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TWO DIMENSIONAL OFF-CENTER WELL MODELING IN RESERVOIR SIMULATION

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Resumo

A diferença de escalas entre o reservatório, da ordem de quilômetros, e o diâmetro do poço, da ordem de centímetros, é um aspecto que deve ser tratado cuidadosamente na simulação de poços e reservatórios de petróleo. Com essas escalas, para captar os gradientes de pressão nas proximidades do poço, seria necessário discretizar suas fronteiras e empregar malhas suficientemente finas na direção radial. Este tipo de abordagem, entretanto, não é usual, já que exigiria elevada capacidade computacional. Desta forma, um modelo matemático analítico que permita determinar uma solução local, para acoplar as variáveis do escoamento no poço, como pressão e vazão, com as variáveis do reservatório, é necessário. Trata-se do conhecido modelo de poço, que, quando baseado em muitas hipóteses simplificativas, pode resultar em aproximações fracas e não realísticas para as variáveis do problema. O objetivo do presente trabalho é avaliar as formulações mais relevantes para poços verticais localizados arbitrariamente em um dado bloco da malha, isto é, poços *off-center*. A avaliação é feita empregando testes onde se comparam as vazões de produção/injeção, pressão no reservatório e pressão de poço. Três modelos selecionados foram implementados e testados, sendo possível identificar suas limitações, restrições e indicar a melhor formulação dentre eles. Neste trabalho, uma gama de conhecimentos fundamentais para a modelagem de poços generalizados foi abordada. O modelo de Y. Ding, entre os modelos avaliados, produziu excelentes resultados com o menor número de restrições, sendo possível utilizá-lo em malhas 2D não-uniformes e não-estruturadas para poços verticais *on-center* e *off-center*.

Abstract

The difference between scales, of order of kilometers for the reservoir, and of centimeters, for the well, requires a very carefully treatment in coupled wellbore-reservoir simulations. With these scales, in order to model the well and to capture the real pressure gradient in its vicinity, an extremely fine grid would be necessary. However, this kind of approach it is not usual, since it would demand high computational effort. Therefore, an analytical mathematical model for calculating a local solution is the usual approach to couple the well variables, as pressure and flow rate, with the reservoir variables. This approach is known as well model, and when formulated with too many restrictions results in poor and non-realistic approximations to the local problem. The present work evaluates some general well models applicable to vertical off-center wells, i.e., a vertical well with arbitrary position inside the gridblock. The evaluation is done comparing reservoir pressure, production/injection flow rates and wellbore pressure for several test problems. Three models were tested and it was possible to identify their limitations, restrictions and also to point the best formulation among the models and problems tested. In this work, some fundamental aspects to model non-conventional wells were presented. Yu Ding's model, among the evaluated models, produced excellent results with minor number of restrictions, being possible to use it in non-uniform and unstructured 2D grids for vertical on-center and off-center wells.

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1. Introduction

Peaceman's well model [3], the most used model by commercial software, is obtained considering that the trajectory of the well is straight and parallel to the main anisotropic axes and intercept the gridblock in its center. In this way, the applicability considering a real wellbore and reservoir applications is restricted. For example, in essentially two-dimensional cases, areal reservoir with vertical wells, the wells can be positioned on-center or off-center in a gridblock. However, in the on-center mathematical model, the well will always be in the gridblock center and, therefore, not respecting its real position. Several authors [1][4][5][6] studied two-dimensional off-center well modeling as illustrated in Figure 1.

This work evaluates the main two-dimensional off-center well models. The evaluation is done with tests comparing the production/injection flow rates, wellbore pressure and reservoir pressure with a reference solution. The models evaluated were developed by Williamson and Chappelear [6], Su [5], Ding and Renard [1] and the classical Peaceman's model [3], being possible to identify their limitations and restrictions. Ding and Renard's [1] model seems to be the best formulation with minor number of restrictions and presenting realistic solutions. In this work, some fundamental aspects to model non-conventional wells are presented.

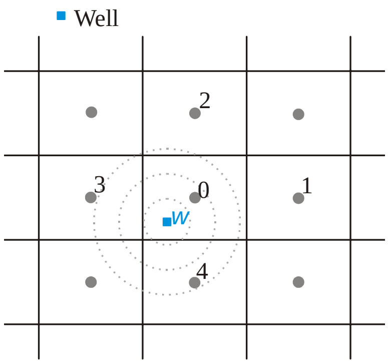


Figure 1. Numerical and areal representation of an off-center vertical well.

