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506

POROSITY AND PERMEABILITY DETERMINATION IN PETROLEUM RESERVOIR SIMULATION METHODS USING UNSTRUCTURED GRIDS

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ABSTRACT

Finite volume methods, also called conservative methods or control volume methods, require the evaluation of the fluxes (mass, momentum, energy etc.) at the interfaces of the control volumes and the mass quantity inside the control volume for the calculation of the transient and source terms. In petroleum reservoir simulation this means the need of calculating mobilities at the control volume interfaces and porosity at the center of the control volume. Considering the geometrical entity defined by the nodes of the computational grid as an element, control volumes in a finite volume method can be classified as a vertex-center construction when they are built around the nodes of the grid, and cell-center when the control volume is coincident with the element.

The widely used method in petroleum reservoir simulation stores the physical parameters, like porosity and absolute permeability, at the center of the control volumes. This means that for a heterogeneous media the interface of the control volumes lies between two media with different properties, and some kind of interpolation in the physical parameters at the interface will be required.

In this work, using a finite volume framework, a novel method is advanced, where all calculations are performed for the element (element-based finite volume method) and properties are stored at the center of the elements, instead of the control volumes. Since in this construction the control volume interfaces lie inside the elements, there is no need of any interpolation. It is shown, by solving two-phase flows in heterogeneous media, that storing the properties at the center of the elements instead at the center of the control volumes gives better results and renders to the methodology flexibility for new and more robust interpolation functions.

NOMENCLATURE

B	volume formation factor
i	mid-points
k	absolute permeability
m	subscript indicating the phase
N	shape functions
NNE	number of element nodes
p	pressure
P	node
q_m	flow-rate per unit of volume at reservoir conditions
s	saturation
t	time
ΔV	total volume of the control volume
ϕ	porosity
λ	mobility
ξ, η	coordinate directions in computational space

INTRODUCTION

This paper briefly discusses two different schemes of storing physical properties, and, as a consequence, of modeling faults using unstructured grids in petroleum reservoir simulation. Depending on the scheme employed, some kind of averaging is needed to estimate the porosity and internodal permeabilities.

The most common procedure, whereby the properties are stored at the control volumes center, employs the harmonic mean. This scheme is often used in commercial simulators, and it can result in an erroneous heterogeneous map [1]. Several authors have studied the impact of numerical bias due to the using of the different weighting formulas in upscaling problems using single-phase flows [1-3].

In this work is related a proposed element-based finite volume method where the concepts of elements and control volumes are used. It is demonstrated that storing the properties in the center of the elements and variables in the center of control volumes there is no need for any permeability averaging.

These different ways of storing the physical properties imply in different ways of geological faults representation. In this work is proposed to model the fault using elements with a small width and zero permeability.

An example involving the simulation of a two-phase flow is also included to demonstrate the potentialities of the proposed method and the difficulties associated with the use of the conventional way of properties storing and fault representation.

THE EbFVM METHOD

The Element-based Finite Volume Method (EbFVM) [4] is the numerical method chosen in this work to solve the system of differential equations. In this method the domain is covered by non-overlapping elements defined by grid nodes: triangles and/or quadrilaterals in 2D. Figure 1 shows an example of a grid used in this method, where one triangular element (formed by nodes 5-3-2) and one quadrilateral element (formed by nodes 3-4-1-2) are depicted. To obtain the approximate equations, the divergent form of the partial differential equation is integrated over control volumes, what is equivalent to make balance conservation on those volumes.

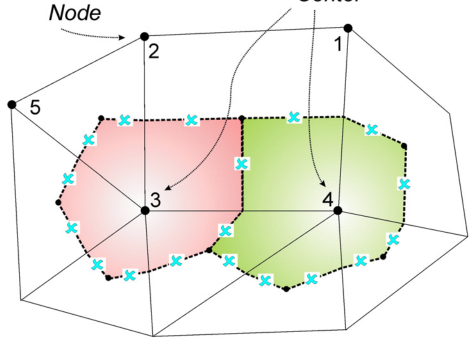


Figure 1 – Example of a grid used in the EbFVM

Differently from the traditional finite volume methods, often called finite difference methods in the petroleum literature, where elements and control volumes are coincident (cell centered construction), in the EbFVM control volumes are built around grid nodes, joining the center of the elements to their medians. The resulting control volume is formed by portions (sub-control volumes) of neighboring elements, as shown in Fig. 1. In this figure the control volumes formed around the nodes 3 and 4 are pointed out, as well as the integration points (labeled by "*x*") located over their boundaries, whereby all fluxes are evaluated using a mid-point approximation. This scheme of control volume construction belongs to the cell vertex category, since the center of the control volumes is a vertex of the element.

