V ENCONTRO NACIONAL DE CIÊNCIAS TÉRMICAS - São Paulo,SP - BRASIL (DEZ. 1994)
II SIMPÓSIO DE ENGENHARIA TÉRMICA - SP - BRASIL (DEZ. 1994)

TRIDIMENSIONAL PETROLEUM RESERVOIR SIMULATION USING GENERALIZED CURVILINEAR GRIDS

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SUMMARY

The present work describes a finite-volume method for the simulation of three-dimensional petroleum reservoirs using nonorthogonal curvilinear grids. The possibility of taking into account geological faults is also included through the especification of fault coefficients in the input data of the problem. The physical model uses the black-oil approximation considering the oil and water phases. Together with the development of the numerical model, considerable effort was spent to create a computacional framework, with pre and post processing facilities, which may allow the introduction of new physical models with relative ease.

INTRODUCTION

The development of three-dimensional methods for simulating petroleum reservoirs is a very difficult task due to the inherent complexity of designing 3D numerical algorithms, as well as because the irregular shape of the reservoir and the need of taking into account the existence of geological faults. The use of curvilinear coordinate systems has becoming an acttractive tool for analysing this type of problems (Sharpe and Anderson, 1991)(Ferguson and Wadsley, 1986).

There are many challenging aspects that need to be carefully addressed during the development of a three-dimensional petroleum reservoir simulator. Among them one can name; a) the need for a strong and versatile grid generator; b) the organization of the imput data with key words for checking the quality of the furnished data; c) the treatment of the geological faults; d) the development of the numerical model itself, that is, to obtain the approximate equations and to solve them, and e) the preparation and full visualization of the output data. All the above aspects have been considered, but in this paper only items c) and d) will be discussed.

GOVERNING EQUATIONS

The black-oil model considering the water and oil phases is used. Since capillarity effects were not included the unknowns are the water and oil saturations and pressure. The governing equations are, therefore,

$$\nabla \cdot [\nabla \lambda_w P] = \frac{\partial}{\partial t}\left[\phi \frac{S_w}{B_w}\right] + q_w \qquad (1)$$

$$\nabla \cdot [\nabla \lambda_o P] = \frac{\partial}{\partial t}\left[\phi \frac{S_o}{B_o}\right] + q_o \qquad (2)$$

$$S_o + S_w = 1 \qquad (3)$$

where P, S, B and q are pressure, saturation, volume formation factor and a source term through which it is specified the flow rate at the injection and production wells. If pressure is specified at the wells these flow rates need to be replaced, in terms of pressure, through a well model. Since one is using the black-oil model with no gas phase, it is immaterial to specifiy the component and the phase, since there is only oil in the oil phase and only water in the water phase.

Eq. (1) and (2), written for a general phase α in the cartesian coordiante system is

$$\frac{\partial}{\partial x}\left(\lambda_x^\alpha \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y^\alpha \frac{\partial P}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z^\alpha \frac{\partial P}{\partial z}\right) = \frac{\partial}{\partial t}\left[\phi \frac{S_\alpha}{B_\alpha}\right] + q_\alpha \qquad (4)$$

Since one is interested in the development of a 3D numerical model in curvilinear coordinates, Eq. (4) needs to be solved in this new coordinate system. There are two alternatives to reach this goal. The first one is to integrate the governing equations, written in the cartesian coordinate system, in the irregular domain, as shown in Fig. 1(a). The other one is to transform the governing equation to the new system and integrate it in the regular transformed domain, as shown in Fig. 1(b). Of course, for unstructured grids only the first alternative is possible. Since one is using structured curvilinear grids, the second alternative is easier and gives rise to transformed equations which keeps the physical information clearly in its terms.

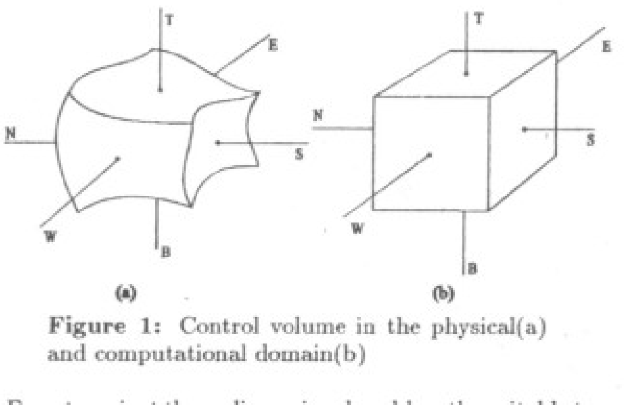

Figure 1: Control volume in the physical(a) and computational domain(b)