The first off-center well model evaluated was proposed by Williamson e Chappelear [6], based on the analytical solution for the one-phase areal flow in an isotropic porous media governed by Laplace's equation. This equation is used as a near well solution to each wellblock in order to model its source/sink term in the mass balance equation. This formulation presented good results for the flow rate when the well is near the wellblock center, reducing to the Peaceman's model [3] when the well is on-center. However, as the well moves away from the center, this technique presents results physically inconsistent. Besides this, since this model puts the entire flow rate to the wellblock and does not modify the transmissibility, no modification in the pressure field [2] is considered.

The second model evaluated, proposed by Su [5], is based on the same one-phase, one-dimensional, analytical near-well Peaceman's [3] solution. The difference is that the well flow rate is composed by portions from the wellblock neighbors. These portions are calculated based on the distances between the well and each gridblock center. Thus, differently from Williamson e Chappelear's [6] model, the pressure field solution takes into account the real well position. The methodology produced good solutions for wells positioned near the grid vertex, but poor results were obtained when the well is positioned near the gridblock center. Indeed, when the well is on-center the solution becomes singular, losing fidelity with the analytical solution and, obviously, not reducing to Peaceman's model [3]. In this way, this model is complementary to Williamson and Chappelear [6], with opposite behavior.

Finally, the last model evaluated, proposed by Ding and Renard [1], provided the best results. The idea is to modify the mass flux over a gridblock face according to well position in the wellblock. In this way, the pressure derivatives consider a logarithmic profile near the well, defining a coefficient to correct the original transmissibility. The transmissibility modification is done in each wellblock faces and can be extended to some wellblock neighbors, where the flow is radial. This model, among the ones that were evaluated, produced excellent results with minor restrictions, being possible to use it in non-uniform and unstructured grids with on-center or off-center wells. A description of the models now follows.

2. Williamson and Chappelear's Model

To present this methodology, the problem as illustrated in Figure 2 is proposed, where an off-center well is located in the gridblock number 0. The assumptions are: single phase flow, homogeneous and isotropic porous medium and steady-state flow. The purpose is to find an analytical expression to the flow near the well. This expression will connect the wellblock reservoir pressure, wellbore pressure through the well flow rate, which is the sink/source term in the gridblock reservoir mass equation. To do this, the authors present a well flow rate equation based on the analytical solution of Laplace's equation ($\nabla^2 p = 0$).

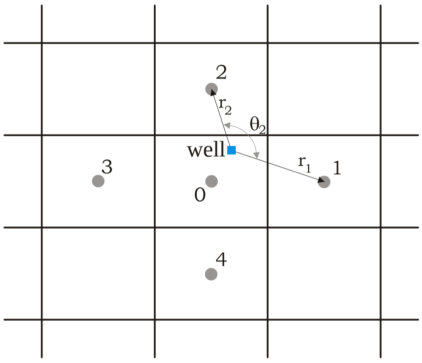


Figure 2. Off-center vertical well representation and neighbor gridblocks.

In cylindrical coordinates, the solution of this equation is

$$p(r,\theta) = a_1 \ln r + c_o + \sum_{n=1}^{\infty} r^n \left(b_n \sin n\theta + c_n \cos n\theta\right) + \sum_{n=1}^{\infty} r^{-n} \left(b_{-n} \sin n\theta + c_{-n} \cos n\theta\right) \tag{1}$$

where a_1 , b_n , b_{-n} , c_o , c_n e c_{-n} are the constants to be defined. Writing the pressure as the arithmetic mean at well surface, the well pressure can be written as

$$p(r_w) = \frac{1}{2\pi}\int_0^{2\pi} p(r_w,\theta)\,d\theta = p_w \tag{2}$$

Introducing Equation (2) in (1), results

$$p(r,\theta) - p_{well} = a_1 \ln\left(r / r_w \right) + \sum_{n=1}^{\infty} r^n \left(b_n \sin n\theta + c_n \cos n\theta\right) \tag{3}$$