It is important that the reader be familiarized with the control volume and element definitions adopted in this work and exemplified in Fig. 1, since these terms will be employed throughout this paper.

The geometrical flexibility achieved with the proposed method resembles the Finite Element Method (FEM), being this the reason why it is usually known as Control Volume Finite Element Method (CVFEM). However, the denomination CVFEM is misleading and conveys the reader to view the methodology as being a finite element technique which uses control volumes for the integration of the equations. Actually, it is a finite volume methodology, whose only similarity with the FEM is the use of elements for the domain geometrical representation and the shape functions for the variables interpolation. A better denomination would be Element-based Finite Volume Method (EbFVM), since it is simply a finite volume methodology that borrows from the finite element technique the process of assembling the equations element by element, and its shape functions [4].

The discretized equations. The problem under consideration involves the simulation of a two-phase porous media flow, whose governing partial differential equations are given by

$$\frac{\partial}{\partial t}\left(\phi \frac{s_m}{B_m}\right) = \vec{\nabla}.\left(\lambda_m \overline{\overline{k}} \vec{\nabla} p\right) + q_m \tag{1}$$

where p is the pressure, λ is the mobility, ϕ is the porosity, s is the saturation, B is the volume formation factor, $\overline{\overline{k}}$ is the permeability tensor, q is the flow-rate per unit of volume at reservoir conditions, and m is the subscript indicating the phase. For convenience, the gravity and capillary effects were neglected.

Integrating Eq. 1 in time and over an elemental control volume P and applying the Gauss divergence theorem, this equation can be written as

$$\frac{\phi_P \, \Delta V_P}{\Delta t}\left(\frac{s_m}{B_m} - \frac{s_m^o}{B_m^o}\right)_P = \int_S \lambda_m \overline{\overline{k}} \vec{\nabla} p \,. d\vec{S} + q_m \, \Delta V_P \tag{2}$$

where ΔV_P is the total volume of the control volume formed by the sum of the sub-control volumes belonging to the elements surrounding node P. The surface integral appearing in the earlier equation, which is over all the edges of a control volume, can be approximated using a mid-point approximation:

$$\int_S \lambda \bar{\bar{k}} \vec{\nabla} p . d\vec{S} = \sum_i \left(\lambda \bar{\bar{k}} \vec{\nabla} p \cdot \Delta \vec{S} \right)_i \qquad (3)$$

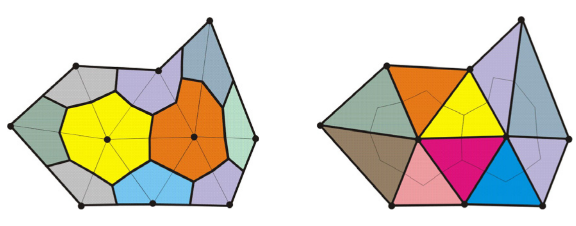
where i represents the mid-points of these edges. These points are called here as "integration points" and they are represented by a "x" in Fig. 1.

The pressure field is approximated inside elements using their nodal values as

$$p = \sum_{j=1}^{NNE} N_j \left(\xi, \eta \right) p_j \qquad (4)$$

where NNE is the number of element nodes, which assumes 3 for triangles and 4 for quadrilaterals, and N_j are the shape functions [4]. Usually, the shape functions are expressed in terms of the local coordinates (ξ, η) in the computational domain, because this coordinates system allows each element to be treated identically, no matter how distorted the element may actually be in terms of the global coordinates.

Different storage schemes. In the EbFVM, the physical properties k (absolute permeability) and ϕ (porosity) of Eq. 2 can be stored either at the center of the control volumes [5] or at the elements center [6]. It is shown in this section that the need for some type of average to calculate these properties will depend on the relative geometrical position of control volumes, elements and the physical properties map. Figure 2 shows the two different ways of storing physical properties in unstructured grids.