For a transient three-dimensional problem the suitable transformation from the cartesian coordinate system to the generalized curvilinear coordinate system, is

$$\xi = \xi(x,y,z,t) \tag{5}$$

$$\eta = \eta(x,y,z,t) \tag{6}$$

$$\gamma = \gamma(x,y,z,t) \tag{7}$$

$$\tau = t \tag{8}$$

Using the inverse function theorem one can find the metrics of the transformation and of its inverse. The expressions of the metrics can be found in Maliska et al(1993), and the Jacobian of the transformation is given by

$$J = \frac{1}{x_\xi y_\eta z_\gamma + x_\eta y_\gamma z_\xi + x_\gamma y_\xi z_\eta - x_\xi y_\gamma z_\eta - x_\eta y_\xi z_\gamma - x_\gamma y_\eta z_\xi} \tag{9}$$

The transformation employed allows for any form of the reservoir, including variable top and bottom, as can be seen in Fig. 2.

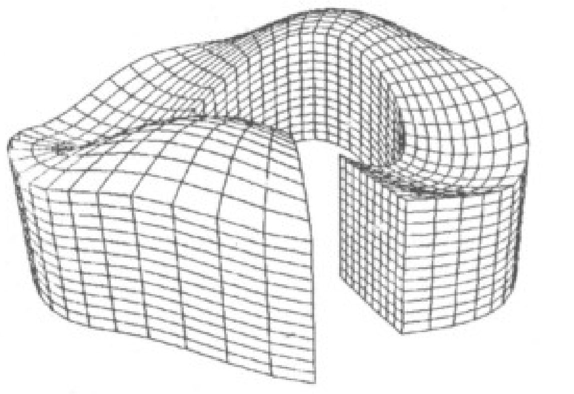

Figure 2: 3D arbitrary reservoir

Using the chain rule, and after some algebraic manipulation, one can get Eq. (4), transformed to the new system, as

$$\begin{aligned}\frac{1}{J}\frac{\partial}{\partial t}\left[\phi\frac{S_\alpha}{B_\alpha}\right] + \frac{q_\alpha}{J} = & \frac{\partial}{\partial \xi}\left[D_{1\alpha}\frac{\partial P}{\partial \xi} + D_{2\alpha}\frac{\partial P}{\partial \eta} + D_{3\alpha}\frac{\partial P}{\partial \gamma}\right] \\ & + \frac{\partial}{\partial \eta}\left[D_{4\alpha}\frac{\partial P}{\partial \xi} + D_{5\alpha}\frac{\partial P}{\partial \eta} + D_{6\alpha}\frac{\partial P}{\partial \gamma}\right] \\ & + \frac{\partial}{\partial \gamma}\left[D_{7\alpha}\frac{\partial P}{\partial \xi} + D_{8\alpha}\frac{\partial P}{\partial \eta} + D_{9\alpha}\frac{\partial P}{\partial \gamma}\right]\end{aligned} \tag{10}$$

The coefficients appearing in Eq. (10) can be found in Maliska et al(1993). According to the finite-volume procedure, Eq. (10) needs to be integrated in time and over the elemental control volume shown in Fig. 1. The integration gives

$$\begin{aligned}\frac{\Delta V}{J\Delta t}\left[\left(\phi\frac{S_\alpha}{B_\alpha}\right) - \left(\phi\frac{S_\alpha}{B_\alpha}\right)^0\right] + \frac{q_\alpha \Delta V}{J} = & \\ \left[D_{1\alpha}\frac{\partial P}{\partial \xi} + D_{2\alpha}\frac{\partial P}{\partial \eta} + D_{3\alpha}\frac{\partial P}{\partial \gamma}\right]_e & \Delta\eta\Delta\gamma \\ - \left[D_{1\alpha}\frac{\partial P}{\partial \xi} + D_{2\alpha}\frac{\partial P}{\partial \eta} + D_{3\alpha}\frac{\partial P}{\partial \gamma}\right]_w & \Delta\eta\Delta\gamma \\ + \left[D_{4\alpha}\frac{\partial P}{\partial \xi} + D_{5\alpha}\frac{\partial P}{\partial \eta} + D_{6\alpha}\frac{\partial P}{\partial \gamma}\right]_n & \Delta\xi\Delta\gamma \\ - \left[D_{4\alpha}\frac{\partial P}{\partial \xi} + D_{5\alpha}\frac{\partial P}{\partial \eta} + D_{6\alpha}\frac{\partial P}{\partial \gamma}\right]_s & \Delta\xi\Delta\gamma \\ + \left[D_{7\alpha}\frac{\partial P}{\partial \xi} + D_{8\alpha}\frac{\partial P}{\partial \eta} + D_{9\alpha}\frac{\partial P}{\partial \gamma}\right]_t & \Delta\xi\Delta\eta \\ - \left[D_{7\alpha}\frac{\partial P}{\partial \xi} + D_{8\alpha}\frac{\partial P}{\partial \eta} + D_{9\alpha}\frac{\partial P}{\partial \gamma}\right]_b & \Delta\xi\Delta\eta\end{aligned} \tag{11}$$

