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The PRIME Method for All Speed Flows Using Non-Staggered Grids

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SUMMARY

An extension of the PRIME method for co-located grids and all-speed flows using co-located grids is reported in this work. The calculation of the velocities at the interfaces are of key importance in this method because the momentum equations are used as velocity correction equations. The findings seem to be useful for other explicit methods and contributes to a better understanding of the role played by the interface velocities in co-located methods. The full methodology, in nonorthogonal coordinates, is described and results for incompressible and compressible flows are reported.

INTRODUCTION

After the publication of the papers by Rhie and Chow (1983), and Perić et al. (1988), many interesting works have been developed about the use of non-staggered grids in finite volume methods for the solution of the governing equations of fluid flows. The SIMPLE-like procedures are normally employed, and the works of Majumdar (1988) and of Choi, Nan and Cho (1993), among others, can be cited. Little attention, however, has been paid to the sequential procedures like PRIME (Maliska 1981) and SUMMIT (Van Doormaal 1985). The latter methods, whatever the variables arrangement employed, seemed to be less efficcient, because they propagate information of momentum transport in a point by point way, througout an explicit velocities calculation.

Albeit this, methods explicit in its nature, like the Runge-Kutta for solving compressible flows presented by Jameson et al. (1981) and the MAPLE (Marek and Straub 1993), have appeared to be attractive due to its effectiveness in exploiting multi-grid techniques and the vectorization resources of modern computers. Therefore, attention is now being given to explicit methods in general.

In the present paper an experience with PRIME, using non-staggered grids is reported. The methodology uses nonorthogonal grids and an all-speed flow algorithm. Interesting findings were observed due to the "explicit" treatment of the momentum equations. A new method is advanced, significantly different from the ones used in SIMPLE-like procedures, for the calculation of the velocities at cell centers and at cell faces.

The results obtained with this methodology in two test cases are compared with other available results.

GOVERNING EQUATIONS

Since one is interested in solving the governing equations in irregular shapes, the appropriate governing equations (Silva and Maliska 1988) are transformed to a general curvilinear coordinate system. Employing the chain rule and after some algebraic manipulations, in order to put the equation in the conservative form (Maliska 1995), they reads

$$\frac{1}{J}\frac{\partial}{\partial t}(\rho\phi)+\frac{1}{y^\kappa}\frac{\partial}{\partial \xi}(y^\kappa \rho U\phi)+\frac{1}{y^\kappa}\frac{\partial}{\partial \eta}(y^\kappa \rho V\phi)= +\frac{\Gamma^\phi}{y^\kappa}\frac{\partial}{\partial \xi}\left(y^\kappa \alpha J\frac{\partial \phi}{\partial \xi}-y^\kappa \beta J\frac{\partial \phi}{\partial \eta}\right) +\frac{\Gamma^\phi}{y^\kappa}\frac{\partial}{\partial \eta}\left(y^\kappa \gamma J\frac{\partial \phi}{\partial \eta}-y^\kappa \beta J\frac{\partial \phi}{\partial \xi}\right)-\hat{P}^\phi+\hat{S}^\phi \quad (1)$$

where,

$$U = y_\eta u - x_\eta v \quad (2)$$

$$V = x_\xi v - y_\xi u \quad (3)$$

$$\alpha = x_\eta^2 + y_\eta^2 \quad (4)$$

$$\gamma = x_\xi^2 + y_\xi^2 \quad (5)$$

$$\beta = x_\xi x_\eta + y_\xi y_\eta \quad (6)$$

$$J = 1/(x_\xi y_\eta - x_\eta y_\xi) \quad (7)$$

The expressions for $\hat{P}^\phi$ and $\hat{S}^\phi$ are presented in Table 1. As can be seen, the cartesian velocity components are kept as dependent variables (Maliska and Raithby 1984).

Table 1: Expressions for $\hat{P}^\phi$ and $\hat{S}^\phi$.