(a) Properties stored at the control volumes center

(b) Properties stored at the elements center

Figure 2 – Different ways of storing the physical properties on the grid

In the first case, the physical properties are stored at the center of the control volumes. This means that for a heterogeneous media the interface of the control volumes lies between two media with different properties, and some kind of interpolation in the physical parameters at the interface will be required. The most used weighting formula is the harmonic average [7], which is physically justified for one-dimensional flow. However, its extension to two and three dimensional situations is not adequate. Several authors have criticized this average in petroleum reservoir simulation [7-9], and the use of the arithmetic and geometric averages have been also proposed [10], but with relevant results applicable only to some specific cases [2]. On the other hand, when the physical properties are stored at the elements center (Fig. 2b), one can assure that there is no need to perform any average to calculate the physical properties at the control volume interfaces. The reason is that the integration points are inside the elements, which are piecewise constant in this scheme. The variables calculated by the model, like pressure and saturations, are still stored in the grid-nodes.

The other advantage of the second scheme is the need of dealing only with one grid (the elements), which is built using triangles and/or quadrilaterals. It results in an easier procedure to build grids that represents the heterogeneities with more fidelity. From a practical point of view of the simulator users, this method demands to "see" only one grid, which is built only by elements, unlike the other storage scheme, Fig. 2a, where there is the need of dealing also with control volumes (the dual grid), built with parts of the elements.

However, as the control volume results heterogeneous in the second scheme, Fig. 2b, the transient term of the partial differential equation should be rewritten, because it is needed to estimate the porosity of the control volume. A weighted average porosity as a function of the volume of each sub-control volume can be employed [6], in an easier and with less influence process than taking averages of the permeability. There are at least two reasons that can support this idea: first, often the range of variation of permeabilities values is greater than the variation of porosity values in a field; second, the permeability is a term appearing in Darcy Law, while porosity is not. We should remember that the Darcy law is the momentum equation for a porous media.

Representing faults. Modeling faults in petroleum reservoir simulation is always a challenging task if an automated procedure is sought. This subject is directly linked to the way as the properties are stored in the computational grid. The common procedure, employed in the scheme where the properties are stored at the

control volumes center, is to consider the fault as a line, neglecting the connection coefficient through the fault. This paper presents a new concept in modeling faults, in which the fault is composed of triangular and/or quadrangular elements with a small width. These elements are easily located to represent the fault, and the permeability values set to them will define the sealing degree of the fault. When zero permeability is used, for instance, the fault will be fully sealant.
In spite of the reduced width of the fault, creating geometrically different elements, and whatever solver used, there will no additional difficulties to solve the linear system of equations, since the magnitude of the coefficients in the matrix remains practically the same.

RESULTS AND DISCUSSION

In this section, the two ways of representing physical properties and faults presented earlier is applied to a test-problem. The boundary conditions used consist of an insulated reservoir having a water injection well and a production well. This test-problem represents a petroleum reservoir subjected to a secondary oil recovery, which is a process often utilized by petroleum companies to enhance the oil production. The geometry of the problem studied is shown in Fig. 3.

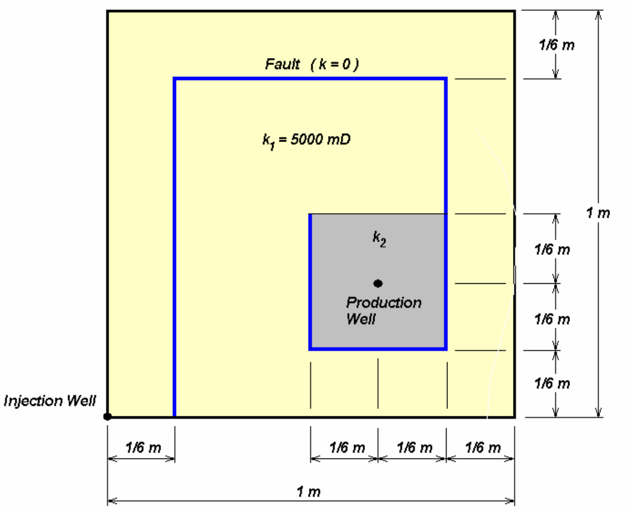


Figure 3 – Depiction of the reservoir of the Test-problem, where k_2 assumes values of 5000 mD (homogeneous problem) or 50 mD (heterogeneous problem)

From now on, due to the similarity of the location of the fault with a labyrinth, this problem will be called "labyrinth problem". In the same figure is shown the position of the wells and the reservoir dimension. The physical properties used are described in Tab. 1.
Two different cases will be analyzed using the same geometry shown in Fig. 3, and their differences are due to the value of absolute permeability set to the region around the production well (5000 or 50 mD). For the case whereby the permeability k_2 is 5000 mD, one has an homogeneous reservoir, otherwise one has an heterogeneous reservoir.