As the numerical stencil considers only five points, as shown in Figure 2, and p_{well} is known, in order to determine the coefficients of Equation (3), it is necessary truncate the low frequency terms of this equation. In this way, it takes the following form

$$p(r,\theta) - p_{well} = a_1 \ln\left(\frac{r}{r_w} \right) + r\left(b_1 \sin\theta + c_1 \cos\theta\right) + r^2 c_2 \cos 2\theta \tag{4}$$

This equation is applied to the four wellblock neighbors and written in a matrix form as

$$\Delta\vec{p} = G\vec{b} \tag{5}$$

where $\Delta\vec{p} = \left\{ p_1 - p_w,\ p_2 - p_w,\ p_3 - p_w,\ p_4 - p_w \right\}$, $\vec{b} = \left\{ a_1,\ b_1,\ c_1,\ c_2 \right\}$,

$$G = \begin{bmatrix} \ln\left(r_1 / r_w \right) & 0 & r_1 & r_1^2 \\ \ln\left(r_2 / r_w \right) & r_2 \sin\theta_2 & r_2 \cos\theta_2 & r_2^2 \cos 2\theta_2 \\ \ln\left(r_3 / r_w \right) & r_3 \sin\theta_3 & r_3 \cos\theta_3 & r_3^2 \cos 2\theta_3 \\ \ln\left(r_4 / r_w \right) & r_4 \sin\theta_4 & r_4 \cos\theta_4 & r_4^2 \cos 2\theta_4 \end{bmatrix} \quad (6)$$

The solution for the vector $\vec{b}$ is given by inverting the matrix G,

$$\vec{b} = G^{-1}\Delta\vec{p} \quad (7)$$

where the G^{-1} elements are denoted by g_{ij}. The wellblock flow rate is given by

$$q_0 = -\frac{K_{eq} h r_w}{\mu} \int_0^{2\pi} \left.\frac{\partial p}{\partial r}\right|_{r=r_w} d\theta = -\frac{2\pi K_{eq} h}{\mu} a_1 \quad (8)$$

where h is the reservoir thickness, r_w is the wellbore radius and K_{eq} is the absolute equivalent permeability [2]. Replacing a_1 by b_1 given in Equation (7), one gets

$$q_0 = \frac{2\pi K_{eq} h}{\mu} \sum_{i=1}^{4} g_{1i}\left(P_i - p_w\right) \quad (9)$$

This model presented good results when the well is near the wellblock center, indeed reducing to Peaceman's well model [3] when it is on-center. However, this technique presents inconsistent results as the well moves away from its center. Since this model attributes the entire flow rate to the wellblock and does not modify the transmissibility, no modification in the pressure field [2] is considered.

3. Ho-Jeen Su's Model

Su's [5] well model uses as analytical solution the same solution used by Peaceman [3]. However, the well flow rate is distributed to the wellblock neighbors as illustrated in Figure 3. In this case, the total flow rate is composed by contributions from the wellblock neighbors, as illustrated. Each portion for the flow rate is calculated according to the distance wx and wy. In this model, differently from Williamson and Chappelear's model [6], the exact well position inside the wellblock is considered for the pressure solution field.

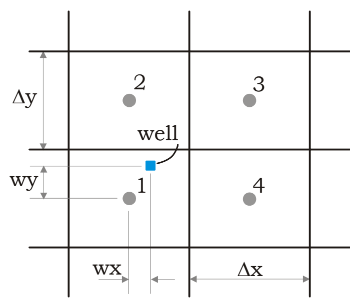


Figure 3. Off-center vertical well representation and wellblock neighbors.

The single phase, one-dimensional analytical solution of the radial flow used is

$$P_i = p_w + \frac{q\mu}{2\pi K_{eq}h}\ln\left(\frac{r_i}{r_w}\right) \tag{10}$$

Collecting the well flow rate in this equation,

$$q = \frac{2\pi K_{eq}h}{\mu \ln\left(\frac{r_i}{r_w}\right)}\left(P_i - p_w\right) \tag{11}$$