Inspecting Eq. (11) one realizes that it is necessary to evaluate six direct derivatives and twelve cross-derivatives at the control volume interfaces. It is not difficult to foresee that it is a very complicated procedure, specially for boundary control volumes. To have different evaluation according to the boundary conditions results in a very cumbersome calculation routine. To avoid this, both boundaries and geological faults are treated in a similar way, that is, a general equation is developed, where the existence of geological faults and the type of the boundary condition is prescribed through coefficients as data input of the problem. This topic is now addressed.

DEVELOPMENT OF A GENERAL EQUATION. To explain the calculation of direct and cross derivatives consider evaluating these derivatives at the east face of the control volume shown in Fig. 3. They are

$$\left.\frac{\partial P}{\partial \xi}\right|_e = FD\xi_P\frac{P_E - P_P}{\Delta\xi} \tag{12}$$

$$\begin{aligned}\left.\frac{\partial P}{\partial \eta}\right|_e = & \frac{CF\xi\eta^P_{SE}P_{SE} + CF\xi\eta^P_E P_E + CF\xi\eta^P_{NE}P_{NE}}{\Delta\eta} + \\ & \frac{CF\xi\eta^P_S P_S + CF\xi\eta^P_P P_P + CF\xi\eta^P_N P_N}{\Delta\eta}\end{aligned} \tag{13}$$

$$\begin{aligned}\left.\frac{\partial P}{\partial \gamma}\right|_e = & \frac{CF\xi\gamma^P_{BE}P_{BE} + CF\xi\gamma^P_E P_E + CF\xi\gamma^P_{TE}P_{TE}}{\Delta\eta} + \\ & \frac{CF\xi\gamma^P_B P_B + CF\xi\gamma^P_P P_P + CF\xi\gamma^P_T P_T}{\Delta\xi}\end{aligned} \tag{14}$$

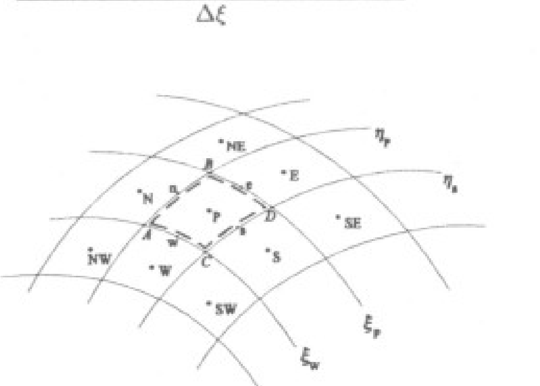

Figure 3: Example of a fault location

To demonstrate the way the fault coefficients are introduced, consider Fig. 3 where a two-dimensional situation is used, for simplicity. If a fault passes through the segment $\overline{BD}$, which is along a constant ξ line, the coefficient $f\xi_P$ is equal to 1. Knowing that the relation between $FD\xi_P$ and $f\xi_P$ is given by

$$FD\xi_P = 1 - f\xi_p \tag{15}$$

one sees that the direct derivative given by Eq. (12) results zero, as it should be, since there is no flow through $\overline{BD}$ if a fault passes through this segment. For faults along $\overline{AB}$, $\overline{AC}$ and $\overline{CD}$, the coefficients are, by analogy, respectively, $f\eta_P = 1$, $f\xi_W = 1$ and $f\eta_S = 1$, with the corresponding FD coefficients given by equations similar to Eq. (15).

Let us now consider the evaluation of a cross-derivative at the east face, given by

$$\left.\frac{\partial P}{\partial \eta}\right|_e = 0.5\left.\frac{\partial P}{\partial \eta}\right|_{Le} 0.5\left.\frac{\partial P}{\partial \eta}\right|_{Re} \tag{16}$$

where

$$\left.\frac{\partial P}{\partial \eta}\right|_{Le} = \frac{P_N - P_S}{2\Delta\eta} \tag{17}$$

$$\left.\frac{\partial P}{\partial \eta}\right|_{Re} = \frac{P_{NE} - P_{SE}}{2\Delta\eta} \tag{18}$$

where the simbol Le and Re means to the left and to the right of e. Lets imagine that a fault exists at the segment $\overline{CD}$, that is in the south face of the control volume. For this case, one has

$$\left.\frac{\partial P}{\partial \eta}\right|_{Le} = \frac{P_N - P_P}{\Delta\eta} \tag{19}$$

If a fault is present also in the segment $\overline{AB}$, then the above derivative is equal to zero.

