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A NONORTHOGONAL MODEL FOR THE SOLUTION OF NATURAL CONVECTION PROBLEMS IN ARBITRARY CAVITIES

C.R. MALISKA AND F.E. MILIOLI
MECHANICAL ENGINEERING DEPARTMENT
FEDERAL UNIVERSITY OF SANTA CATARINA
P.O.BOX 476
FLORIANÓPOLIS - SC - BRASIL

ABSTRACT

A finite-volume method, described in boundary fitted nonorthogonal coordinates, to deal with natural convection problems of a Boussinesq fluid in arbitrary cavities is presented. The method uses the approach of transforming the conservation equations to a general curvilinear coordinate system, performing all the computations in a fixed rectangular domain. The basis of the model is described in [1,2] and the present work extends the model to deal with heat transfer calculations.

Another feature of this work is the continuation of a series of tests which are being conducted in order to have a complete assesment of the model described in [1,2] in the solution of pure hydrodynamic, forced, natural and combined forced/natural convection problems with different types of boundary conditions. The model, based on the tests realized up to now has demonstrated excelent performance, and in this paper it is applied for the solution of pure natural convection problems, solving the thermal driven flow in a square cavity. The problem is solved for Rayleigh numbers ranging from 10^3 to 10^6 and the results are compared with the bench mark solution of de Vahl Davis [7].

The solution of the buoyancy flow between a square cylinder placed inside a circular cylinder is also reported and the results compared with

those available in the specialized literature.

1. INTRODUCTION

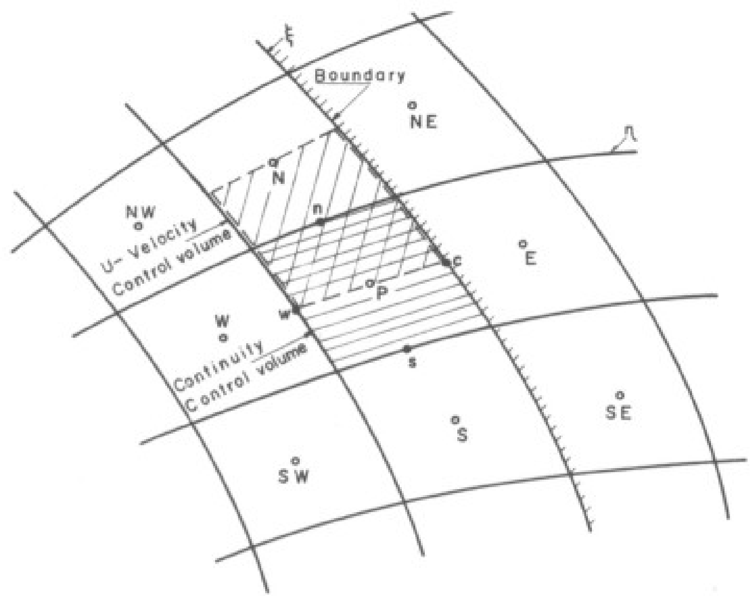
The use of boundary fitted nonorthogonal coordinate systems requires special attention in arranging the dependent variables in the grid when the Cartesian velocity components (or the velocity components in any other orthogonal system) are kept as dependent variables in the transformed plane. This very important question is discussed in detail in [1] but its is important to report here the outcome of the analysis. If mass conservation is to be enforced, through a pressure equation, for both contravariant velocity components in each face of the elemental control volume, as done in [3], an extra set of pressure points (relative to the pressure points used in the standard staggered grid) must be introduced in the grid arrangement. This procedure does not improve accuracy, but it is a practice which merely sweeps the solution domain twicely with mass conservation control volumes. Indeed, it is an undesirable feature of the model since the solution of the pressure equation, which is time consuming, will involve twice as many pressure points, slowing down the convergence rate. Even worse is the fact that when and orthogonal grid is used the pressure field from the original pressure points and the one from the extra set are loosely tied through the metrics.

For the case of an orthogonal system with the metrics kept constant in the domain, as is the case for uniform Cartesian meshes, the two pressure fields are totally uncoupled. In this paper the strategy used in [1,2], which avoids the aforementionated problems, is employed.

The grid layout used is shown in Fig. 1, where the temperatures are located at the middle of the continuity control volumes. There are no pressure neither temperature nodes lying on the boundaries.

The nonorthogonal boundary fitted coordinate is generated using the well known method described in [5], where details are given.