ϕ	$\hat{P}^\phi$	$\hat{S}^\phi$
1	0	0
u	$\frac{\partial p}{\partial \xi}y_\eta - \frac{\partial p}{\partial \eta}y_\xi$	$\frac{\mu}{3}\left[y_\eta \frac{\partial}{\partial \xi}(\nabla\cdot\vec{V}) - y_\xi \frac{\partial}{\partial \eta}(\nabla\cdot\vec{V})\right]$
v	$\frac{\partial p}{\partial \eta}x_\xi - \frac{\partial p}{\partial \xi}x_\eta$	$\frac{\mu}{3}\left[x_\xi \frac{\partial}{\partial \eta}(\nabla\cdot\vec{V}) - x_\eta \frac{\partial}{\partial \xi}(\nabla\cdot\vec{V})\right] - \frac{\kappa\mu v}{Jy^2}$
T	$-\frac{1}{Jc_p}\left[\frac{\partial p}{\partial t} + \nabla\cdot(p\vec{V}) - p(\nabla\cdot\vec{V})\right]$	$\frac{\mu}{c_p}\left\{2J\left[\left(y_\eta\frac{\partial u}{\partial \xi}-y_\xi\frac{\partial u}{\partial \eta}\right)^2 + \left(x_\xi\frac{\partial v}{\partial \eta}-x_\eta\frac{\partial v}{\partial \xi}\right)^2\right] + \frac{2\kappa}{J}\left(\frac{v}{y}\right)^2 + J\left[\frac{\partial}{\partial \xi}(vy_\eta - ux_\eta) + \frac{\partial}{\partial \eta}(ux_\xi - vy_\xi)\right]^2 - \frac{2}{J3}(\nabla\cdot\vec{V})^2\right\}$

DISCRETIZED EQUATIONS

Eq. (1) is integrated over a square control volume in the transformed space. Following the usual procedure in finite volume methods for incompressible flows (Patankar 1980) (Silva 1991), one obtains, after the integration,

$$\frac{M_p\phi_p - M_p^0\phi_p^0}{\Delta t} + \dot{M}_e\phi_e - \dot{M}_w\phi_w + \dot{M}_n\phi_n - \dot{M}_s\phi_s = +\left[D_{de}^\phi\frac{\partial\phi}{\partial\xi}+D_{ce}^\phi\frac{\partial\phi}{\partial\eta}\right]_e - \left[D_{dw}^\phi\frac{\partial\phi}{\partial\xi}+D_{cw}^\phi\frac{\partial\phi}{\partial\eta}\right]_w + \left[D_{dn}^\phi\frac{\partial\phi}{\partial\eta}+D_{cn}^\phi\frac{\partial\phi}{\partial\xi}\right]_n - \left[D_{ds}^\phi\frac{\partial\phi}{\partial\eta}+D_{cs}^\phi\frac{\partial\phi}{\partial\xi}\right]_s - L\left[\hat{P}^\phi\right]_p\Delta V + L\left[\hat{S}^\phi\right]_p\Delta V \quad (8)$$

where some of the coefficients appearing in the above equation are

$$M_e = \frac{y^\kappa \rho \Delta\xi \Delta\eta}{J_e \Delta t}$$

$$\Delta V = y^\kappa \Delta\xi \Delta\eta$$

$$\dot{M}_e = (y^\kappa \rho U)_e \Delta\eta$$

$$D^\phi_{de} = \Gamma^\phi (y^\kappa \alpha J)_e \Delta\eta$$

$$D^\phi_{ce} = \Gamma^\phi (y^\kappa \beta J)_e \Delta\eta$$

The remaining coefficients can be derived by analogy. The $L[\ ]$ denotes numerical approximation of the expression inside the brackets. Due to the presence of cross-derivatives in the diffusion term the WUDS scheme (Raithby and Torrance 1974) is applied only to the direct derivatives (Maliska 1981). The cross derivatives are evaluated using central differencing. Therefore,

$$\phi_e = (\tfrac{1}{2} + \bar{\alpha}_e)\,\phi_P + (\tfrac{1}{2} - \bar{\alpha}_e)\,\phi_E \tag{9}$$

and

$$\left(\frac{\partial\phi}{\partial\xi}\right)_e = \bar{\beta}_e \frac{\phi_E - \phi_P}{\Delta\xi} \tag{10}$$

Expressions for $\bar{\alpha}$ and $\bar{\beta}$ can be found in Minkowycz (1988).

