	
X_{wp}	$350m$
Y_{wp}	$783.334m$
Z_{wp}	$30m$
L_{wp}	$300m$

5.2.2 Modified Ding's Model

The reservoir domain data can be seen in Tab. 4 and flow rate erros in Tab. 5.

Table 3. Refinement Test Flow Rate Errors for Modified Peaceman's Model.

Grid	Flow Rate (m^3/s)	Error(%)
Reference Solution	1.2968×10^-2	-
30x30x3	1.2811×10^-2	1.21
90x90x9	1.2637×10^-2	2.56
270x270x27	1.2633×10^-2	2.59

Table 4. Refinement Test Well Position for Modified Ding's Model.

Well Injection	
X_{wi}	$38.75m$
Y_{wi}	$38.75m$
Well Production	
X_{wp}	$358.75m$
Y_{wp}	$798.75m$
Z_{wp}	$38.75m$
L_{wp}	$300m$

Table 5. Refinement Test Flow Rate Errors for Modified Ding's Model.

Grid	Flow Rate (m^3/s)	Error(%)
Reference Solution	1.1273×10^-2	-
50x50x3	1.1075×10^-2	1.76
100x100x6	1.1106×10^-2	1.48
200x200x12	1.1136×10^-2	1.22

5.3 Spherical Portion Test

The reservoir domain data can be seen in Tab. 6, flow rate erros in Tab. 7 and flow rate curve along of horizontal well in Fig. (2).

Table 6. Spherical Portion Test Well Position.

Well Injection	
X_{wi}	$55m$
Y_{wi}	$55m$
Well Production	
X_{wp}	$315m$
Y_{wp}	$595m$
Z_{wp}	$35m$
L_{wp}	$380m$

Table 7. Spherical Portion Test Flow Rate Errors.

Grid	Flow Rate (m^3/s)	Error(%)
Reference Solution	1.3240×10^-2	-
Peaceman's Model	1.3073×10^-2	1.26
Modified Peaceman's Model	1.3078×10^-2	1.23
Ding's Model	1.3042×10^-2	1.50
Modified Ding's Model	1.3053×10^-2	1.42

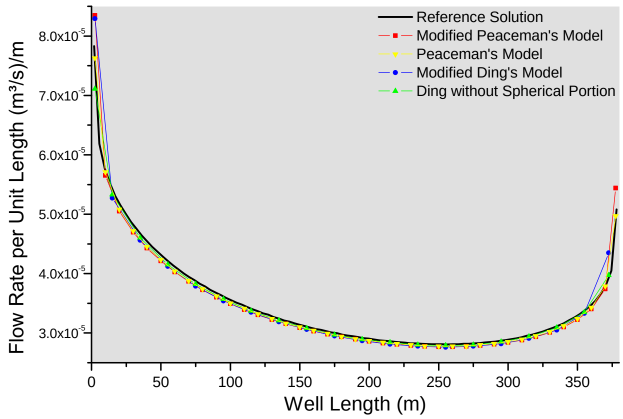

Figure 2. Horizontal Well Flow Rate Gradient - Spherical Portion Test

6. CONCLUSIONS

The modified models presented good results. It was expected because in real flow in horizontal well has the spherical portion on extremities. Note that the spherical radial flow adds a very small value on the total flow rate, because the semi-spherical area is much smaller than cylindrical area along the well. The computational implementation is relatively simple, it is not necessary a reimplementation just add the correction factors in the well index calculation. However is necessary to perform further new tests to better evaluate the incorporated changes.

There is a result on Tab. 3 which shows an increase in the total flow rate error with grid refinement. The origin of this problem was not determined and more tests must be made for a better analysis.

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