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS



○ PRESSURE AND TEMPERATURE
• CARTESIAN AND CONTRAVARIANT VELOCITY COMPONENTS

FIG.1 - GRID LAYOUT FOR THE DEPENDENT VARIABLES

To obtain the system of equations which governs the laminar natural convection problem of a Boussinesq fluid in the general curvilinear coordinate system, the conservation equations are transformed from the Cartesian coordinate system to the general curvilinear coordinate system, to give

$$\frac{\partial}{\partial t}(\rho\Phi)+\frac{\partial}{\partial\xi}(\rho U\Phi)+\frac{\partial}{\partial\eta}(\rho V\Phi)+\hat{P}^{\Phi} = \frac{\partial}{\partial\xi}\left(C_1\frac{\partial\Phi}{\partial\xi}+C_2\frac{\partial\Phi}{\partial\eta}\right)+\frac{\partial}{\partial\eta}\left(C_3\frac{\partial\Phi}{\partial\eta}+C_4\frac{\partial\Phi}{\partial\xi}\right)+\hat{S}^{\Phi} \qquad (1)$$

where U and V are the contravariant velocity components given by

$$U = y_\eta u - x_\eta v \qquad (2)$$

$$V = x_\xi v - y_\xi u \qquad (3)$$

where C_1 C_2 C_3 and C_4 are the transformed diffusion coeficcients. Details can be found in [6].

3. NUMERICAL PROCEDURE

3.1 DISCRETE EQUATIONS

The finite volume equations are obtained by integration of the conservation equations over the elemental control volume. Approximations are introduced to evaluate the values and the gradients of the transported property in the control volume faces. The weighted upstream differencing scheme is applied, where the cell Peclet number is calculated using the contravariant velocity component in the respective direction.

For the general variable ϕ the discrete equations have the form

$$A_P\phi_P^{n+1} = A_e\phi_E^{n+1} + A_w\phi_W^{n+1} + A_n\phi_N^{n+1} + A_s\phi_S^{n+1} + \frac{A_P}{1+E}\phi_P^{n} - L\left[P^{\phi}\right]\Delta V + L\left[\hat{ST}^{\phi}\right]\Delta V \tag{4}$$

where $L[\;]$ means the approximation of the term inside the brackets and $\hat{ST}^{\phi}$ is given by

$$\hat{ST}^{\phi} = \hat{S}^{\phi} + \frac{\partial}{\partial\xi}\left(C_2\frac{\partial\phi}{\partial\eta}\right) + \frac{\partial}{\partial\eta}\left(C_4\frac{\partial\phi}{\partial\xi}\right) \tag{5}$$

The momentum equations, for the contravariant velocity components which enters the mass conservation balance, for the control volume centered at P in Fig. 1, are

$$U_e = \hat{U}_e - \left(\frac{\Delta\Gamma}{A_p^u}\alpha\right)_e(P_E-P_P) + \left(\frac{\Delta\Gamma}{4A_p^u}\beta\right)_e(P_{NE}+P_N-P_{SE}-P_S) \tag{6}$$

$$V_n = \hat{V}_n - \left(\frac{\Delta\Gamma}{A_p^v}\gamma\right)_n(P_N-P_P)+\left(\frac{\Delta\Gamma}{4A_p^v}\beta\right)_n(P_{NE}+P_E-P_{NW}-P_W) \tag{7}$$

Similar equations are obtained for U_w and V_s. Substituting the above equations and U_w and V_s in

the mass conservation equation a Poisson-like equation for pressure is obtained whose source term is the divergence of the vector $\hat{V}$.

The same procedure is applied for the continuity control volumes close to the boundaries. The enforcement of mass conservation, obeying the velocity boundary condition existing at the respective boundary, eliminates the need for boundary conditions on pressure. The application of boundary conditions for pressure is a cumbersome problem, specially when there is pressure points lying on the boundaries. The usual approach, when using staggerd grid, for the application of the pressure boundary conditions, is to use fictitious pressure points with homogeneous Neumann boundary condition where the velocity vector is prescribed. With the approach adopted here the influence coefficients that would connect pressure point P with its neighbouring fictitious