Introducing equations (9), (10), and the corresponding ones for the remaining faces of the control volume, in Eq. (8), and after some algebraic manipulation one gets,

$$\begin{aligned} A_P\phi_P &= A_E\phi_E + A_W\phi_W + A_N\phi_N + A_S\phi_S \\ &+ A_{NE}\phi_{NE} + A_{NW}\phi_{NW} + A_{SE}\phi_{SE} + A_{SW}\phi_{SW} \\ &+ \frac{M^0_P\phi^0_P}{\Delta t} + B \end{aligned} \tag{11}$$

where the coefficients are shown in Marchi and Maliska (1994)

PRESSURE EQUATION

In discrete form the mass conservation equation can be written as (Marchi and Maliska 1994),

$$\frac{M_P - M^0_P}{\Delta t} + \dot{M}_e - \dot{M}_w + \dot{M}_n - \dot{M}_s = 0 \tag{12}$$

Using the linearization of mass fluxes described in Van Doormaal (1985), applied for curvilinear coordinate systems (Silva and Maliska 1988), one has, for the east face of a control volume

$$\dot{M}_e = y^\kappa_e \Delta\eta\,(\rho U^* + \rho^* U - \rho^* U^*)_e \tag{13}$$

Substituting Eq. (13) and its counterparts in Eq. (12), one gets

$$\begin{aligned} &\frac{M_P - M^0_P}{\Delta t} \\ &+ \Delta\eta\,[(y^\kappa \rho U^*)_e - (y^\kappa \rho U^*)_w + (y^\kappa \rho^* U)_e - (y^\kappa \rho^* U)_w] \\ &+ \Delta\xi\,[(y^\kappa \rho V^*)_n - (y^\kappa \rho V^*)_s + (y^\kappa \rho^* V)_n - (y^\kappa \rho^* V)_s] \\ &- \Delta\eta\,[(y^\kappa \rho^* U^*)_e - (y^\kappa \rho^* U^*)_w] \\ &- \Delta\xi\,[(y^\kappa \rho^* V^*)_n - (y^\kappa \rho^* V^*)_s] = 0 \end{aligned} \tag{14}$$

In order to obtain a equation for pressure from Eq. (14) it is necessary to write U, V and ρ as functions of pressure. First, one shall write the cartesian components of velocity at cell faces. Again taking the east face as example one gets

$$u_e = \hat{u}_e - d_e L\left[\hat{P}^u\right]_e \tag{15}$$

$$v_e = \hat{v}_e - d_e L\left[\hat{P}^v\right]_e \tag{16}$$

The way $\hat{u}_e$, $\hat{v}_e$ and d_e are calculated is the matter of the next section of this paper. Using the relation between the cartesian and the contravariant (without metric normalization) velocity components, equations (2) and (3), one obtains

$$U^*_e = \hat{U}^*_e - d_e \left(\alpha \frac{\Delta p}{\Delta\xi} - \beta \frac{\Delta p}{\Delta\eta}\right)_e \tag{17}$$

For the remaining faces similar equations are obtained.

The above expression is the so-called velocity correction equation. The way the uncorrect velocities ($\hat{U}$ and $\hat{V}$) are calculated depends on the pressure-velocity coupling method used. In this paper the main goal is to extend the sequential method PRIME (Maliska 1981) for co-located arrangement. In this case Eq. (17) is exactly the combination of the u and v momentum equations for the east face.

The density is obtained through a linearization of the state equation as

$$\rho = \rho_0 + C^\rho p \tag{18}$$

Here the ideal gas assumption is used, giving $\rho_0 = 0$ and $C^\rho = (RT)^{-1}$.

Since ρ is located at the cell center, at cell faces it is obtained using an interpolating function. An upwind interpolation was used in this work (Van Doormaal 1985), making the weighting parameter, $\bar{\gamma}$, equals to $+1/2$ or $-1/2$ according the signal of U (or V) that is employed in the calculation of the mass flux across each particular face. For the east face one gets,

$$\rho_e = (1/2 + \bar{\gamma})\,\rho_P + (1/2 - \bar{\gamma})\,\rho_E \tag{19}$$

Introducing Eqs. (17) and (19) with its analogues for the remaining faces in Eq. (14) one obtains the equation for pressure as

$$\begin{aligned} A_P p_P &= A_E p_E + A_W p_W + A_N p_N + A_S p_S \\ &+ A_{NE} p_{NE} + A_{NW} p_{NW} + A_{SE} p_{SE} + A_{SW} p_{SW} + B \end{aligned} \tag{20}$$

where the coefficients are straightforwardly deduced.

VELOCITY FIELD CALCULATION

Expressions for velocity calculation at cell faces, based on the work of Rhie and Chow (1983), are proposed in Marchi and Maliska (1994) and in the present work. Both can be written, for calculating a cartesian component u at an east face, as,

$$u_e = \frac{R_P + R_E + (A^t_P + A^t_E)\,u^0_e - 2L\,[P^u]_e\,\Delta V_e}{I_P + I_E + A^t_P + A^t_E} \tag{21}$$

In Marchi and Maliska (1994) it is proposed that

$$R_E = \sum_{nb} (A_{nb} u_{nb})_E + L\,[S^u]_E \tag{22}$$

with $I_E = (A_P)_E$, whereas, in the present work, R_E is given by

$$R_E = \sum_{nb} [A_{nb}\,(u^*_{nb} - u^*_P)]_E + L\,[S^u]_E + I_E u^*_e \tag{23}$$

with $I_E = \varepsilon \sum_{nb} (A_{nb})_E$.