This flow rate is composed by the sum of the partial flow q_i of each “i” gridblock, that is

$$q = \sum_{i=1}^{4} q_i = \sum_{i=1}^{4} N_i q \quad \rightarrow \quad q_i = N_i q \tag{12}$$

where N_i are the shape functions,

$$\begin{cases} N_1 = \left(1 - x_D\right)\left(1 - y_D\right) \\ N_2 = \left(1 - x_D\right) y_D \\ N_3 = x_D y_D \\ N_4 = x_D\left(1 - y_D\right) \end{cases}, \qquad x_D = \frac{2\text{wx}}{\Delta x_1 + \Delta x_4}, \quad y_D = \frac{2\text{wy}}{\Delta y_1 + \Delta y_2} \tag{13}$$

Introducing equation (11) into (12), the partial flow to each gridblock is

$$q_i = N_i \frac{2\pi K_{eq}h}{\mu \ln\left(\frac{r_i}{r_w}\right)}\left(P_i - p_w\right) \tag{14}$$

This methodology produced good solutions for wells positioned near the grid vertex, but poor results were obtained when the well is positioned near the gridblock center. Indeed, when the well is on-center the solution becomes singular, losing fidelity with the analytical solution and, obviously, not reducing to Peaceman’s model [3]. In this way, this model is complementary to Williamson and Chappelear [6], with opposite behavior.

4. Ding and Renard’s Model

The last evaluated model was proposed by Ding and Renard [1], providing the best results. The author’s idea is to use the single phase, one-dimensional, radial flow used by Peaceman [3] as analytical solution and to modify the mass flux over a gridblock face according to well position inside the gridblock. In this way, the pressure derivatives consider a logarithmic profile near the well, and the pressure field solution considers the exact well position. To show their methodology, an off-center well positioned as illustrated in Figure 4 is considered.

The mass flux at an east face is written as a portion of the well flow rate as

$$f_1 = \frac{\theta_1}{2\pi} q = \frac{1}{\mu} T_{eq,1}\left(P_1 - P_0\right) \tag{15}$$

where $T_{eq,1}$ is the equivalent transmissibility to be defined in the east face.

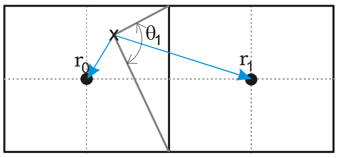


Figure 4. Transmissibility angle correction.

The single phase, one-dimensional radial flow equations, applied between the well and wellblock position and between the gridblock “1” and wellblock “0”, are

$$P_0 - p_{well} = \frac{q\mu}{2\pi Kh}\ln\left(\frac{r_0}{r_w}\right) \tag{16}$$

$$P_1 - P_0 = \frac{q\mu}{2\pi Kh}\ln\left(\frac{r_1}{r_0}\right) \tag{17}$$

where p_{well} and r_w are, respectively, the well pressure and the well radius.

The sink/source term to the reservoir mass equation is modeled by equation (16). Following, the mathematical development is used to define the equivalent transmissibility.

Introducing equation (17) into equation (15), results

$$\frac{\theta_1}{2\pi}q = T_{eq,1}\frac{q}{2\pi Kh}\ln\left(\frac{r_1}{r_0}\right) \tag{18}$$

Collecting $T_{eq,1}$, the equivalent transmissibility is defined as

$$T_{eq,1} = Kh\frac{\Delta y_0}{L_{eq,1}} \tag{19}$$

where $L_{eq,1}$ is defined as the equivalent distance between the gridblocks, given by

$$L_{eq,1} = \Delta y_0 \frac{\ln\left(\frac{r_1}{r_0}\right)}{\theta_1} \tag{20}$$

Expressions (19) and (16) are the basis of the Ding and Renard’s [1] well model. The first expression models the sink/source term of the mass balance equation, and the second calculates the exact wellblock face transmissibility that considers the exact well position. It is convenient to define the coefficient α_i for each gridblock face “i” that multiplies the standard transmissibilities, providing the exact transmissibility for each face, as in Equation (19). This coefficient, for an east face is

$$\alpha_1 = \frac{T_{eq,1}}{T_1} = \frac{\Delta x_{0-1}}{\Delta y_0} \frac{\theta_1}{\ln\left(r_1 / r_0 \right)} \tag{21}$$

This model, among the ones evaluated, produced excellent results with minor restrictions, being possible to use it in non-uniform and unstructured grids with on-center or off-center wells. In additional, when the well is on-center, the methodology reduces to Peaceman's model [3], whereas the radius r_0 in Equation (16) is the Peaceman's equivalent radius [3].