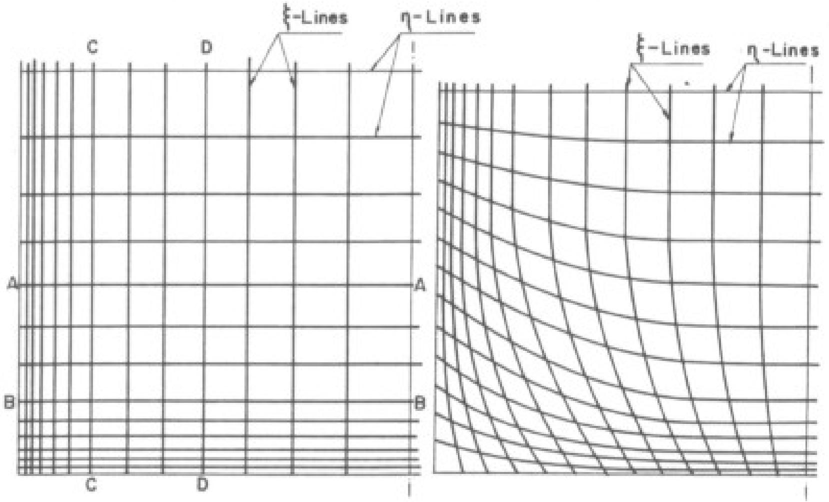


FIG.2 - CARTESIAN AND NONORTHOGINAL GRID

points vanish.

Another numerical detail of the model is the evaluation of the pressure gradient for the velocity control volumes close to the boundaries. The need of evaluating the pressure gradient in both directions for each momentum equation, and because there is no pressure points available for a second order evaluation using central differencing, a special way for the gradient evaluation needs to be developed. Refering to Fig. 1, the pressure gradient at the point n is calculated by.

$$\left.\frac{\partial P}{\partial \xi}\right|_n = \frac{P_N + P_P}{4} - \frac{P_{NW} + P_W}{4} + \frac{1}{2} \cdot \left.\frac{\partial P}{\partial \xi}\right|_{wall} \qquad (8)$$

where the pressure gradient at the wall is obtained with the pressure field from the previous iteration level. If the pressure gradient at the wall is neglected a first order approximation is, then, adopted.

The boundary conditions for temperature are incorporated in the source term for the energy equation. This permits to change thermal boundary conditions with very small changes in the computer code.

3.2 SOLUTION PROCEDURE

The equation set which comprises the momentum equations, energy equation and the pressure equation is solved iteratively.

The PRIME technique is used to handle the pressure-velocity coupling. In this procedure the momentum equations are solved in a Jacobi iteration fashion, while the equation for pressure is solved only once in each iteration cycle by a point S.O.R technique. The pressure field obtained is used for correcting the velocity field, in order to satisfy continuity and, in the same time, it is understood as the pressure field for that iteration level. This procedure saves computing time compared with methods which solve one Poisson equation for a velocity correction function and another one for establishing the pressure level.

The temperature equation is solved every cycle momentum is. Under-relaxation is applied in order to avoid instabilities in the source term form the momentum equations. Iteration is performed until an error of 5.E-5 is satisfied for every velocity and temperature in the domain.

4. NUMERICAL RESULTS

The buoyancy driven flow in a square cavity is an excellent test problem for checking numerical models because of the strong recirculating flow obtained with high Rayleigh numbers. Also, the results reported in [7] offer to the numerical analyst the possibility of testing new numerical models with accuracy.

In this paper the nonorthogonal model was used to solve the square cavity problem employing an orthogonal and a nonorthogonal grid, as shown in Fig. 2. The problem was solved for Ra numbers

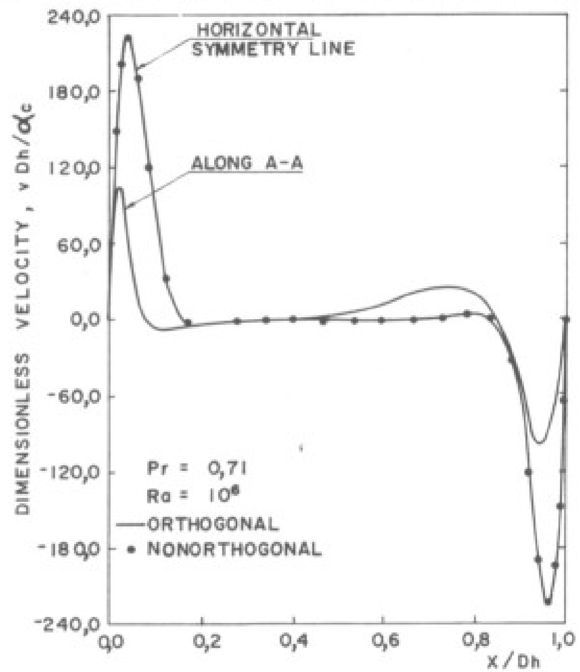


FIG.3 - VELOCITIES ALONG HORIZONTAL LINE

ranging from 10^3 to 10^6 with Prandlt number equal to 0,71.