In the above expressions the subscripts nb indicates summing over the neigbouring points of P and E. It is worth to note that here the transient part $A^t_E = M^0_E/\Delta t$ was removed from the coefficient $(A_P)_E$. It must be kept in mind that we want to obtain one equation for the cell face velocity from the equations of the velocities at nodal points P and E. The ε coefficient in the expression of the "inertia" term I_E is introduced for controlling inertia effects. This will be explained bellow.

Although these two expressions for R_E seems to be very similar, the former has shown to be inadequate for the use with "explicit" methods like PRIME. This has led to the development of the new method proposed here.

It is important to remember that the cell face velocity, herein reported, is the one which will enter the contravariant velocity component used for calculating the mass flow. When u assumes the role of ϕ in the Eq. (11) the cell face values required for ϕ remain been calculated through the WUDS or another interpolating function.

To explain Eq. (23) it is easier to consider a unidimensional version of Eq. (8). Substituting Eq. (9) and Eq. (10) in

Eq. (23), one gets

$$\frac{M_P\phi_P - M_P^0\phi_P^0}{\Delta t} =$$
$$\dot{M}_w\left[\left(\tfrac{1}{2}+\bar{\alpha}_w\right)\phi_W + \left(\tfrac{1}{2}-\bar{\alpha}_w\right)\phi_P\right]$$
$$-\dot{M}_e\left[\left(\tfrac{1}{2}+\bar{\alpha}_e\right)\phi_P + \left(\tfrac{1}{2}-\bar{\alpha}_e\right)\phi_E\right]$$
$$+\bar{\beta}_e D_{de}^{\phi}\left(\frac{\phi_E-\phi_P}{\Delta\xi}\right) - \bar{\beta}_w D_{dw}^{\phi}\left(\frac{\phi_P-\phi_W}{\Delta\xi}\right)$$
$$+L\left[\hat{S}^{\phi}\right]\Delta V - L\left[\hat{P}^{\phi}\right]\Delta V \qquad (24)$$

In an "explicit" technique, like the PRIME method, all quantities at the right hand side of this equation, except the pressure term, will be evaluated with quantities known from the last iteration. In semi-implicit methods all the ϕ values would be obtained from the solution of the linear system formed by the set of equations like (24), one for each grid volume. To simplify Eq. (24) it is wise to subtract the mass conservation equation, Eq. (12) in its unidimensional form, multiplied by ϕ_P^*. The resulting equation is

$$\frac{M_P\phi_P - M_P^0\phi_P^0}{\Delta t} - \phi_P^*\left(\frac{M_P - M_P^0}{\Delta t}\right) =$$
$$\left[\dot{M}_w\left(\tfrac{1}{2}+\bar{\alpha}_w\right) + \bar{\beta}_w\frac{D_{dw}^{\phi}}{\Delta\xi}\right]\left(\phi_W^* - \phi_P^*\right)$$
$$+\left[\dot{M}_e\left(\tfrac{1}{2}-\bar{\alpha}_e\right) + \bar{\beta}_e\frac{D_{de}^{\phi}}{\Delta\xi}\right]\left(\phi_E^* - \phi_P^*\right)$$
$$+L\left[\hat{S}^{\phi}\right]\Delta V - L\left[\hat{P}^{\phi}\right]\Delta V \qquad (25)$$

Observe that in Eq. (24) ϕ_E and ϕ_W were made equal to ϕ_E^* and ϕ_W^*. If ϕ^* is set equal to ϕ one recovers the usual equation for ϕ. Our objective is to obtain an interface velocity ($\phi = u, v$) using Eq. (25), which is written for the cell center velocities.

The usual procedure is to write Eq. (25) for the control volumes P and E and then average them to obtain the velocities at the interfaces. In the procedure advanced here the average of the equations is made different for each term, this difference being according to the physical interpretation of each term.

The transient term is averaged using the interface velocity, since it represents an information about the control volume which would be related to that velocity. The pressure term to avoid the even-odd decoupling, must be evaluated at the cell interfaces too.