5. Results

In this section, the results of the methodology which presented the best results will be presented, that is the results of Ding and Renard's well model [1]. Two test problems are proposed. The first with one on-center well in an infinite medium and the second, more general, with three off-center wells, one injector and two producers, in a square medium with no-flux boundaries. The wells are close to the boundary, providing a small radial flow area.

5.1. On-center well – Infinite medium

This hypothetical test is made up of a square domain with 15 x 15 x 1 meters, isotropic and homogeneous medium with one on-center injector well (Figure 5). The problem is single phase, incompressible, with pressure prescribed at the boundaries arising from the analytical solution of the same problem in an infinite medium. The problem is numerically solved using three different well models, in a grid composed by 15 x 15 x 1 gridblocks. The first solution is obtained applying Peaceman's model [3], the second is using Ding and Renard's model [1] correcting the all domain's transmissibilities. The last evaluation is, again, using Ding and Renard's model [1], but applying the correction only in the wellblock faces.

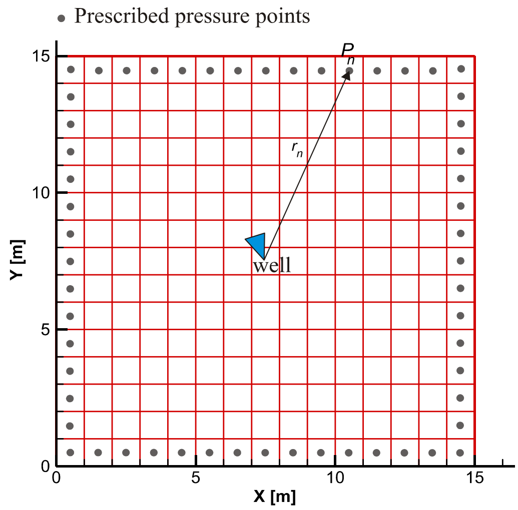
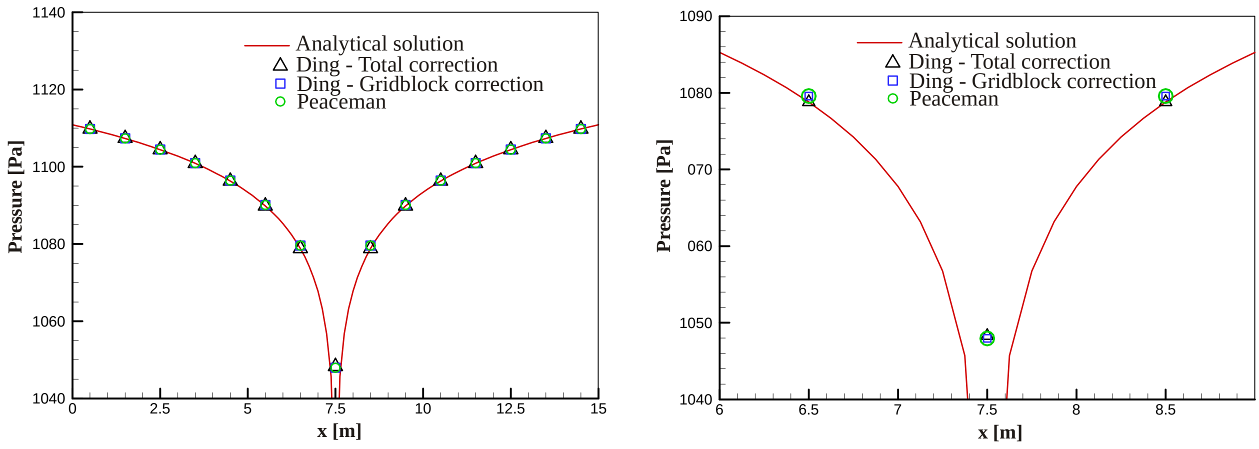


Figure 5. Problem domain with one on-center well.

The solution is presented in Figure 6 and Table 1, where is shown that, when Ding and Renard's model [1] is applied only in the wellblock, it reproduces Peaceman's model [3], and that this model reproduces the analytical solution when applying the correction in the whole domain.



(a) (b)

Figure 6. On-center problem solution: (a) pressure along x coordinate; (b) zoom near well position.

Table 1. On-center problem solution.