For both coordinate systems a better grid resolution was used close to the boundaries with a 26x26 mesh for both cases. Figures 3 and 4 show the dimensionless velocity profiles for the u and v computed using orthogonal and nonorthogonal grids. The results are in good agreement. The convergence rate for both grids was similar. In

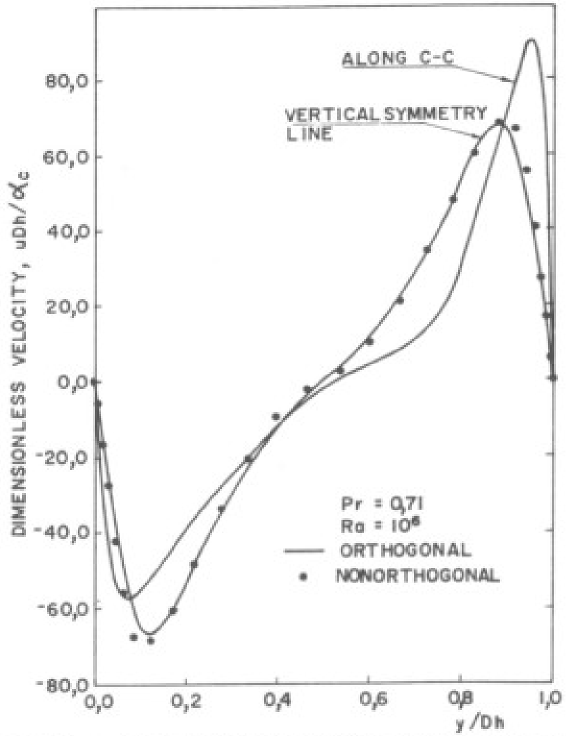


FIG.4 - VELOCITIES ALONG VERTICAL LINE

fact, in all calculations the CPU-time was slightly less for the nonorthogonal mesh case.

Figure 5 shows the dimensionless temperature profile using orthogonal and nonorthogonal grids. Again the agreement, for all Rayleigh numbers, is very good. These results indicate that the model performs well using nonorthogonal coordinates.

The results obtained with this problem are summarized in Table 1 and compared with the results of de Vahl Davis [7] . It is seen that the results compare well. The comparison exercise [7] also shows that several contributions reported results with higher errors than the ones obtained with this work. Considering the 37 contributions the results reported in this paper can be classified as good ones.

It is expected that even better results can be obtained with a more extensive numerical experiments considering changing grid resolution at the boundaries.

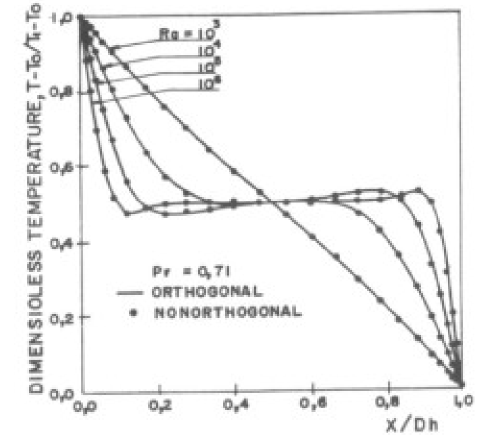


FIG.5 - DIMENSIONLESS TEMPERATURE

TABLE 1

Ra	u^*_{max} y^*	v^*_{max} x^*	Nu_{max} y^*	Nu_{min} y^*	Nu_o	
10^3	3,679 0,8384	3,724 0,1477	1,466 0,1301	0,747 1	1,116	
	3,649 0,813	3,697 0,178	1,505 0,092	0,692 1	1,117	*
10^4	16,091 0,8476	20,090 0,1040	3,555 0,1440	0,603 1	2,238	
	16,178 0,823	19,617 0,119	3,528 0,143	0,586 1	2,238	*
10^5	35,65 0,8827	68,95 0,0548	7,714 0,0807	0,806 1	4,468	
	34,73 0,855	68,59 0,066	7,717 0,081	0,729 1	4,509	*
10^6	67,58 0,8975	224,62 0,0307	16,132 0,0364	1,214 1	8,743	
	64,63 0,850	219,36 0,0379	17,925 0,0378	0,989 1	8,817	*

The observation has shown that the minimum Nusselt number is a very sensitive parameter to be calculated. A numerical experiment, where an error of 0.1 per cent was introduced in the temperature field, produced an error of 23 per cent in the minimum Nusselt number.