The remaining terms of the equation (representing the divergence of the convective and diffusive fluxes, and the source term), can be evaluated with great simplicity, througout an avarage of the values of such quantities at the cell centers.

It is also needed to eliminate the unknown M_P of Eq. (25) if one is interested in extract from it an equation for velocities. The fact that the additional term on the left hand side of the equation involving ϕ_P^* is part of the approximation of the divergence of the fluxes, and so it should be evaluated using an average of the values obtained at cell centers, precludes the obvious device of make ϕ_P equal to ϕ_P^*. A way to avoid this difficulty would be to work with the products ρu and ρv as unknowns in the momentum conservation, as done in the ICE (Harlow and Amsden 1971), but it will precludes the use of the linearization here utilized for the mass conservation equation. Another solution would be substitute M_P by its last iteration value, M_P^*. Here M_P was made equal M_P^0 because only steady state solutions was sought. The equation for ϕ_e then reads

$$\frac{1}{2}\left(\frac{M_P^0}{\Delta t} + \frac{M_E^0}{\Delta t}\right)\phi_e =$$
$$\frac{1}{2}\left\{\left[\dot{M}_w\left(\tfrac{1}{2}+\bar{\alpha}_w\right) + \bar{\beta}_w\frac{D_{dw}^{\phi}}{\Delta\xi}\right]\left(\phi_W^* - \phi_P^*\right)\right.$$
$$\left.+\left[\dot{M}_e\left(\tfrac{1}{2}-\bar{\alpha}_e\right) + \bar{\beta}_e\frac{D_{de}^{\phi}}{\Delta\xi}\right]\left(\phi_E^* - \phi_P^*\right)\right\}_P$$
$$+\frac{1}{2}\left\{\left[\dot{M}_w\left(\tfrac{1}{2}+\bar{\alpha}_w\right) + \bar{\beta}_w\frac{D_{dw}^{\phi}}{\Delta\xi}\right]\left(\phi_W^* - \phi_P^*\right)\right.$$
$$\left.+\left[\dot{M}_e\left(\tfrac{1}{2}-\bar{\alpha}_e\right) + \bar{\beta}_e\frac{D_{de}^{\phi}}{\Delta\xi}\right]\left(\phi_E^* - \phi_P^*\right)\right\}_E$$
$$+\frac{1}{2}\left(L\left[\hat{S}^{\phi}\right]_P\Delta V_P + L\left[\hat{S}^{\phi}\right]_E\Delta V_E\right)$$
$$-L\left[\hat{P}^{\phi}\right]_e\Delta V_e + \frac{1}{2}\left(\frac{M_P^0}{\Delta t} + \frac{M_E^0}{\Delta t}\right)\phi_e^0 \qquad (26)$$

It is quite known that explicit methods works on a Δt range inferior of that of the semi-implicit methods. To approximate these ranges, an additional inertia term is introduced in the equation resulting in Eq. (21) and (23).

If Eq. (21) is written in the form of Eq. (15) one gets

$$\dot{u}_e = \frac{R_P + R_E + \left(A_P^t + A_E^t\right)u_e^0}{I_P + I_E + A_P^t + A_E^t} \qquad (27)$$

and

$$d_e = \frac{2\Delta V_e}{I_P + I_E + A_P^t + A_E^t} \qquad (28)$$

When dealing with co-located variables and implicit methods, it is not necessary the approach just described. The reason is that in the PRIME procedure the correcting equations are exactly the momentum equations and not auxiliary equations as in the SIMPLE-like procedures. So, the average of the velocities equation at the cell centers, to obtain the velocities at the interfaces, must be an accurate representation of momentum conservation.

In the SIMPLE-like methods there's no pressure equation, but a pressure correction equation that drives the velocity field to mass conservation according to a mass imbalance initially detected. The mass and momentum conservation are treated separately with its respectives linear systems. The velocities introduced in mass conservation are interpolated and not calculated at cell faces. The important fact is that the role played by the momentum conservation equations in the calculation of the cell face velocities like proposed in Marchi and Maliska (1994) is not too crucial. Some propositions for this calculation for semi-implicit methods, in wich the expression of Marchi and Maliska (1994) was inspired, can be viewed as a linear interpolation of velocities with a third order pressure smoothing term (Lien and Leschziner 1994). The velocities, in these methods, are really calculated at cell centers, through the appropriate linear system. In the PRIME procedure, due to its explicit nature we could say that the velocities are really calculated at the interfaces. The velocities at cell centers are obtained by interpolation of the contravariant components at the interfaces and use of the inverses of equations (2) and (3):

$$u = J\left(Ux_\xi + Vx_\eta\right) \qquad (29)$$

$$v = J\left(Uy_\xi + Vy_\eta\right) \qquad (30)$$

RESULTS

To asses the performance of the methodology described, two test cases are shown: the incompressible (Mach 3.2×10^{-5}) and laminar ($Re = 100$) lid driven cavity flow with lateral cavity walls tilted of 45° (Demirdžić et al. 1992), and the inviscid supersonic flow over an infinite cylinder.