The final general equation will be function of the f coefficients, already described, with the following form

$$\begin{aligned}\left.\frac{\partial P}{\partial \eta}\right|_{Le} = {} & \frac{(1-f\eta_S)(1-f\eta_P)[P_N - P_S]}{2\Delta\eta} + \\ & \frac{f\eta_S(1-f\eta_P)[P_N - P_P]}{\Delta\eta} + \\ & \frac{f\eta_P(1-f\eta_S)[P_P - P_S]}{\Delta\eta}\end{aligned} \tag{20}$$

Eq. (20) can be written in a more elegant form putting together the pressures with the same index. The resulting equations is

$$\left.\frac{\partial P}{\partial \eta}\right|_{Le} = \frac{(2CF\eta_N P_N + 2CF\eta_S P_S + 2CF\eta_P P_P)}{\Delta\eta} \tag{21}$$

The same procedure can be done for the derivate at Re. Using Eq. (16) the general equation, Eq. (13), is obtained. The equation obtained is valid for any control volume independently if the volume is located at the boundaries or close to geological faults, inside the reservoir. In other words boundaries are treated as geological faults. The final equation for pressure is

$$\begin{aligned}\frac{1}{J\Delta t}\left[\left(\phi\frac{S_{\alpha P}}{B_{\alpha P}}\right) - \left(\phi\frac{S_{\alpha P}}{B_{\alpha P}}\right)^0\right] + \frac{q_{\alpha P}}{J} = {} & -A_P P_P + A_E P_E \\ & + A_W P_W + A_N P_N + A_S P_S + A_T P_T + A_B P_B \\ & + A_{SE} P_{SE} + A_{SW} P_{SW} + A_{NE} P_{NE} + A_{NW} P_{NW} \\ & + A_{BE} P_{BE} + A_{BW} P_{BW} + A_{TE} P_{TE} + A_{TW} P_{TW} \\ & + A_{TS} P_{TS} + A_{TN} P_{TN} + A_{BS} P_{BS} + A_{BN} P_{BN}\end{aligned} \tag{22}$$

Eq. (22) can be written in a more compact form, as

$$\frac{\Delta V}{J\Delta t}\left[\left(\phi\frac{S_{\alpha P}}{B_{\alpha P}}\right) - \left(\phi\frac{S_{\alpha P}}{B_{\alpha P}}\right)^0\right] + \frac{q_{\alpha P}\Delta V}{J} = -A_P^\alpha P_P + \sum A_{NB}^\alpha P_{NB} \tag{23}$$

NUMERICAL SCHEME

As already stated in the begining of the paper the major preocupation of this work was to create a full framework for developing numerical methods using 3D boundary-fitted grids. To move from an explicit scheme to an implicit one is a minor change compared with the complex structure required by a 3D petroleum simulation method.

In the present work the two-phase black-oil model is solved using the IMPES method. Capillarity effects are not considered, what amounts in solving the equations for pressure and water and oil saturations. To accomplish this, Eq. (22), written for the water and oil saturations, are added, according to Eq. (3), to obtain the implicit equation for pressure. Having pressure calculated the saturations can be found explicitly.

TESTING PROCEDURES

To test a 3D numerical simulation method is a difficult task because there is no bench mark solutions, available in the literature, which considers simple problems. In general the 3D calculations involve real reservoirs with real conditions, by-passing many steps necessary in checking a computational model. In this paper one has followed some step-by-step solutions in order to assess the correctness of the model implementation.

To accomplish this the two-phase flow in a cube, with only one injection well located at the cube center, is solved. The flow is allowed to leave the domain through the boundaries. At this point it is worthwhile to mention that the model was constructed such that any vertical or horizontal well can have many different completations. Fig. 4 and Fig. 5 show the iso-pressure and iso-saturation lines after 0.3 porous volume injected. The idea of solving this problem was to check the symmetry of the solution, an important aspect that can only be assessed with simple problems.

The second test problem involves a more general reservoir. In this case the two-dimensional solution must be recovered, if the 3D simulation is realized considering zero derivatives as boundary conditions along the depth of the reservoir.