Table 1
Physical properties and numerical parameters used

Porosity	0.2
Permeability (isotropic) – homogeneous case	5000 mD
Reservoir thickness	1 m
Viscosity of water and oil	1 cp
Compressibility of water and oil	0
Formation volume factor of water and oil	1
$s_{wi} = s_{or}$	0
$k_{rw} @ s_{wi} = k_{ro} @ s_{or}$	1
Curve of k_{rw} and k_{ro}	linear
Water flow-rate at the injection well	0.1 m^3/day
Bottom-hole pressure at the production well	100 kPa
Well index	1 mD.m
Time step	0.01 day

The geological fault of Fig. 3 is modeled using the two schemes of representation presented in the previous section. Following, these schemes are related in details.

First scheme: In the first scheme, in which the physical properties are stored at the control volume center and the fault is represented by canceling the flux at the integration points, it is used the grid shown in Fig. 4. This grid is composed by 9 quadrilateral elements and 16 nodes.
The computational procedure of modeling the fault in Fig. 4 is simply nullifying all fluxes through the control volumes interfaces geometrically coincident with the fault. In this scheme, the permeability region around the node 7 (region k_2) is represented with a homogeneous control volume according to the process explained in Fig. 2a.

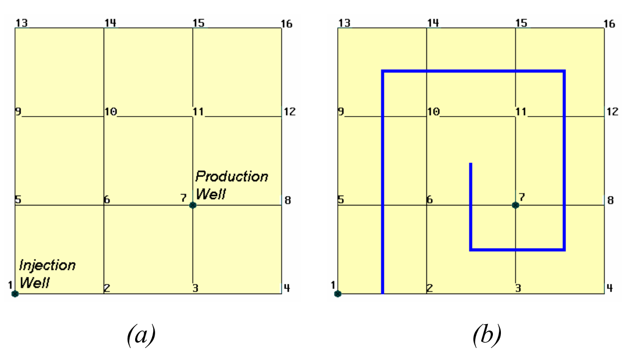
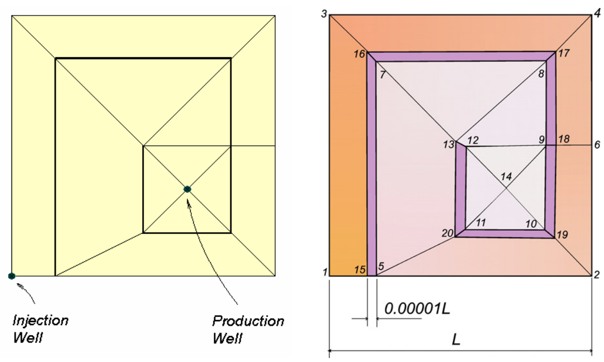


Figure 4 – 'Labyrinth problem' in the first case: *(a)* grid used, and *(b)* location of the fault

Second scheme: The other alternative studied in this work consists in using piecewise constant physical properties over elements instead of control volumes. In this case, the fault is modeled using elements with small width and zero permeability. The reservoir in Fig. 3 is discretized using a grid composed by triangular and quadrilateral elements shown in Fig. 5*a*, with 20 nodes and 18 elements.



(a) *(b)*

Figure 5 – 'Labyrinth problem' in the second case: *(a)* grid used, and *(b)* detail of the fault with its width enlarged to better the visualization

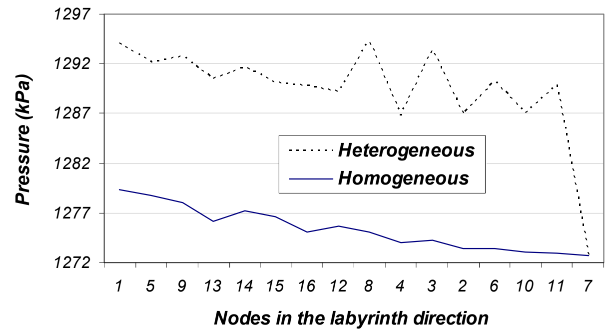
The fault is also visualized in Fig. 5b not in scale, in order to permit visualization, otherwise it would be impossible to visualize it (cf. Fig. 5a), because the width used is 0.00001*L*, where *L* is the reservoir dimension.

Results. Due to the geometrical configuration of the reservoir, this problem can be considered practically one-dimensional throughout the labyrinth. Hence, according to this work purposes, it is enough to analyze the pressure values at the grid nodes in that direction, as shown in Fig. 6, where the results obtained by EbFVM in the two schemes of modeling physical properties in the grid are compared.