In the first case, a 40×40 uniform grid was used. The profile of the nondimensional vertical cartesian velocity component along the horizontal middle line obtained with the methodology described in this work is compared with the benchmark solution of Demirdžić et al. (1992), and with the results obtained replacing the calculation of the cell face velocity as suggested in Marchi and Maliska (1994) for the SIMPLEC procedure (Van Doormaal 1985).

The results of Fig. 1 show the inadequacy of using expressions developed for SIMPLE-like methods in sequential methods like PRIME.

A large time step was taken (10^6 times the lid length per lid velocity) making the transient term neglegible in the equations. This calculation was successfull due to the beneficial effects of the artifficial inertia term introduced in Eq. (23).

For the cylinder, grids with 20 volumes in the radial direction and 26 in the circunferencial one were used. They were

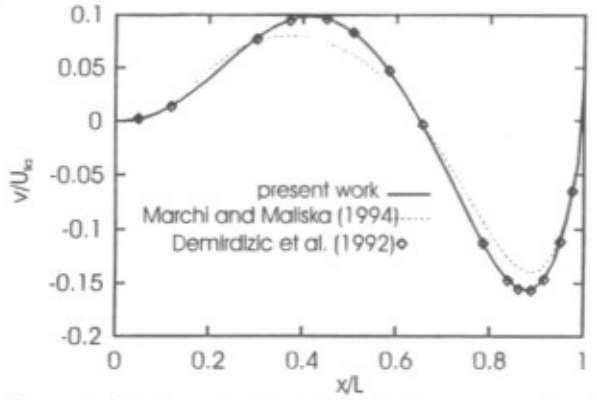

Figure 1: Nondimensional vertical velocity component profile along the middle horizontal line of the cavity.

based on that used to solve the same problem in Silva (1991). A special care was taken to make the grid orthogonal at the symmetry line and smooth near there. The results are not good if this is not done. The reason for that is the need of the grid to be consistent with the physics, that is, the grid needs to be also symmetric, which implies orthogonality, at the symmetry line. It can be seen, in Fig. 2, that the results agree well with the experimental ones shown in Liepmann and Roshko (1957).

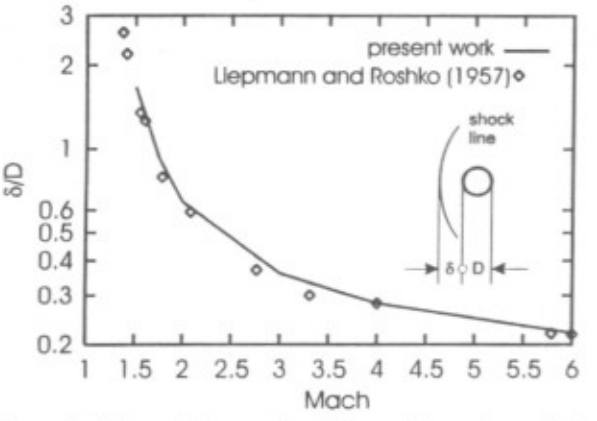

Figure 2: Distance between shock line and stagnation point.

A comparision between the pressure coefficient over the cylinder surface for Mach 4, given by this methodology and that proposed in Silva (1991), is shown in Maliska (1995). The nondimensional time step $\Delta\tau = \Delta t D/U_\infty$ employed for supersonic flows was of 0.2 in the present methodology and of 0.3 in that of Silva (1991). These was near the maximum attainable. The CPU time were also similar for both methodologies.

CONCLUDING REMARKS

This paper presented a non-staggered finite-volume method for handling the pressure-velocity coupling in sequential procedures. Due to the nature of the velocity calculation in sequential procedures, the cell face velocities is an important part of the method. The method is easier to implement than the usual ones. The outcome of this work can be also applied to explicit, like MAPLE (Marek and Straub 1993), and artificial compressibility methods. It also may be applied when the velocities used in mass flow calculation are not stored at the middle of the cell faces, as in ALE (Hwang 1993).

The results obtained for incompressible and compressible flow problems are encouraging.

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