Fig. 6 shows the water iso-saturation lines after 0.263 porous volume injected for the 3D and 2D calculations. The results are exactly the same. Using the same geometry as in the second

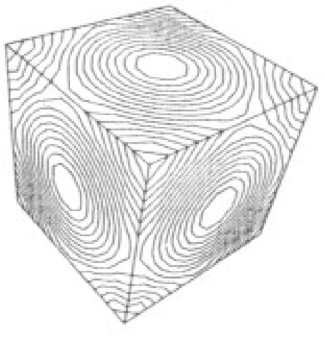

Figure 4: Lines of constant pressure after 0.3 PVI

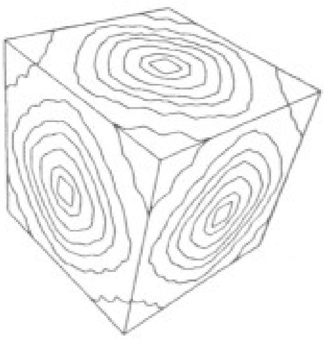

Figure 5: Lines of constant water saturation after 0.3 PVI

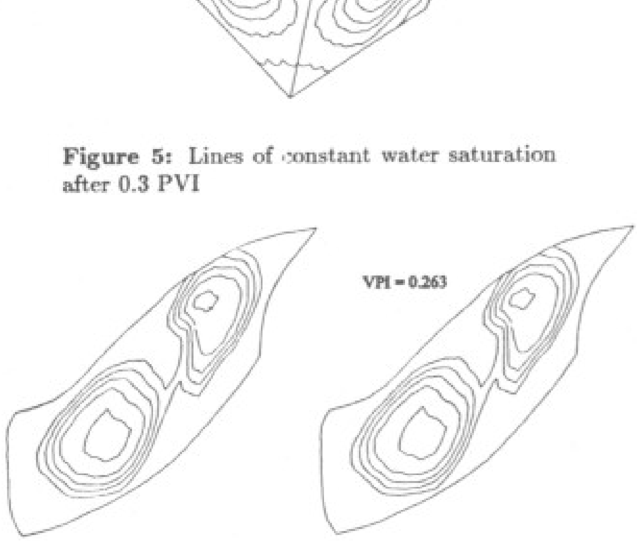

Figure 6: Results for the 3D and 2D calculations - irregular reservoir

test problem, the two-phase flow is now solved in a fully three-dimensional situation. Six layers are used with one injection well completed in two middle layers andother along all layers. Fig. 7 shows the velocity vector for the surface of the reservoir and Fig. 8 depicts the iso-saturation lines. Many other 3D situations were solved considering blocks of geological faults inside the reservoir. These results are reported in Maliska et al(1993).

CONCLUSIONS

This work presented a 3D numerical model for simulating two-phase flow in petroleum reservoirs. The model uses boundary-fitted grids creating a single equation applied for any control volume, which simplifies enormously the procedure for creating the system of algebraic equations. The computational model is prepared to accomodate explicit or implicit schemes,

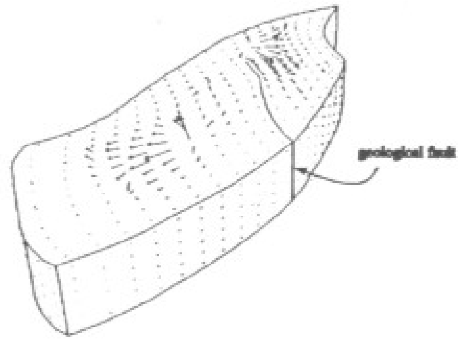

Figure 7: Velocity vectors for the 3D reservoir

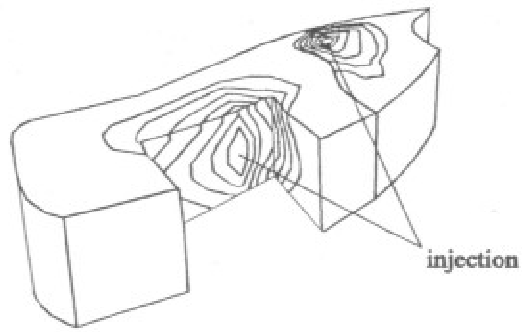

Figure 8: Iso-saturation lines for the 3D reservoir

having the possibility of specifying vertical, horizontal and inclined wells, with different completations. Faults can be represented through fault coefficients given to the model as input data.

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ACKNOWLEDGEMENTS

The authors are grateful to CENPES/PETROBRÁS for supporting the research work in petroleum reservoir simulation at the Computational Fluid Dynamics Laboratory/SINMEC.