Solution	***Injected volume [m3]***	***Error [%]***
Analytical	1.000E-10	
Peaceman	9.932E-11	6.837E-01
Ding wellblock	9.932E-11	6.837E-01
Ding total	1.000E-10	1.000E-03

5.2. Off-center well – No-flow Boundaries

In this problem, the domain and properties are the same of the previous problem. The boundary conditions are no mass flux and pressure prescribed in each well. The off-center wells are positioned as shown in Figure 7 and Table 2.

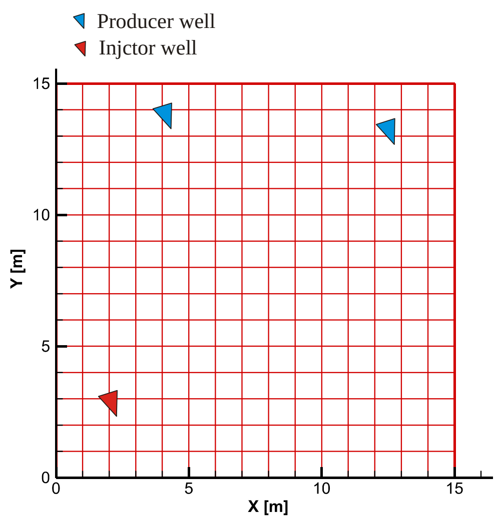


Figure 7. Test problem domain with three off-center wells.

Table 2. Off-center wells position.

Well	***Type***	***Position***
1	Injector	x = 2.25; y=2.25
2	Producer	x = 4.25; y=13.25
3	Producer	x = 12.75; y=12.75

The reference solution chosen is a numerical solution with a refined grid (30 x 30 x 1), where the wells lie on the center of the gridblocks. Therefore, the on-center model used in the reference solution is Peaceman's model [3] that, as shown previously, is a good approximation for on-center wells. The solution in the coarse grid uses Ding and Renard's model [1] with correction in the wellblocks only. The flow rate obtained is shown in Table 3.

Table 3. Off-center wells problem solution.

Solution	***Injected/produced volume [m3]***			***Error [%]***		
	Well 1	Well 2	Well 3	Well 1	Well 2	Well 3
Reference	4.419E-08	1.9865E-08	2.433E-08			
Ding wellblock	4.431E-08	1.9983E-08	2.432E-08	0.260	0.592	0.012

It is important to notice that in this particular case (single phase, incompressible, homogeneous and isotropic medium) the grid refinement has no role in the solution obtained. This was extensively tested during this work. Therefore, the error obtained and shown in Table 3 is only due to the approximations of the off-center well model.

The pressure solution is shown along the diagonal, $x = y$, in Figure 8, where it is noticeable the excellent results obtained.

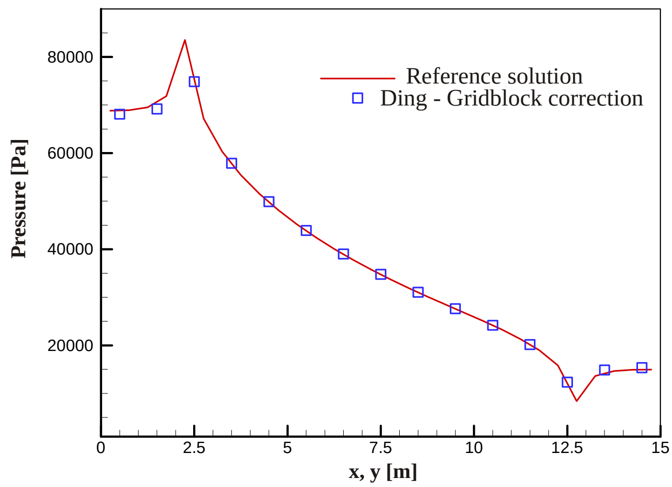


Figure 8. Pressure along the domain diagonal with three off-center wells.

6. Conclusion

This work evaluated the main methodologies in modeling two-dimensional off-center wells, presenting some fundamental aspects of modeling non-conventional wells. Williamson and Chappelear [6], Su [5], Ding and Renard [1] and the classical Peaceman's model [3] were used. The best results were obtained with Ding and Renard's [1] well model. Their methodology has minor number of restrictions and the pressure field considers the exact well position, being possible to use it in non-uniform and unstructured 2D grids with vertical on-center and off-center wells.

7. References

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