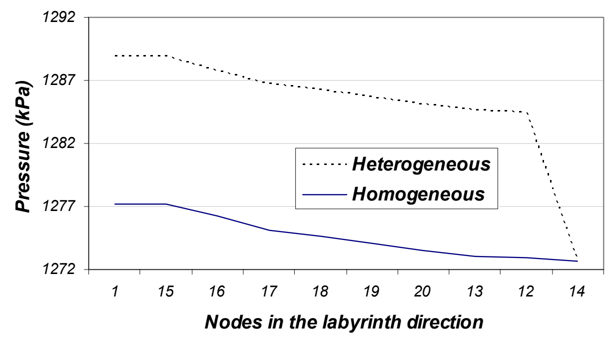
Note that in the first case, Fig. 6a, in which the physical properties are stored at the center of control volumes and the fault is modeled canceling the flux at the control volumes interfaces, the pressure values present oscillations that increase when the reservoir is heterogeneous.

On the other hand, the results obtained representing the physical properties using piecewise constant elements, Fig. 6b, do not show these difficulties. Indeed, it is found the same behavior verified in Cartesian grids for this case [6], characterized for having a linear pressure drop through the 'laryrinth'.

Discussion. One possible cause of the undesirable results shown by using the first scheme, Fig. 6a, can be related to the interpolation functions that are including inadequate nodal values in the calculation of flux in some control volume interfaces.



(a) First scheme



(b) Second scheme

Figure 6 – Comparison between the values of pressure through the 'labyrinth' obtained by two schemes of representing physical properties

To exemplify it, consider the quadrilateral element defined by nodes 3, 4, 8 and 7 in Fig. 7. In this element, the fluxes at two integration points (symbolized with ⊗) were nullified so that the fully sealant geological fault could be represented. Thus, the resulting flux, which is coming from the control volume built around the node 8 to the control volume built around the node 3, must pass only through the interfaces of the control volume built around the node 4, in each one of integration points identified with the symbol ***X*** in this figure.

The first analysis tell us that a good interpolation function used to obtain the discretized expression of the flux at the integration points (***X***) could not involve any physical parameter stored in the node 7, since this node correspond to a control volume without direct link with the flux flowing at these integration points. However, in the numerical method chosen, the shape functions necessarily relate any interpolated value inside the element to its four nodal values (or three in case of triangles). Hence, in the element pointed out in Fig. 7, the pressure gradient of the discretized expression of the flux (Eq. 2) is approximated using four nodal values, including the node 7. Therefore, as the value of the pressure at this node gets more and more different from the other node values, like what occurred in the heterogeneous case analyzed, more physically inconsistent the results obtained.

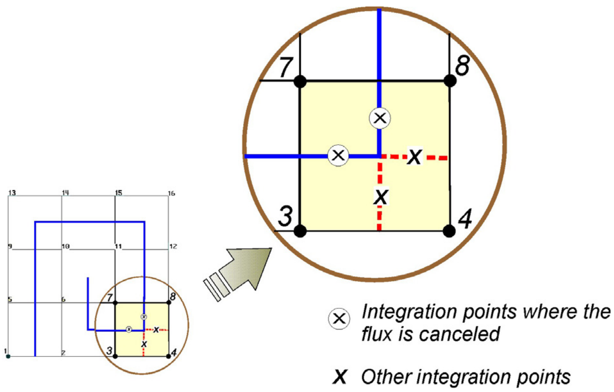


Figure 7 – Grid element showing the physical inconsistency in using the value of node 7 in the flux discretized equation at the integration points (***X***)

The second scheme proposed in this paper does not have this drawback because the elements are piecewise constant, thus without discontinuities inside elements. In addition, using this scheme there is no need for any permeability averaging, disappearing the uncertainties associated to the use of the harmonic mean in two and three dimensional situations. It is possible to show [6] that, even in non-faulted heterogeneous reservoir, the storing the physical properties in the control volumes associated with the use of the harmonic mean can result in undesirable oscillations in the pressure field.

CONCLUSIONS

The results presented here allow us to conclude that any scheme that creates properties discontinuities inside elements should be avoided when using unstructured grids. Therefore, the choice of storing the physical propertied in the center of the elements seems to be the proper one, contrary what is normally used in the petroleum reservoir simulators, which stores the propertied at the center of the control volumes.

The approach advanced in this paper also allows an easier representation of faulted reservoirs.

KEYWORDS

Reservoir simulation, physical properties, finite volume method, unstructured grids.

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