* Results from Ref. [7]

As a second test problem the natural convection problem in the cavity shown in Fig. 6

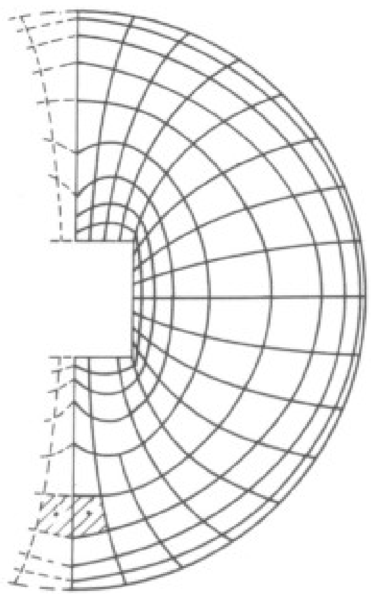


FIG.6 - NONORTHOGONAL GRID FOR THE SECOND TEST PROBLEM

was solved. The nonorthogonal grid was generated for only half of the domain. The nonorthogonal metrics were set to zero by reflection, at the symmetry lines, in order to avoid fictitious shear stress at that lines. The results are shown in Fig.7 and compared with those of Chang [8], in terms of Keq, defined by

$$Keq = \frac{\int q \, ds}{\int q_c \, ds} \tag{9}$$

where q is the local heat transfer flux and q_c is the local heat transfer flux by pure conduction for the same geometry. The results show good agreement. More details about the numerical results can be found in [6].

5. - CONCLUDING REMARKS

A nonorthogonal model for the solution of two dimensional natural convection problems inside

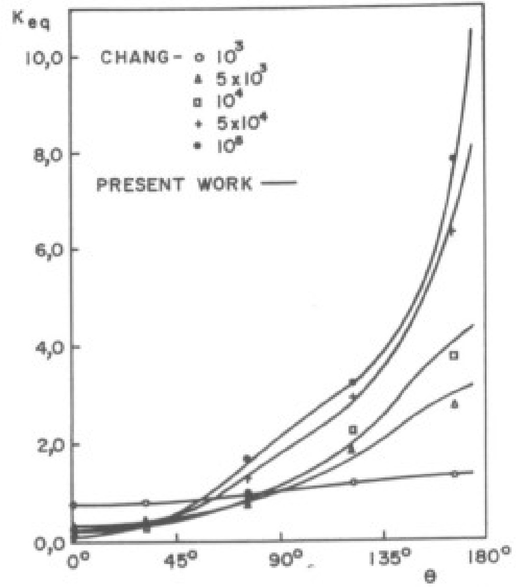


FIG. 7 - EQUIVALENT CONDUCTIBILITY

arbitrary cavities was presented. The introduction of the energy equation did not altered the strong stability already observed when solving pure hydrodynamic problems. The results of the two test problems solved were compared with the available results in the literature. Good agreement was observed. Computer time required for the solution using a nonorthogonal grid is the same as needed for orthogonal grids. Extension of the method to 3-D elliptic problems is encouraging.

This demonstrates that the use of finite volume methods, developed in a nonorthogonal framework using the conservation principles, is a viable route for the complete generalization of numerical techniques for the prediction of fluid flow and heat transfer in arbitrary geometries.

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THE NUMERICAL COMPUTATION OF THREE-DIMENSIONAL NATURAL CONVECTION IN A VERTICAL ENCLOSURE

E.K. Glakpe, C.B. Watkins, Jr. and B. Kurien
Department of Mechanical Engineering
Howard University
Washington, D.C. 20059, U.S.A.

ABSTRACT

Numerical solutions are presented for the steady-state, laminar natural convection in a vertical enclosure formed by a vertical square rod inside a concentric circular cylinder with horizontal top and bottom surfaces. Solutions are obtained for Rayleigh number up to 10^6 with air (Pr=0.703), and radius and aspect ratios of 1. Velocity vector and contour plots of isotherms show the effects of natural convection on the flow pattern and temperature distribution. Local and average Nusselt numbers are derived and presented in forms useful for the design of such a heat removal system.

INTRODUCTION

Natural convection in enclosed spaces has been studied extensively in recent years, particularly, in response to demands from energy-related applications. A configuration of practical importance that simulates a shipping container for the transportation of spent fuel is a vertical enclosure with an inner square rod bounded by a circular enclosure. The natural convection flow phenomena in this enclosure geometry is three-dimensional in nature and, numerically, involves the simultaneous solution of the Navier-Stokes equations.

Natural convection in a cylindrical annulus configuration has attracted much attention, and a review of the works concerning this geometry was presented by Kuehn & Goldstein [1]. However, most of the works reported were for horizontal concentric cylinders with infinite axial length, enabling the convective flow to be regarded as two-dimensional.

It has become feasible recently to numerically solve three-dimensional problems due to improvements in processing speed and memory capacity of digital computers. Several